

ELEMENTARY TREATISE
ON
NATURAL PHILOSOPHY.

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TRANSLATED AND EDITED, WITH EXTENSIVE ADDITIONS,

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IN FOUR PARTS.

Part I.

MECHANICS, HYDROSTATICS, AND PNEUMATICS.

ILLUSTRATED BY

181 ENGRAVINGS ON WOOD, AND ONE COLOURED PLATE.



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AUTHOR'S PREFACE.

THE importance of the study of Physics is now generally acknowledged. Besides the interest of curiosity which attaches to the observation of nature, the experimental method furnishes one of the most salutary exercises for the mind—constituting in this respect a fitting supplement to the study of the mathematical sciences. The method of deduction employed in these latter, while eminently adapted to form the habit of strict reasoning, scarcely affords any exercise for the critical faculty which plays so important a part in the physical sciences. In Physics we are called upon, not to deduce rigorous consequences from an absolute principle, but to ascend from the particular consequences which alone are known to the general principle from which they flow. In this operation there is no absolutely certain method of procedure, and even relative certainty can only be attained by a discussion which calls into profitable exercise all the faculties of the mind.

Be this as it may, physical science has now taken an important place in education, and plays a prominent part in the examinations for the different university degrees. The present treatise is intended for the assistance of young men preparing for these degrees; but I trust that it may also be read with profit by those persons who, merely for purposes of self-instruction, wish to acquire accurate knowledge of natural phenomena. Having for nearly twenty years been charged with the duty of teaching from the chair of Physics in one of the lyceums of Paris, I have been under the necessity of making continual efforts to overcome the inherent difficulties of this branch of study. I have endeavoured to turn to account the experience thus acquired in the preparation of this volume, and I shall

be happy if I can thus contribute to advance the taste for a science which is at once useful and interesting.

For the convenience of candidates for the Bachelor's degree, I have appended to this treatise a number of problems, most of which have been taken from the examinations of the Faculty of Sciences of Paris or of the departments. With the same view I have made it my object to omit from the work none of the formulæ which are usually required for the solution of such questions. Beyond this point I have made very limited use of algebra. Though calculation is a precious and often indispensable auxiliary of physical science, the extent to which it can be advantageously employed varies greatly according to circumstances. There are in fact some phenomena which cannot be really understood without having recourse to measurement; but in a multitude of cases the explanation of phenomena can be rendered evident without resorting to numerical expression. In such cases calculation is of secondary importance, and may be said to be merely practical.

The physical sciences have of late years received very extensive developments. Facts have been multiplied indefinitely, and even theories have undergone great modifications. Hence arises considerable difficulty in selecting the most essential points and those which best represent the present state of science. I have done my best to cope with this difficulty, and I trust that the reader who attentively peruses my work, will be able to form a pretty accurate idea of the present position of physical science. I shall be happy in a second edition to avail myself of any observations which may be communicated to me on this or any other point.

TRANSLATOR'S PREFACE.

THE "TRAITÉ ÉLÉMENTAIRE DE PHYSIQUE" of Professor Deschanel, though only published in 1868, has already obtained a high reputation in France, and has been adopted by the Minister of Instruction as the text-book for Government Schools.

I did not consent to undertake the labour of translating and editing it till a careful examination had convinced me that it was better adapted to the requirements of my own class of Experimental Physics than any other work with which I was acquainted; and in executing the translation I have steadily kept this use in view, believing that I was thus adopting the surest means of meeting the wants of teachers generally.

The treatise of Professor Deschanel is remarkable for the vigour of its style, which specially commends it as a book for private reading. But its leading excellence, as compared with the best works at present in use, is the thoroughly rational character of the information which it presents. There is great danger in the present day lest science-teaching should degenerate into the accumulation of disconnected facts and unexplained formulæ, which burden the memory without cultivating the understanding. Professor Deschanel has been eminently successful in exhibiting facts in their mutual connection; and his applications of algebra are always judicious.

The peculiarly vigorous and idiomatic style of the original would be altogether unrepresentable in English; and I have not hesitated in numerous instances to sacrifice exactness of translation to effective rendering, my object being to make the book as useful as possible to English readers. For the same reason I have not scrupled to suppress or modify any statement, whether historical or philosophical, which I deemed erroneous or defective. In some instances I have endeavoured

voured to simplify the reasonings by which propositions are established or formulæ deduced.

As regards weights and measures, rough statements of quantity have generally been expressed in British units; but in many cases the numerical values given in the original, and belonging to the metrical system, have been retained, with or without their English equivalents; as it is desirable that all students of science should familiarize themselves with a system of weights and measures which affords peculiar facilities for scientific calculation, and is extensively employed by scientific men of all countries. For convenience of reference, a complete table of metrical and British equivalents has been annexed.

The additions, which have been very extensive, relate either to subjects generally considered essential in this country to a treatise on Natural Philosophy, or to topics which have in recent years occupied an important place in physical discussions, though as yet but little known to the general public.

The sections distinguished by a letter appended to a number are all new; as also are all foot-notes, except those which are signed with the Author's initial "D."

In many instances the new matter is so interwoven with the old that it could not conveniently be indicated; and I have aimed at giving unity to the book rather than at preserving careful distinctions of authorship.

Comparison with the original will however be easy, as the numbering of the original sections has been almost invariably followed.

The chief additions in Part I. (Chap. i.-xviii.) have been under the heads of Dynamics, Capillarity, and the Barometer. The chapter on Hydrometers has also been recast.

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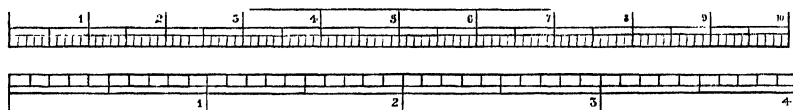
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INCHES AND TENTHS.

TABLE FOR THE CONVERSION OF FRENCH INTO ENGLISH MEASURES.

MEASURES OF LENGTH.

1 Millimetre	=	·03937079 inch, or about $\frac{1}{25}$ inch.
1 Centimetre	=	·3937079 inch.
1 Decimetre	=	3·937079 inches.
1 Metre	=	39·37079 inches, or 3·2809 feet nearly.
1 Kilometre	=	39370·79 inches, or 1093·6 yards nearly.

MEASURES OF AREA.

1 sq. millimetre	=	·00155006 sq. inch.
1 sq. centimetre	=	·155006 sq. inch.
1 sq. decimetre	=	15·5006 sq. inches.
1 sq. metre	=	1550·06 sq. inches, or 10·7643 sq. feet.

MEASURES OF VOLUME.

1 cubic centimetre	=	·0610271 cubic inch.
1 cubic decimetre	=	61·0271 cubic inches.
1 cubic metre	=	61027·1 cubic inches, or 35·3166 cubic feet.

The Litre (used for liquids) is the same as the cubic decimetre, and is equal to 1·76172 imperial pint, or ·220215 gallon.

MEASURES OF WEIGHT (or MASS).

1 milligramme	=	·015432349 grain.
1 centigramme	=	·15432349 grain.
1 decigramme	=	1·5432349 grain.
1 gramme	=	15·432349 grains.
1 kilogramme	=	15432·349 grains, or 2·20462125 lbs. avoird.

MEASURES INVOLVING REFERENCE TO TWO UNITS.

1 gramme per sq. centimetre	=	2·048098 lbs per sq. foot.
1 kilogramme per sq. metre	=	·2048098 " "
1 kilogramme per sq. millimetre	=	204809·8 " "
1 kilogrammetre	=	7·23314 foot-pounds.

1 force de cheval = 75 kilogrammetres per second, or 542½ foot-pounds per second nearly, whereas 1 horse-power (English) = 550 foot-pounds per second.

TABLE FOR THE CONVERSION OF ENGLISH INTO FRENCH
MEASURES.

MEASURES OF LENGTH.

1 inch	=	25·39954 millimetres.
1 foot	=	·30479449 metre.
1 yard	=	·91438347 metre.
1 mile	=	1·60932 kilometre.

MEASURES OF AREA.

1 sq. inch	=	645·137 sq. millimetres.
1 sq. foot	=	·0928997 sq. metre.
1 sq. yard	=	·8360973 sq. metre.
1 sq. mile	=	2·589895 sq. kilometres.

SOLID MEASURES.

1 cubic inch	=	16386·6 cubic millimetres.
1 cubic foot	=	·0283153 cubic metre.
1 cubic yard	=	·7645131 cubic metre.

MEASURES OF CAPACITY.

1 pint	=	·5676275 litre.
1 gallon	=	4·54102 litres.
1 bushel	=	36·32816 litres.

MEASURES OF WEIGHT.

1 grain	=	·064799 gramme.
1 oz. avoird.	=	28·3496 grammes.
1 lb. avoird.	=	·453593 kilogramme.
1 ton	=	1·01605 tonne = 1016·05 kilos.

MEASURES INVOLVING REFERENCE TO
TWO UNITS.

1 lb. per sq. foot	=	4·88261 kilos. per sq. metre.
1 lb. per sq. inch	=	·0703095 kilos. per sq. centimetre.
1 foot-pound	=	·138253 kilogrammetre.

ELEMENTARY TREATISE

ON

NATURAL PHILOSOPHY.

CHAPTER I.

PRELIMINARY NOTIONS.

• 1. **Origin of Natural Philosophy.**—The object of Natural Philosophy or Physics (*φύσις*, nature) is the study of the material world, including the phenomena which it presents to us, the laws which govern them, and the applications which can be made of them to our various wants.

In its widest sense, the study of physics must be traced back to the origin of the human race; for ever since man came into being, he must necessarily have been struck by the spectacle of the heavens and the continually changing aspect of terrestrial phenomena. But isolated and vague observations, and the barren admiration of phenomena which provoke attention or excite curiosity, do not constitute science; this can only exist where there is a mass of accurate knowledge in which the facts are related to each other and studied in connection with the causes which produce them. This process of co-ordination is only possible after a considerable collection of facts has been accumulated; but it then becomes inevitable, from the very constitution of the human mind. Thus, in examining the history of the nations among whom we place the cradle of our civilization, we find constant efforts of philosophers to explain the mechanism of the external world,—to bring all the facts which nature presents to us under one theory—one system. The Greek philosophers, especially, who appear to have borrowed the greater part of their physical knowledge from the Egyptian priests, have left us different systems, by the aid of which they profess to explain all natural phenomena. Thus Thales, the most celebrated of the seven wise men of Greece

(640 B.C.), made water a universal principle, which nourishes at once the sun, the earth, and the planets. Plato (398 B.C.) assumed two distinct principles, matter and form, which by their combination give birth to five elements—earth, water, fire, air, and ether. According to Anaximander, there is but one principle, the infinite, which gives birth to all bodies. According to Anaxagoras, air is the sovereign of nature. We need not stay to examine the exact meaning of these propositions, which, taken in their literal sense, appear at the present day sufficiently unintelligible. While acknowledging that these illustrious philosophers knew and taught some important facts of general physics, we are bound to remark that in the elaboration of their systems experiment played no part; that observation itself only held a secondary place; and that their theories were veritable *à priori* conceptions, to which facts had to be accommodated. Hence there is nothing in their works approaching the experimental method which serves as the foundation of modern physics. Some faint foreshadowings of this method may be traced in the works of Aristotle (383 B.C.), who was a disciple of Plato, but far superior to his master in scientific genius, besides being an eminent naturalist, and author of a history of animals, which alone would constitute an imperishable monument to his memory. Thus, to investigate the weight of air, he had recourse to a direct experiment, which consisted in weighing a skin empty and inflated. Finding no difference in the weights, he concluded wrongly that air is destitute of weight, and was thus led in his attempts at the explanation of certain phenomena to the famous principle that nature abhors a vacuum, which was universally admitted down to the time of Galileo.

It was especially in the hands of Archimedes (287 B.C.), and the philosophers of the school of Alexandria, who may be regarded as his successors, that the method of scientific observation took a distinct form, and led to important results. Every one has heard of the admirable discoveries of Archimedes respecting the theory of the lever, the determination of centres of gravity, and the measurement of specific gravities by means of the principle which bears his name; discoveries founded upon experiments which were doubtless not very accurate, but were regarded by him as necessary in order to furnish a solid basis for his investigations. After him Hipparchus (140 B.C.), by means of persevering observations, methodically directed, changed the face of astronomy, and arrived at brilliant discoveries, among which the most notable was that of the

precession of the equinoxes. At a later period, Ctesibius, Hero, Posidonius, &c., following in the traces of their illustrious predecessors, advanced the boundaries of the exact knowledge already acquired, and originated several inventions displaying more or less ingenuity. To the first of these philosophers the invention of pumps appears to be due, and the fountain of Hero still finds a place in all collections of physical apparatus.

Among philosophers belonging more or less directly to the school of Alexandria, who have enriched science by important discoveries, we will only mention Plutarch, who is said to have discovered the refraction of light in its passage from air into water; and Ptolemy, the author of various works on optics and celestial physics, which constitute a better title to glory than the astronomical system which bears his name, a system which only served to retard the progress of science.

We will carry this historical review no further, but will content ourselves with remarking that, starting from the seventh century, the period of the conquest of Alexandria by the Arabs, and the burning of its celebrated library, until the time of Galileo (1564), science may be said to have been stationary. Still, some discoveries of importance belong to this period; for example, that of the mariner's compass, which was known from the thirteenth century. Shortly before Galileo, the thermometer, the microscope, and telescope were invented; but it is unquestionably to this distinguished philosopher that we owe the true scientific method—the method of experiment. His treatises upon falling bodies, the pendulum, &c., furnish admirable examples of the manner in which the physical investigator should interrogate Nature by the aid of experiment. It was by the introduction of this method that physical science became finally disentangled from the prejudices and *à priori* assumptions which had hitherto impeded its progress.

At the present day, after numberless discoveries which have introduced most material changes in our social condition, physical science has attained a very high degree of perfection. It is to the experimental method that we owe this result, and it is by remaining true to this method that we must hope to achieve fresh progress.

2. The Experimental Method.—The experimental method can easily be described in general terms: it consists in observing facts instead of trying to divine them; in carefully examining what really happens, and not in reasoning as to what ought to happen.

It is therefore entirely independent of metaphysics, which has always proved a false ally; in fact, as long as the dominion of metaphysics lasted, science continued to run in the old ruts, which it did not leave till, thanks to the teaching of Bacon and Galileo, the conviction became established that there is no way of arriving at physical truths but by the help of observation and experiment.

The experimental method is usually called by logicians the method of *observation* and *induction*. From the observation of particular facts it ascends to the general law which embraces them; being very different in this respect from the method of *deduction* employed in mathematics, in which we always descend from a certain and absolute principle to the different consequences which flow from it. Let us enter into some details upon this point.

3. Phenomena—Physical Law.—A phenomenon is any change that takes place in the condition of a body; the fall of a stone, the flowing of water, the melting of lead, the combustion of wood, for example, are phenomena. When we study the characteristics which belong to phenomena of the same class, we soon perceive that the various circumstances of their production have a mutual dependence, so that if one of them varies, the others undergo a corresponding variation. The expression of this connection constitutes a physical law.

Sometimes the law appears of itself and without difficulty, by means of observation alone. Such, for example, is the following: *All bodies left to themselves fall to the surface of the earth.* But more frequently the law is disguised by disturbing causes, whose influence should, as far as possible, be eliminated. This elimination is the object of experiment. Experiment differs from observation in this respect,—that the phenomenon is produced under conditions previously determined and regulated by the experimenter. If we wish to know, for example, what are the velocities which gravity produces in different bodies falling freely, we must not let them fall in air, because this fluid retards their movement, and that in unequal degrees for different bodies; we must operate *in vacuo*, and thus we arrive at the law, which observation alone could never have discovered, that *gravity produces the same velocity in all bodies.* It will be readily understood then that the art of experimenting, that is, of regulating the special conditions under which phenomena shall take place, and of measuring their constituent elements, is absolutely necessary to the physical investigator; and that a genius for physical

science mainly consists in the possession of this aptitude in a more or less eminent degree.

We may remark that when the general law of a class of phenomena is known, the expression of this law is often called the physical cause of the particular phenomena which it includes. A phenomenon is said to be explained or accounted for, or traced to its cause, when we show that it is contained in the enunciation of a known law.

Thus, when we have once laid down the principle that the volumes of gases under different pressures vary inversely as these pressures, we are in a position to explain a crowd of facts depending on the action of a gas whose volume and pressure vary simultaneously.

When the law of observed phenomena admits of numerical statement, calculation becomes a valuable instrument for making known all its consequences, and the experimental verification of these consequences constitutes a confirmation of the physical law itself. In this way mathematical methods become powerful auxiliaries to physical science.

4. Physical Theory.—The enunciation of any one physical law, and the rational development of its consequences, constitute a partial physical theory. The assemblage of all the laws which belong to one class of phenomena, forms a more general physical theory; but it will be readily understood that these different laws may be merely corollaries of a single law.

The discovery of this single law, when it exists, marks a decided step in the progress of physical science. Thus Newton traced to the single law of gravitation all the movements of our planetary system, as well as those of bodies which fall to the surface of the earth.

In like manner the different partial theories of optics are rigorous consequences of the properties attributed to a fluid called *ether*, with which we suppose space to be filled, and whose vibrations serve for the propagation of light and heat.

This work of synthesis, however, has as yet made little progress, though these last few years have been marked by very successful efforts in this direction; but it should be considered as the true object of physical science in general, and the highest generalization will have been attained when it has been demonstrated that all the physical agents which have hitherto been regarded as distinct, are merely transformations of one and the same primordial agent.

5. Divisions of Physical Science.—Physical or natural science in

general comprises the aggregate of all the phenomena of the external world; but the accumulation of discoveries in different parts of this mighty whole has necessitated the division of it into several branches, which at present constitute distinct sciences.

Natural History comprises all those facts which have reference to the different beings, organic or inorganic, which are found upon the surface of the globe; it is further subdivided into several parts. *Zoology* is occupied with the organization and habits of animals, with their regular classification, and with all the phenomena connected with their development and reproduction. *Botany* treats of the same questions with respect to vegetables. *Mineralogy* has for its object the description and methodical classification of the different inorganic bodies (minerals) which nature presents to us; the knowledge of the peculiar characteristics which serve to distinguish them from one another; and the enumeration of their principal properties as well as of the various applications that can be made of them.

Geology is the history of the earth; it recounts the different revolutions which have modified the surface of the globe and finally brought about its present configuration, the arrangement and nature of the rocks that enter into its composition, and the description of those ancient animals and vegetables whose fossil remains are still in existence, belonging in many cases to types which have since become extinct. It is the basis of the art of the mining engineer, and enables him to follow a regular method in searching for the various metals or combustible substances which are hid in the depths of the earth, and which we employ to satisfy our various requirements.

Astronomy is occupied with the laws of the movements of the heavenly bodies; thanks to the perfection to which our measuring instruments have been brought, to the progress of mathematical science, and to the discovery of the universal law of gravitation, astronomy has arrived at such a degree of perfection that it may be classed among the exact sciences.

Besides natural history and astronomy, there is room further for distinguishing physics from chemistry. This latter science, in fact, has for its object the study of phenomena in which the essential character of materials seems to be changed; phenomena in which matter seems to be destroyed, or at least metamorphosed. If we take a piece of sulphur and heat it, it will melt; if we rub it with a piece of wool, it will acquire the power of attracting light bodies, and will present the peculiar and curious properties which are charac-

teristic of electrical excitation; but the sulphur will not have lost its proper nature, and when the different influences to which it has been submitted cease to act, it will resume all its original characteristics. The sulphur under these circumstances has displayed *physical* phenomena. If, on the other hand, we place this same body in a fire, we shall see it burn with a blue flame; at the end of some time it will have entirely disappeared, or at least will have been transformed into a gaseous substance which is dissipated with the other products of combustion. In this case the sulphur has ceased to exist as sulphur; a *chemical* phenomenon has taken place.

In a more restricted sense, then, physics or natural philosophy is understood as embracing the study of all the phenomena of the material world except those which consist in the action of vital forces or of chemical affinities. It is in this restricted sense that physics forms the subject of the present treatise. We may remark, however, that the two kinds of phenomena are often produced by the same causes, and that each is frequently the necessary consequence of the other. Thus in heating a body we render it better adapted to undergo chemical transformations; and, on the other hand, such transformations often produce a great quantity of heat. Physics and chemistry, though pursuing different ends, should yet afford each other mutual assistance. For example, our ideas of electricity would be very imperfect without a knowledge of the curious and often useful chemical phenomena which it is capable of producing.

A similar remark may also be made with regard to all the other branches of natural science. How, for example, can we separate mineralogy and chemistry, when it so often happens that the only means of recognizing a mineral is by making a chemical analysis of it? and when, on the other hand, a complete description of the substances which the chemist produces in his laboratory, must necessarily include an account of their external characteristics, such as their crystalline form, which specially belongs to the province of mineralogy?

To take another instance. Can we draw a sharp line of demarcation between zoology and botany on the one side, and physical science and chemistry on the other? Does not the tissue of organic beings undergo various chemical reactions which are a necessary accompaniment of vital phenomena? Do not physical agents in their turn produce phenomena of such a nature as completely to embarrass the physiologist and the physician, unless they are armed

with a knowledge of the laws which regulate the action of these agents upon inorganic bodies?

Finally, though astronomy may seem to form a totally distinct science, consisting of the geometry and mechanics of the movements of the heavenly bodies, must it not avail itself of all the resources of physical science if it would arrive at any rational conjectures respecting their constitution? We may say, then, that all the parts of natural science are interwoven together; they form one connected whole, and the division into distinct sciences has simply arisen from the vastness of the subject, which renders it impossible for any one mind adequately to follow the development of its various branches.

CHAPTER II.

MECHANICS.

6. Principle of Inertia.—The fundamental principle of physics is the inertia of matter. Inertia does not consist in the inactivity of material particles, nor in the impossibility of changes being produced in their states of rest or motion by their mutual action; for a glance at nature is sufficient to show that repose nowhere exists, and that motion changes in an endless variety of ways. The principle of inertia is an abstract principle which must be considered as applicable to a single isolated particle. It may be enounced in the following terms:—

An isolated material point cannot change its state, whether of rest or motion. That is to say, if it be at rest it will remain at rest; if it be in motion it will continue to move in the same direction and with the same velocity.

If, then, we see a material point which was at rest begin to move, or if we observe any change in the motion of a point, we say that it has been acted on by a *force*.

Without entering upon the very obscure subject of the intimate nature of forces—without seeking to know whether they form an essential part of bodies or have a separate existence, but only regarding them in the effects which they produce, we may define them in the following manner:—

*A force is any cause which tends to urge a material point in a definite direction with a definite velocity.*¹

7. Manifestations of Inertia.—The principle of inertia, as above enounced, does not admit of direct experimental verification; for we cannot observe a material point, which is a mere abstraction;

¹ The words *with a definite velocity* only imperfectly express the idea intended to be conveyed. The correct phrase would be *with a definite acceleration*. See Chap. v.

still less an isolated material point. The principle of inertia is one of those ultimate and abstract principles which presented themselves to the minds of the founders of the science of mechanics—of Newton especially—as the key and reason of the manifold and complex characters of external phenomena. But if it is impossible to verify the principle of inertia directly, it is easy to show its influence in external phenomena, this influence reducing itself evidently to the tendency of bodies to continue in their state of rest or motion.

The tendency to continue in a state of rest is manifest to the most superficial observation. The tendency to continue in a state of uniform motion can be clearly understood from an attentive study of facts. If, for example, we make a pendulum oscillate, the amplitude of the oscillations decreases more and more; and ends, after a longer or shorter time, by becoming nothing. This is because the pendulum experiences resistance from the air, due to the successive displacement of the particles of this fluid; and because the axis of suspension rubs on its supports. These two circumstances combine to produce a diminution in the velocity of the apparatus until it is completely annihilated. If the friction at the point of suspension is diminished by suitable means, and the apparatus is made to oscillate *in vacuo*, the duration of the motion will be immensely increased.

Analogy evidently indicates that if it were possible to suppress entirely these two causes of the destruction of the pendulum's velocity, its motion would continue for an indefinite time unchanged.

This tendency to continue in motion is the cause of the effects which are produced when a carriage or railway train is suddenly stopped. The passengers are thrown in the direction of the motion, in virtue of the velocity which they possessed at the moment when the stoppage occurred. If it were possible to find a brake sufficiently powerful to stop a train suddenly at full speed, the effects of such a stoppage would be identical with those which would result from collision with another train of the same weight coming in a contrary direction with equal velocity.

Inertia is also the cause of the severe falls which are often received in alighting incautiously from a carriage in motion; all the particles of the body have, in fact, a forward motion, and the feet alone being reduced to rest, the upper portion of the body continues to move, and is thus thrown forward.

When we fix the head of a hammer on the handle by striking the end of the handle on the ground, we utilize the inertia of matter.

In fact, at the moment of the shock, and of the stoppage which results, the head continues to move, and ends after some blows by becoming firmly fixed.

8. Mechanics.—All physical phenomena fundamentally consist in motions; but these motions are in many cases too minute to admit of direct observation, and are only inferred from their effects. Thus when a solid body is heated and melted, it is certain that the liquid state results from a particular displacement of the molecules, and perhaps also from a change of their form—that is to say, from circumstances which are reducible to motions; but the liquid body thus formed has acquired peculiar properties, which form a subject of study in themselves apart from the motions to which they are due.

When motions are considered in themselves, according to their geometrical relations, and in connection with the forces which produce them, they form the subject of the science of mechanics, which must be regarded as an indispensable introduction to physics. We shall give in this chapter enunciations and illustrations of some fundamental propositions, referring the reader to special treatises on this subject for fuller information.

9. Elements required to specify a Force.—The material point submitted to the action of force is called the *point of application of the force*. It tends, in virtue of this action, to move in a certain direction, which is called the *direction of the force*, and which can be represented geometrically by a straight line drawn from the material point. It is obvious also that the force must act with some definite intensity, which is different in different cases. This intensity may manifest itself, for example, by a greater or less velocity of the point, a greater velocity corresponding to a greater force.

When two forces separately applied to the same point at rest give it the same motion, they may be called equal. The union of a number of equal forces gives a force which is a corresponding multiple of one of them, and thus the intensities of forces can be numerically compared. Forces then can be represented either by numbers or by lines; in the latter case a certain length (as an inch) being taken to represent a certain force

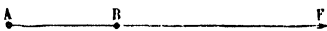


Fig. 1.

(as the weight of a pound). It is usual to indicate the direction of a force by a line AF with which the direction of the force coincides, and to lay off on this a length AB representing (on the scale chosen) the intensity of the force.

For accuracy, it is to be observed that the pound, ounce, and other units of weight are essentially units of mass, not of force. In order to render them available as accurate units of force, the *locality* must be specified, inasmuch as the force requisite to support a pound of matter is different in different localities, being for example greater at the poles of the earth than at the equator by about 1 part in 190.

10. Resultant.—When a material point or a system of points is urged by a certain number of forces, it will be readily understood that a single force of determinate magnitude, and applied at a suitable point, may be capable of producing the same effect as all the given forces acting together. This single force is called the *resultant* of the given forces, and they are called its *components*.

Thus, for example, a vessel descending a river, whether propelled by steam or wind, provided its motion be rectilinear, is really urged forward by a great number of forces applied at different points; but it is evident that a single force of proper magnitude and line of action would produce the same effect.

It is not every system of forces that has a resultant; but, in the case of those which have, it is very important to determine its magnitude and position, for the study of the body's motion will thus be evidently simplified. The following is an important case in which this determination is easily made.

11. Parallelogram of Forces.—*If a material point A is acted on by two forces represented in magnitude and direction by AB and AC, there is a resultant, which is exactly represented by the diagonal AD of the parallelogram of which AB and AC are sides.*

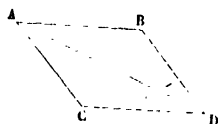


Fig. 2—Parallelogram of Forces.

This proposition can be verified experimentally by the aid of the following apparatus due to Gravesande. ABDC (Fig. 3) is a parallelogram jointed at its four corners. To the points

B and C cords are fixed, which, passing over the pulleys M and N, support at their extremities weights P and P', of 90 and 60 ounces respectively.

The lengths of the sides AB and AC are themselves proportional to the numbers 90 and 60. To the corner A is attached a weight P'' of 120 ounces. In these circumstances, the parallelogram will take a position of equilibrium, in which the cords attached to B and C will be found to form prolongations of the sides AB, AC,

and the diagonal AD will be vertical. But the forces P and P' have a resultant acting vertically at A, since their resultant must be equal and opposite to the weight P'' which balances them. The diagonal

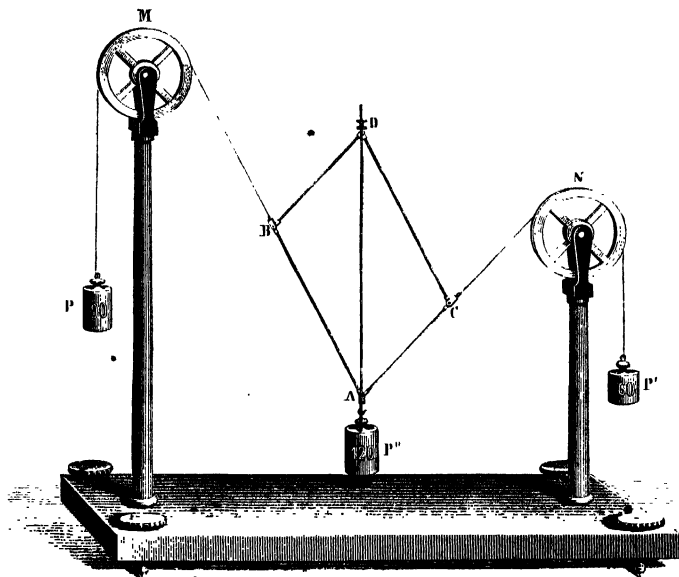


Fig. 3.—Gravesande's Apparatus.

AD therefore agrees with the resultant in direction; and if this diagonal is measured, its length will be found to be 120 on the same scale on which the lengths of AB and AC are 90 and 60.

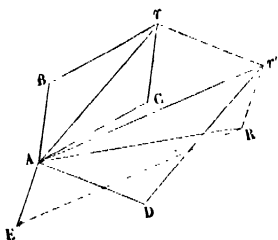


Fig. 4.—Composition of any Number of Forces.

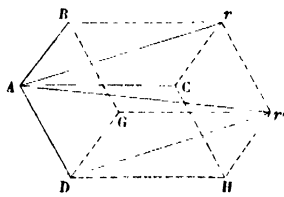


Fig. 5.—Parallelopiped of Forces.

12. Composition of Forces.—Knowing how to find the resultant of two forces, that is to say, to *compound* two forces, applied to the same point, it is easy to compound any number.

Let there be, for example (Fig. 4), four forces applied to the

be at the middle of AB ; if the force F is double of F' , the segment CA will be equal to half of CB .

This proposition can be verified by the aid of the following apparatus called the *arithmetical lever* (Fig. 7).

The lever AB supports two equal weights P at its extremities; it is suspended at its middle by a cord which, passing over the pulley M , sustains a weight P' . It will be found that, when the weight P' has a certain value, the lever is in equilibrium; whence it follows that the two weights P and the weights of the different parts of the lever, which we may suppose distributed two and two at equal distances from the middle point, have a resultant acting at this point, equal and opposite to the force P' . It will also be found that P' is equal to the sum of the two weights P together with the weight of the lever.

In the case of the second figure, a single weight P is placed at one of the extremities B , whilst two equal weights are suspended at the middle point C of the second half of the lever. It will be found that there is still equilibrium, provided that the weight P' is the sum of the three weights P and the weight of the lever.

To interpret this result, we may remark that the lever being balanced directly by an equal portion of P' , we may neglect its weight; there remain then only two forces, of which one, that on the left, is double of the other. Now the resultant evidently passes through the point of suspension O , which is exactly twice as far from B as from C .

14. When the parallel forces F and F' act in opposite directions, there is still a resultant parallel to the components, acting in the same direction as the greater, and equal to their difference $F - F'$. Moreover its point of application C is so placed that the distances CA and CB are inversely proportional to the forces, a result analogous to that which holds when the forces act in the same direction.

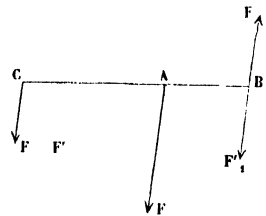


Fig. 8.—Parallel forces in Opposite Directions.

We see, as a consequence of this proposition, that if the two parallel and opposite forces differ little from one another, their resultant has a very small value, but its point of application is very remote. In the particular case in which the two forces are equal, the rule of composition is absolutely inapplicable. Such a system,

consisting of two equal and parallel forces acting in opposite directions, is called a *couple*. It cannot be equilibrated or replaced by a single force, but obviously tends to produce a motion of rotation. Now in nature we frequently see bodies possessing at once a motion of translation and a motion of rotation. We may assume that the translation has been produced by a force and the rotation by a couple; this latter then presents itself as a sort of *natural element* in mechanics. The idea of couples originated with the geometrician Poinso, and has greatly simplified many mechanical problems.

The perpendicular distance between the lines of action of the two equal forces which constitute a couple is called the *arm* of the couple. The product of one of the two equal forces by the arm is called the *moment* of the couple, and is the measure of the power of the couple to produce rotation. It is proved in treatises on mechanics that two couples acting on a body and tending to turn it in opposite directions will equilibrate each other if their moments are equal, even though they be applied at different parts of the body. Two or more couples acting on a body and tending to turn it in the same direction, may be replaced by a single couple whose moment is the sum of their moments; and any number of couples acting on a body and tending to turn it in any directions whatever, are always, except when they are in equilibrium, equivalent to a single couple.

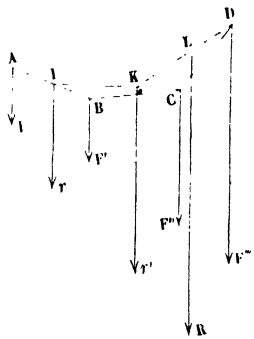


Fig 9. — Composition of any Number of Parallel Forces.

15. Composition of any Number of Parallel Forces. To compound a given number of parallel forces, F, F', F'', F''' , we may first compound the first with the second, which gives a partial resultant r ; this compounded with the third force F'' , gives a second resultant r' , which combined with F''' , gives the complete resultant R . It is clear that this procedure is applicable to any number of forces, and that the resultant is always equal to the sum of all the forces. As to its point of

application, we may remark, that the point of application of the first partial resultant is obtained by dividing AB into two parts AI and BI inversely proportional to F and F' . In like manner the point of application K of r' is obtained by dividing IC into parts IK and KC , inversely proportional to r and F'' ; and lastly, the point of application L of the complete resultant is obtained by performing

an analogous operation on the line KD. Now it will be remarked that this series of constructions is independent of the absolute direction of the forces, and only supposes that they are parallel; if then the forces were turned about their points of application in such a manner as still to remain parallel to one another, their resultant would still pass through the same point L. This point is on this account called the *centre of parallel forces*.

16. Resolution of Forces.—As any number of forces can be compounded into a single force, so a given force can be (decompounded or) resolved into two or any number of forces which would produce the same effect. Thus, for example, the force AD (Fig. 2) can be replaced by the two forces AB and AC, inasmuch as it is their resultant. It is obvious that, to resolve a force applied to a material point into two others having given directions, it is necessary to draw, through the extremity D of the line which represents the given force, lines parallel to the given directions. They determine by their intersections the points B and C, and consequently the magnitudes AB and AC of the two forces which can be substituted for the given force.

The composition of several forces into a single force constitutes an obvious simplification. As to resolution (or decomposition), its utility is not so obvious. It may appear at first sight to be a complication. Such however is not always the case, and in the course of this treatise we shall find it of continual use. It will be readily conceived, in fact, that when a force is applied to a point whose movements are constrained by guides or physical connections, the effect which the force can produce is not easily perceived. But it is sometimes possible, by resolution, to replace it by components, of which some are destroyed by the conditions of constraint, and the others can act in a manner directly appreciable.

We will confine ourselves to a single example of this, namely, the explanation of the opposite courses which can be taken by two sailing boats with the same wind.

Consider, for example, the case of the first figure. The wind, blowing in the direction Vm, tends to exert a force which can be resolved at the point *m* into two components, one of them *tt'* tangential to the sail, which has no effect, the other *mn* perpendicular to the sail. This latter tends to urge the boat obliquely towards the left of the figure. But as the boat can be moved much more easily in the direction of its length than in any other direction, we

may resolve this latter force into two components: one perpendicular to the length, which has little effect; the other in the direction of the length, which produces the forward movement of the boat in the direction of the arrow placed below the figure.

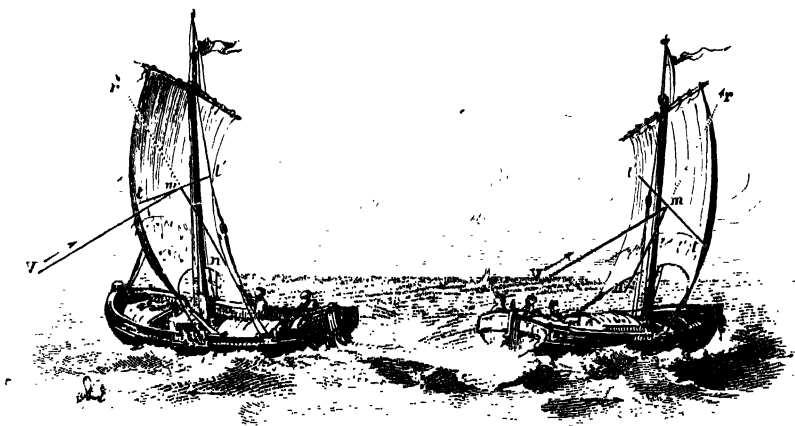


Fig. 10.—Resolution of Pressure of Wind on Sails.

In the second figure it will be seen that the same methods of resolution lead to an opposite result, and that the direction of motion is opposite, though the direction of the wind is the same. In reality, the two motions are not directly opposite, because the action of the component perpendicular to the length of the boat cannot be altogether neglected, but produces what is called *leeway*.

17. Work done by a Force.—In the different operations to which forces are applied, such as the raising of burdens, the compressing, piercing, and pulverizing of solid bodies, it is clear that it is always necessary to overcome a certain resistance and produce a certain displacement. Hence we are led to a special mechanical element, involving the joint consideration of force and the displacement of its point of application. This element is called *work*.

The work done by a force is the product of the force into the displacement which it produces in its point of application.

In this definition the force is supposed to be constant, and the motion of the point of application is supposed to coincide with the direction of the force.

In stating the work done by a machine it is usual to take as unit of work the foot-pound; that is to say, the work done in raising a pound through a height of a foot.

The notion of work is itself independent of time; but it is evident that in practice it is advantageous for a machine to employ little time in producing a given amount of work.

The unit employed for expressing the rate of working of a machine is the *horse-power*, and denotes 33,000 foot-pounds of work done per minute or 550 foot-pounds per second. Thus a machine which can raise 12 tons through a height of 10 feet in 2 minutes is a machine of rather more than 4 horse-power; since it does $12 \times 2240 \times 10 = 268,800$ foot-pounds in 2 minutes, or 134,400 in 1 minute; and 134,400 is rather more than 4 times 33,000.

17A. The above definition of *work* is only applicable to the case in which the displacement of the point of application of the force is in a direction precisely coincident with the direction of the force. It is often necessary to consider cases where (owing perhaps to circumstances of constraint, or to the action of other forces besides the one considered, or to previous motion) the point of application of a force moves in a direction oblique to that of the force. In this case the force may be resolved into two components, of which one is perpendicular and the other either coincident with or directly opposite to the direction of displacement. The former of these components is to be neglected in estimating the work done by the force; and the product of the latter component by the displacement is the work done *by* or *against* the force according as the direction of this component *coincides with* or is *opposite to* that of the displacement.¹

The necessity of having a name to denote the idea thus defined is obvious from the following proposition, which is called the *principle of work*.

Every machine may be regarded as an instrument for transmitting work; and if we neglect friction, we may assert that *the work thus transmitted is unaltered in amount*. If, for example, the machine is driven by forces applied at points $A_1, A_2, \&c.$, and if the machine overcomes resistances at points $B_1, B_2, \&c.$, then the whole work done by the forces at $A_1, A_2, \&c.$, estimated according to the fore-

¹ Or the work done by a force is the product of the force by the projection of the displacement of its point of application on the direction of the force, or is the continued product of force, displacement, and cosine of included angle. The three definitions are obviously equivalent. Work done *against* a force is to be regarded as *negative work done by the force*.

going definition, will be precisely equal to the whole work done against the resistances at B_1 , B_2 , similarly estimated.

The numerous vain schemes for producing perpetual motion are founded on ignorance of this law. They are attempts to make work increase in its transmission through a machine.

Practically, work is always diminished in its transmission through a machine, owing to friction. The work thus lost leaves an equivalent in the shape of heat (see Chap. xxxii.)

CHAPTER III.

CONSTITUTION OF BODIES.

18. Different States of Matter.—As the object of physics is the study of the general properties of bodies, it is necessary for us to form some idea of the constitution of the different kinds of matter. Matter presents itself in three different states: the solid, liquid, and gaseous. Solid bodies are characterized by a kind of invariability of form; that is to say, their form cannot be changed without an effort, more or less considerable. Hence a solid body forms a firmly connected whole, so that the movement of one of its parts produces motion in the rest.

Liquids, on the contrary, appear to be formed of particles which are independent of each other and can obey individually the action of the forces which urge them, being able to slide past each other with the greatest facility. From this property the name *fluids*, by which, in common with gases, they are often designated, is derived (*fluere*, to flow). This also is the reason why a liquid moulds itself to the form of the containing vessel. Liquidity, consisting essentially in the perfect mobility of the constituent parts of a body, may evidently be met with in different degrees of perfection. Thus sulphuric ether and alcohol are more perfectly liquid than water; water itself is more liquid than oil, and so on. *Viscosity* is a name used to denote the want of independence between the particles of a liquid, which establishes a kind of intermediate state between these bodies and solids. Thus we may say that there is an insensible passage from liquids more or less perfect to viscous liquids, from these to plastic substances such as putty or moist clay, and from these last to solid bodies.

Gaseous bodies, of which the atmosphere offers us an example, are formed, like liquids, of independent particles: but these particles

appear to be in a continual state of repulsion, so that a gaseous mass has a continual tendency to expand to a greater and greater volume. This property, called the expansibility of gases, is commonly illustrated by the following experiment:—

A bladder, nearly empty of air, and tied at the neck, is placed under the receiver of an air-pump. At first the air which it contains and the external air oppose each other by their mutual pressure,

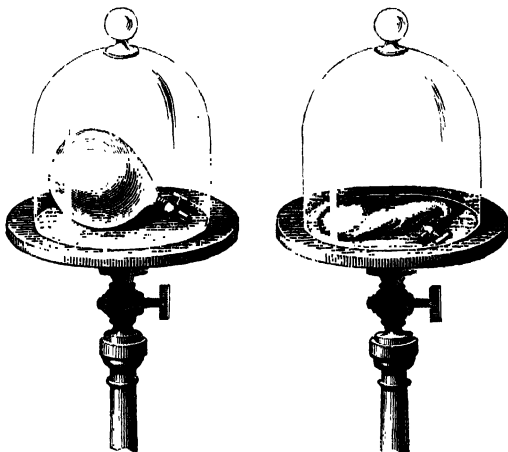


Fig. 11.—Expansibility of Gases.

and are in equilibrium. But if we proceed to exhaust the receiver, and thus diminish the external pressure, the bladder gradually becomes inflated, and thus manifests the tendency of the gas which it contains to occupy a greater volume.

It follows from this property that, however large a vessel may be, it can always be filled by *any quantity whatever* of a gas, which will always exert pressure against the sides. It is in consequence of the existence of this pressure, which is itself a result of expansibility, that the name of *elastic fluids* is often given to gases.

It is necessary to remark that the same substance may, according to its temperature, assume any one of the three states. Thus water in the cold of winter assumes the solid state and becomes ice; and, on the other hand, there is always more or less water diffused through the air in the gaseous state, called aqueous vapour. If the thermal conditions existing at the surface of the earth were to receive a notable change in either direction, some of the bodies which we habitually see in the liquid state would become either solids or vapours.

19. Molecular Constitution.—Whatever be the state under which a body presents itself, it is the general opinion of physicists that it is not composed of continuous matter, but is an aggregation of distinct parts held at a distance from each other. These constituent parts are called particles or molecules. They must be regarded as exercising two kinds of mutual actions, the one attractive, the other repulsive, which balance each other in the case of solids and liquids. In the case of gases this equilibrium does not subsist; there is a permanent repulsive force between the particles, which gives rise to expansibility or elastic force.

The molecules¹ of solids and of liquids ought not to be considered as similar. In the latter, in fact, each molecule can turn on its axis without producing any modification in the equilibrium; in other words, equilibrium depends only on the molecular distances and not at all on the form or relative disposition of the molecules. An approximate idea of this physical constitution will be obtained by assuming that the molecules of liquids are spherical, and hence that molecular equilibrium depends only on the distances between the centres of the spheres.

In solids, much depends upon the form and relative disposition of the molecules. It would seem as if these molecules (according to the ideas of some ancient philosophers) were formed with hooked projections which become locked together and so give a determinate figure to the mass. It is not, however, necessary to fall back upon such a gross image as this for the explanation of rigidity. It is sufficient to conceive that when an effort is exerted against any part of a solid body, its molecules turn on their axes, assume new directions, and take up a new position of equilibrium. Such a supposition corresponds with that invariability of form which we are accustomed to connect with the solid state. In reality this invariability is not absolute. The smallest force applied to a solid body produces some change of form, but frequently this change is only appreciable when the force is very intense.

20. Divisibility.—This hypothesis regarding the constitution of bodies amounts to assuming that matter is not infinitely divisible, but that, whatever be the means employed to produce division, there is for each body a limit below which it never descends. These

¹ The hypotheses broached in the remainder of this section must be received with caution, as being merely conjectural explanations of the distinction between solids and liquids.

parts which always remain undivided are called atoms (α , privative, $\tauέρω$, to cut); and we are to understand by this designation not elements which it is *impossible* to divide in an absolute or metaphysical sense, but elements which are not susceptible of division by any known forces.

It is in chemistry especially that reasons are found for assuming a limit to the division of matter. In fact in chemical phenomena we see this division attain limits which, though doubtless very remote, are yet fixed. The composition of compound bodies is invariable, whatever may be the circumstances of their production;¹ their properties, which could hardly fail to be altered by a change in the size of the constituent particles, are also the same; whence it seems necessary to conclude that the elements between which chemical affinity is exerted are absolutely alike and unchangeable—have a definite existence. They are the veritable *individuals* of the mineral kingdom. These are what we mean by atoms.

The notion of an atom does not involve the idea of size; but experience teaches us that their size must be excessively minute; for we can in several different ways divide matter into extremely small parts, without finding any reason to think that we have attained or even approached the limit of division. We will cite some examples which prove the extreme divisibility of matter.

Wollaston succeeded in obtaining threads of platinum of a diameter not exceeding $\frac{1}{100000}$ of an inch. The method which he employed for preparing them consisted in drawing a silver wire with a platinum core, and dissolving the shell of silver in nitric acid. In this way threads can be obtained so fine that they are actually invisible to direct view, and that their existence can only be detected by the aid of certain special optical phenomena. In the art of beating gold, leaves are obtained whose thickness cannot exceed $\frac{1}{200000}$ of an inch. A square inch of this leaf would weigh less than the $\frac{1}{200000}$ of an ounce, and as a square whose side is $\frac{1}{100}$ of an inch is visible to the naked eye, it follows that this square inch of leaf contains more than 60,000 visible parts.

The diffusion of colouring matters and perfumes affords a notable instance of the extreme divisibility of matter. A cubic millimetre of indigo (about $\frac{1}{100000}$ of a cubic inch) dissolved in sulphuric acid,

¹This is called the law of "definite proportions." The laws of "multiple proportions" and "equivalent proportions" furnish perhaps a still stronger argument for the atomic hypothesis.

can colour to an appreciable extent more than 10 litres (about 2 gallons) of water. Now, a litre contains a million cubic millimetres; the cubic millimetre of indigo, therefore, in this experiment is divided into ten million visible parts.

The diffusion of odoriferous substances is still more astonishing. It is well known that a grain of musk will continue for years to supply the air, which is continually being renewed around it, with a sufficient number of particles to communicate to it its odour. The mind can hardly form an idea of the degree of tenuity which such particles must have.

21. Porosity.—Porosity is an immediate consequence of the hypothesis of molecular constitution. It consists in the existence, in the interior of all bodies, of intervals or pores between their material particles. This porosity is often so marked as to permit the passage of liquids or gases through the substance of solids. It then receives the name of *permeability*. Permeability is the property which is utilized in the employment of stone filters; the pores are large enough to allow the water to pass, and small enough to prevent the passage of the small solid bodies held in suspension in the water.

It is by means of permeability that the communication and contact of liquids takes place in organized bodies; for the vessels which contain them are nowhere open, and it is always through the substance of their walls that the final changes of elements are made which are necessary to vital action.

By using great pressure, liquids can be made to pass through metals. These latter, or at least some of them, iron and platinum for example, when raised to a high temperature, allow ready passage to different gases. Thus it is that cast-iron stoves, when red-hot, allow some of the deleterious products of combustion to pass out, and sometimes occasion serious accidents.

But even when no permeability can be detected, pores must still be assumed to exist; and the proof is found in the fact that all bodies can have their volumes increased or diminished,—that they are dilatable and compressible. The dilatation of bodies by the action of heat is a general phenomenon which will be studied further on. We will here confine ourselves to compressibility.

22. Compressibility.—Compressibility consists in the reduction of volume which bodies experience under the action of external pressure. The compressibility of solids is extremely small; that is to say, a very considerable pressure is required to produce any sensible diminution

of volume. The existence of such an effect has, however, been well established, and it is found necessary to allow for it in structures.

The compressibility of liquids, though greater than that of solids, is still very small. Hence, in comparison with gases, they have often been called incompressible fluids. It is easy with the aid of

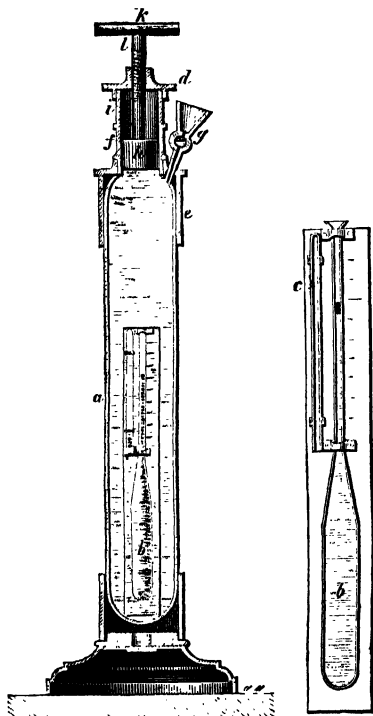



Fig. 12.—Ersted's Piezometer.

Ersted's apparatus, represented in section in Fig. 12, to show that liquids are more compressible than solids, and to measure approximately the degree of their compressibility. The liquid to be compressed is contained in a kind of large thermometer *b*, called a piezometer, whose tube has been carefully divided, and its reservoir gauged so as to determine how many divisions of the tube its volume is equal to. The tube is open at the top, and a globule of mercury, placed above the liquid column, serves for index. The apparatus is placed in a vessel of water *a*, having very thick sides.

When pressure is exerted by means of the screw-piston *klh*, the index of mercury is seen to descend, showing a diminution of the volume of the liquid. The amount of the pressure is known from the volume of air contained in the tube *c*, which serves as a manometer. In this experiment the number of divisions through which the extremity of the liquid column moves indicates the *apparent* diminution of volume; that is to say, the excess of the diminution of volume of the liquid above that of the envelope. It is easy, in fact, to understand that the piezometer itself must, under the pressure to which it is subjected, undergo a diminution of capacity, which must be taken into account. Ersted supposed that this diminution was insensible, or nothing, since the pressure is exerted on the interior as well as the exterior of the piezometer. But this conclusion is erroneous; for

if the piezometer were solid and submitted to compression, the interior shells would react with a force precisely equivalent to that which is produced when the instrument is hollow and the liquid occupies its interior. The piezometer then, in Ørsted's experiment, undergoes a diminution of volume equal to that which a solid piezometer would undergo in the same circumstances. In order, then, to find the true diminution of volume of the liquid, it is necessary to increase the apparent contraction by the contraction of the envelope. This latter element varies according to the quality of the glass of which the envelope is composed, but may be estimated at about $\cdot 000029$ per atmosphere of pressure. The true compressibility of water, according to recent experiments conducted under the direction of M. Jamin by Messrs. Amaury and Descamps, is, at the temperature of 15° centigrade, $\cdot 0000457$ per atmosphere. It diminishes when the temperature increases. Alcohol and ether are rather more compressible, and their compressibility (unlike that of water) increases with the temperature. Mercury is much less compressible than water. Its variations of volume may therefore, in ordinary experiments, be neglected.

 Gases are enormously more compressible than solids and liquids. This is easily shown by the pneumatic syringe, Fig. 13. It is a cylinder of very thick glass, closed at one end. A piston, which exactly fits the tube, is made to enter the other end, and can be forced in until the air is reduced to a half, a third, or a tenth of its original volume.

Hence it would seem to follow that in gases the spaces between the particles are much greater than in liquids and solids, and consequently that there is much less matter in the same volume. The same conclusion is established by the comparison of specific gravities. To give an idea of the difference, it may suffice to mention that water, when converted into steam, at the ordinary atmospheric pressure, and at the ordinary temperature of boiling water, expands 1700 times, so that a cubic inch of water gives about a cubic foot of steam.

23. Elasticity.—This term, when applied to solids, is used in modern physics to denote the property in virtue of which a body tends to recover its form and dimensions when these are forcibly changed. The great majority of solid bodies possess almost perfect



Fig. 13.—
Pneumatic
Syringe.

elasticity for small deformations; that is to say, when distorted, extended, or compressed within certain small limits, they will, on the removal of the constraint to which they have been subjected, return instantly to their original form and dimensions. These limits (which are called the limits of elasticity) are different for different substances; and when a body is distorted to an extent exceeding these limits, it takes a set, the form to which it returns being intermediate between its original form and that into which it was distorted.

When a body is distorted (using this word to include extension or compression as well as change of shape) within the limits of its elasticity, the force with which it reacts is simply proportional to the amount of distortion. For example, the force required to make the prongs of a tuning-fork approach each other by a tenth of an inch, is precisely double of that required to produce an approach of a twentieth of an inch; and if a chain is lengthened a twentieth of an inch by a weight of 1 cwt., it will (if we may neglect the weight of the chain itself) be lengthened $\frac{1}{10}$ th of an inch by a weight of 2 cwt., the chain being supposed to be sufficiently elastic to experience no permanent set from this greater weight. Also (within the limits of elasticity) equal and opposite distortions are resisted by equal reactions; for example, the same force that suffices to make the prongs of a tuning-fork approach by $\frac{1}{20}$ th of an inch, will suffice, if applied in the opposite direction, to make them separate by the same amount.

An important consequence which can be mathematically deduced from the laws just stated, is that when a body is distorted within its limits of elasticity, the vibrations which ensue when the constraint is removed have periods which are independent of the magnitude of the distortion. For example, a common C tuning-fork makes about 528 vibrations in a second whether vibrating strongly or feebly; by whatever amount the prongs are made to approach each other, the time which elapses from their being released to the attainment of their greatest separation is $\frac{1}{528}$ of a second, and the same time elapses from their greatest separation to their nearest approach. The sum of these two intervals, $\frac{1}{264}$ of a second, is the period of a complete vibration; and during the whole time that the vibrations are dying away until the fork finally comes to rest, this period remains unaltered, the diminution of distance moved being exactly compensated by the increasing slowness of the motion.

India-rubber has very wide limits of elasticity. Glass has sensibly perfect elasticity up to the limit at which it breaks.

Putty and wet clay are instances of bodies which are almost entirely destitute of elasticity.

The resistance of a cylindrical or prismatic bar to elongation or flexure is measured by a number called "Young's modulus of elasticity." The following are examples of its value for different substances, in kilogrammes per square millimetre:—

Flint-glass,.....	5,851	Iron (wrought),.....	19,994
Brass,.....	10,948	Do. (cast),.....	13,741
Steel,.....	21,793	Copper,.....	12,558

To illustrate the meaning of this table by the case of steel; a steel wire whose section is a square millimetre will be elongated by $\frac{1}{21793}$ of its length by a weight of one kilogramme. The elongation is inversely proportional to the section and directly proportional to the stretching weight; so that a steel wire whose section is half a millimetre will, when stretched by a weight of six kilogrammes, receive twelve times the elongation above specified.

The resistance of a cylindrical or prismatic bar or beam to bending (called its flexural rigidity) is proportional to the value of Young's modulus of elasticity for the material of the bar or beam; so that from the dimensions of the bar, the value of this modulus, and the magnitudes and directions of the externally applied forces, the amount of bending could be calculated.

The resistance of a cylindrical rod to twisting (called torsional rigidity) does not depend upon the value of Young's modulus, but upon an entirely distinct element, an element which is sometimes called simply "rigidity," and which expresses the resistance which a square of given thickness would oppose to being changed by external forces into a rhombus of the same area, having angles differing by a given small amount from right angles.

Elasticity being a molecular phenomenon, it is to be expected that all circumstances which modify the molecular constitution of a body will alter its elasticity; but in the present state of science it is impossible to predict *à priori* the nature and direction of the change, the effects being sometimes opposite for different substances. Thus tempering (that is to say heating followed by sudden cooling), which, as is well known, augments in a high degree the hardness and elasticity of steel, produces a reverse effect on the bronze of which gongs are made. This alloy, in fact, when cooled slowly, possesses the fragility of glass; whilst, when cooled suddenly, it can be wrought with the hammer.

The elasticity of springs furnishes a simple means of comparing forces. Fig. 14 represents an apparatus designed for this purpose and called a *dynamometer*. It is formed of two plates of steel, AB and A'B', jointed at their extremities to two metallic bridles which connect them. To the middle of the upper plate is attached a ring, by means of which the apparatus can be suspended from a fixed

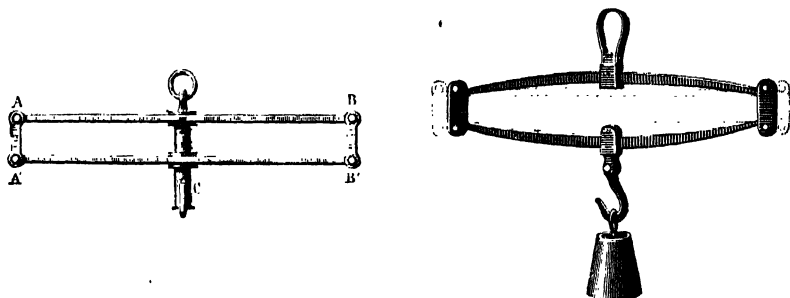


Fig. 14.—Dynamometer.

point. To the middle of the lower plate is attached a hook, which can either receive a weight or serve as a point of application for the force which is to be tested. Under the action of the force thus applied the spring plates bend, the distance of the middle points increases, and this increase serves to measure the force itself; or the force may be measured by observing what weight must be suspended from the hook to produce the same effect.

The spring-balance is another apparatus of the same kind. In its most common form it contains a spiral spring which is elongated by the application of the force which is to be measured. The equality of the graduations illustrates the law above stated of the proportionality of distortions to the forces producing them. The resistance of a spiral spring to elongation depends chiefly (as shown by Professor James Thomson) on the torsional rigidity of the wire which composes it.

It is important to remark, that whereas a pair of scales is essentially a measure of mass, a spring-balance is essentially a measure of force. Hence if a spring-balance be graduated so as to show weights correctly at a medium latitude, it will indicate too little if carried to the equator (where the force of gravity is feebler), and too much at the poles (where gravity is more intense).

CHAPTER IV.

GRAVITY.

24. Terrestrial gravity is the force in virtue of which all bodies fall to the surface of the earth. This force is general; its effects are observed in all places and for all bodies. If some of these latter, as smoke and hydrogen gas, appear to be exceptions, it is because they are sustained by the air in the same manner as cork is sustained by water. In space deprived of air, not only do all bodies fall, but, as we shall see later, they fall with equal velocities.

25. **Direction of Gravity.**—The direction of gravity is called the *vertical*. It is easily determined by the aid of the simple apparatus called a plumb-line, which consists of a thread fixed at one end and carrying a heavy body at the other. When the system is in equilibrium it is clear that the resultant of the actions of gravity on all the parts of the heavy body has exactly the same direction as the thread, since it is this which prevents the fall. But it can be shown that this direction does not change when the form and volume of the heavy body are altered, it must therefore be the same as the direction of the force which would act upon one of the elementary particles if suspended alone at the extremity of the thread.

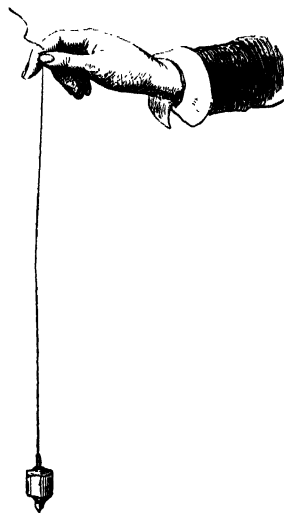


Fig 15.—Plumb-line.

It can be shown by experiment that the direction of gravity is perpendicular to the surface of a liquid in equilibrium, or to use the common expression, to the surface of still water. For this purpose a plumb-line OA is suspended over the

surface of a fluid in equilibrium (which should be slightly opaque,

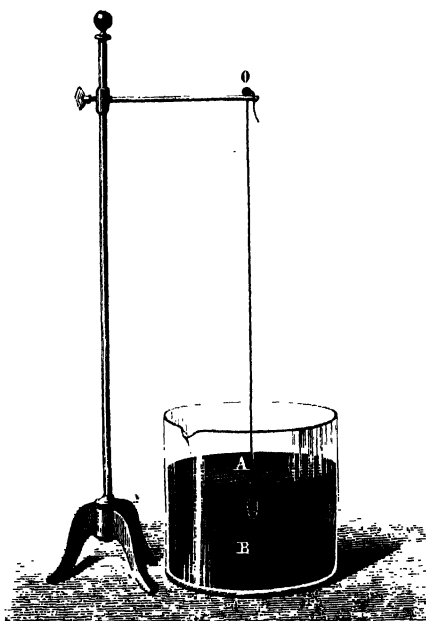


Fig. 16.—Experiment for showing that the Plumb-line is perpendicular to the surface of a fluid at rest.

as blackened water), and the plummet is allowed to plunge in the liquid. The image AB of the thread produced by reflection at the surface of the liquid will be seen with great distinctness, and will be observed to be exactly in a line with the thread itself. Now we shall see in a subsequent part of this treatise, that whenever reflection takes place at a plane mirror, each point of the object and the corresponding point in the image are on the same perpendicular to the mirror and at equal distances from the mirror on opposite sides. Since, then, in the experiment here described the thread and its image are

in one straight line, this line must be perpendicular to the surface.

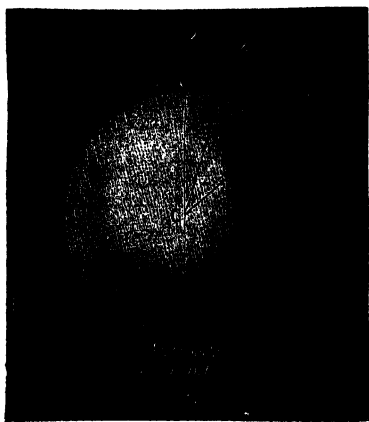


Fig. 17.—Verticals at different places.

The surface of still water defines in each locality what is called the surface of the earth. This expression denotes the surface of an imaginary ocean of calm water supposed to cover the whole earth. This surface is known to be sensibly spherical. It follows that the different verticals will nearly meet in the centre of the earth. The figure shows the relative position of some verticals CZ, CZ', CZ''; it is evident that they contain angles equal to the an-

gular distance which separates the corresponding places.

At any one locality all verticals may be treated as parallel, on account of the immense distance of the centre of the earth. Let us calculate, for example, the angle contained between two verticals a metre (39·37 inches) apart. Ten millions of metres correspond to a quarter of the earth's circumference, that is to say, to 90° . A length of a metre, therefore, represents 90° divided by ten millions, that is to say, about $\frac{1}{100000}$ of a second, a quantity quite inappreciable even with our most perfect instruments. It should be remarked, however, that the parallelism of the verticals at any one place is a physical fact, completely independent of all previous knowledge of the figure of the earth, and can be established by direct observation.

We may remark in passing, that the latitudes of places on the earth's surface are determined by the directions of the verticals. What is commonly called the latitude of a place is the angle which a vertical at the place makes with a plane perpendicular to the earth's axis of rotation. As distinguished from this, the geocentric latitude (which is required in a few astronomical problems) is the angle which a line drawn from the place to the earth's centre makes with a plane perpendicular to the axis of rotation. The difference between common and geocentric latitude generally amounts to some minutes, and attains its greatest value ($11' 29''$) at latitude 45° .

26. Point of Application of Gravity—Centre of Gravity.—Gravity being a property of matter, its points of application must evidently be the different material particles which compose each body. Though a body be divided into as many parts as we please, and even reduced to the state of impalpable powder, each of the grains thus obtained will be subject to the action of gravity. The total force which urges a body to fall is the sum, or more strictly, the resultant of all the forces which are thus actually applied to its several elements. Now these forces are parallel, as has just been stated, and act in the same direction; their resultant is therefore equal to their sum, and it constitutes what is called the *weight* of the body; that is to say, the force with which it presses the obstacle which prevents it from falling. The point of application G of this resultant

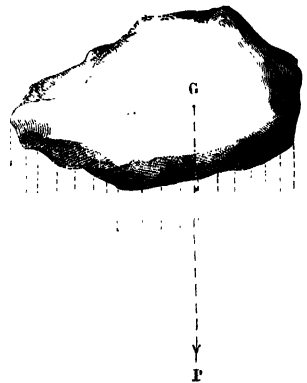


Fig. 18.—Parallelism of the Forces of Gravity on the different Points of a Body.

(Fig. 18) is called the *centre of gravity*. It follows from the property indicated in section 15, that the position of this point does not vary when the direction of the components is made to vary. The body can therefore be turned about in any manner without the centre of gravity changing its position in the body. It is a fixed point depending only on the form of the body and the distribution of the matter which composes it.

If the body has the same density throughout, the position of the centre of gravity depends only on the figure, so that in this case bodies of similar form have their centres of gravity similarly situated.

The determination of the position of the centre of gravity is a problem of mechanics which is solved by appropriate methods of general application, founded on the principle (to which we can here barely allude), that if a body be divided into a great number of equal elements, the sum of their distances from any one plane, divided by the number of elements, is equal to the distance of the centre of gravity from the same plane.

Whenever a body of uniform density contains a point which is a centre of symmetry (that is, a point which bisects all straight lines drawn through it), this point must be the centre of gravity. Hence:

1. The centre of gravity of a straight line is its middle point;
2. The centre of gravity of a circle, or of the circumference of a circle, is the centre;
3. The centre of gravity of a parallelogram is the intersection of the diagonals;
4. The centre of gravity of a sphere is its centre;
5. The centre of gravity of a cylinder is the middle point of its axis;
6. The centre of gravity of a parallelopiped is the common intersection of the diagonals, &c.

It may naturally be asked how we can speak of centres of gravity of lines and surfaces, which, being only of one or two dimensions, cannot possess weight. The answer is, that in so speaking we make an abstraction analogous to that which gives us the idea of a material point. We suppose lines or surfaces composed of elements possessing weight; and the results thus obtained can be utilized in investigating the centres of gravity of real bodies. Consider, for example, a triangular prism. It can be conceived as decomposed into elements which would be, so to speak, heavy triangles. The centre of gravity of the solid would therefore be in the line which joins the centres of gravity of all these triangles, and would be its middle point. We

may say then that the centre of gravity of a triangular prism, and in general of any prism, if composed of homogeneous material, is the middle point of the line which joins the centres of gravity of the two ends.

We must guard against the error of supposing that the force of gravity on a body acts at the centre of gravity. Gravity really acts equally on all the particles of the body; but its effect is in many respects the same as that of a single imaginary force supposed to act at the centre of gravity.

The centre of gravity sometimes lies outside the body, as in the case of a ring or a hollow sphere. When this is the case, it must be regarded as rigidly connected to the body.

27. Physical Definition of the Centre of Gravity.—The centre of gravity, regarded from the mechanical point of view, is in reality merely the centre of parallel forces distributed in a determinate manner. Its position can therefore be found by a purely geometrical investigation, and apart from any physical idea of the nature of bodies. Nevertheless, it is certain that the discovery of the existence of centres of gravity had its origin in the consideration of the phenomena of equilibrium, which are exhibited by bodies under the influence of gravity. Experiment, in fact, shows that in most bodies there is a point such that, if it be supported, the body will be in equilibrium, and if it be not supported, the body will move under the action of gravity. This point is the centre of gravity, and the property in question is an obvious consequence of the mechanical definition already given. But this property may itself be used as the definition of the centre of gravity (though with some want of precision). We shall thus use it in the examination of some important cases of equilibrium.

28. Equilibrium of a Body capable of turning about an Axis or a Fixed Point.—Consider, for example, a triangular plate movable about an axis of rotation O , and let G be the position of the centre of gravity. In order that there may be equilibrium, the centre of gravity must be supported; that is to say, the vertical drawn through it must meet the axis. This condition may be fulfilled by two very different positions of the body: the centre of gravity may be either above or below the axis. In the former case (Fig. 20) it is evident that if the body be ever so little displaced from its position of equilibrium, the effect of gravity will be to make it fall still further away. In the second case, on the contrary (Fig. 19), the action of

gravity tends to restore the body to the position of equilibrium. In the former case the equilibrium is *unstable*; in the latter it is *stable*.

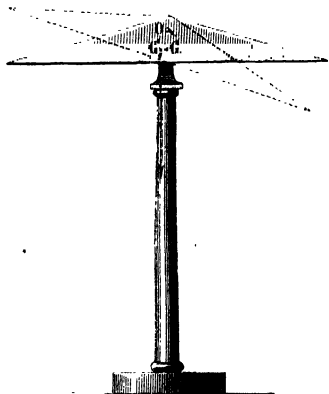


Fig. 19.—Stable Equilibrium.

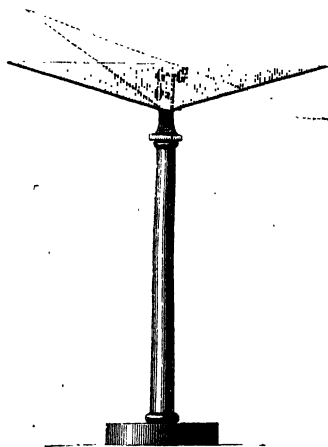


Fig. 20.—Unstable Equilibrium.

We see then that the condition of stable equilibrium is that the centre of gravity be below the axis or point of suspension.

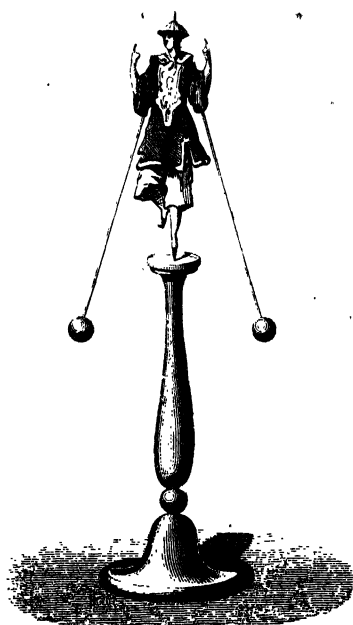


Fig. 21.—Balancer.

The toy called the balancer is an application of this principle. It consists of an ivory figure resting by one point on a small horizontal stand. Two stiff wires fixed to the figure terminate below in leaden balls. The centre of gravity of the system is thus brought below the point of support; the equilibrium is consequently stable. If we draw the figure to one side and then release it, it will perform a series of oscillations, and will end by taking a position of equilibrium such that the vertical through the centre of gravity passes through the point of support.

If a body were traversed by an axis through its centre of gravity, its equilibrium would be neutral, and the body would remain in

equilibrium in all positions. This condition ought to be rigorously fulfilled by the wheels of pieces of mechanism which only serve to transmit motion.

29. Equilibrium of a Body resting on a Horizontal Plane which touches it in one Point.—Consider (Figs. 22 and 23) a body of

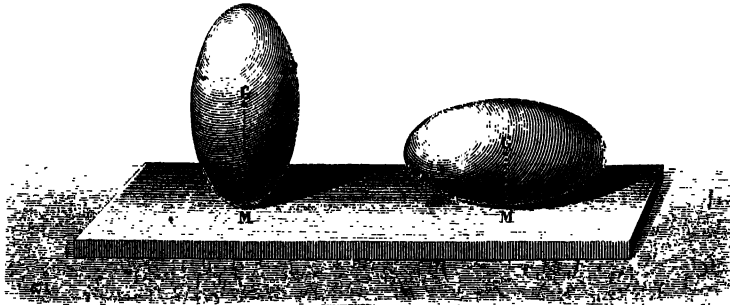


Fig. 22.—Unstable Equilibrium.

Fig. 23.—Stable Equilibrium.

ellipsoidal form resting on a horizontal plane. In order that there may be equilibrium it is evidently necessary and sufficient that the vertical through the centre of gravity G should meet the horizontal plane at the point of contact. We see by the figure that this condition may be realized in two ways.

The first figure corresponds to *unstable*, the second to *stable* equilibrium. The figure shows that in the latter case the *centre of gravity occupies its lowest possible position*.

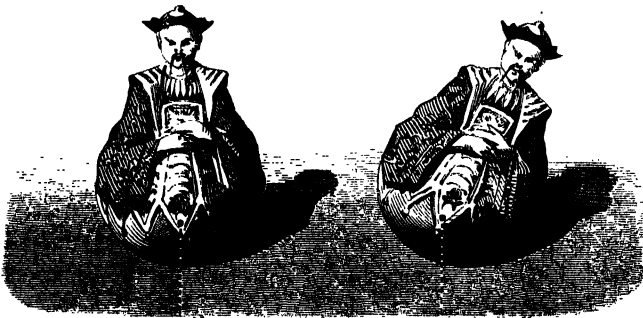


Fig. 24.—Tumblers

The tumbler (Fig. 24) is founded on this principle. The centre of gravity being near the lower side in consequence of the accumulation of matter in this region, the apparatus is in stable equilibrium. If

it be removed from its position and subjected even to very wide displacements, it always rises again and returns to its position of equilibrium after a number of oscillations.

If the centre of gravity were always at the same distance from the plane—if, for example, the body were spherical, there would be equilibrium in all positions,—the equilibrium would be neutral.

We may remark in general that a position of unstable equilibrium is only mathematically possible; it never can have a physical existence; for the smallest derangement destroys it, and in nature a multitude of causes, such as the movement of the air, the flexibility of supports, &c., introduce displacements which violate the conditions of equilibrium. If there be any actual cases of such equilibrium—if, for example, it is possible to make an egg stand on its end, it is by the help of friction, which constitutes a new force tending to prevent displacement.

'30. Equilibrium of a Body resting on a Horizontal Plane at several Points.—When a body rests on a horizontal plane at several points,

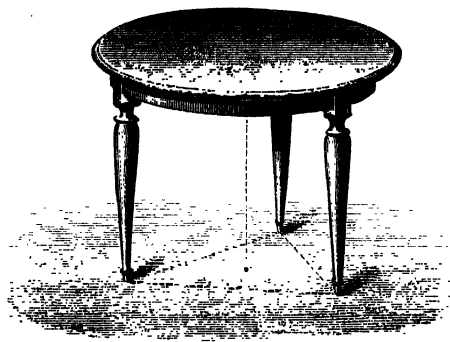


Fig. 25.—Equilibrium of a Body supported on a Horizontal Plane at three or more Points.

it is necessary for equilibrium that the vertical through the centre of gravity fall within the convex polygon which can be formed by joining the points of support. It is clear that in this case gravity will have no effect but to press the body against the plane. It is also obvious that the equilibrium will be the more stable as the centre of gravity is lower, and the distance of the vertical through it from the nearest side of the polygon greater. If this vertical is very near one side, a small force will be sufficient to overturn the body on that side, although a very great force may be required to overturn it in the opposite direction.

§1. Practical Method of finding the Centre of Gravity.—The different methods which are employed in practice for the experimental determination of the centre of gravity are dependent on the principles above explained. Whatever be the particular nature of the proceeding, it always consists in placing the body in a position of equilibrium from which it can be inferred that the centre of gravity lies in a certain line or surface.

Thus, for example, if we suspend a body by one point, it is clear that the centre of gravity must lie in the prolongation of the suspending thread. If we then suspend the body by another point, a similar inference follows. Consequently, the centre of gravity must be the intersection G of the two directions thus indicated.

If we wish, for example, to pierce a plate or board by an axis which is to pass through its centre of gravity, we may begin by balancing it in a horizontal position upon two points near its circumference. The line joining them will pass vertically under the centre of gravity.

By repeating the operation we may find a second line which possesses the same property, and the required axis must pass through their intersection and be perpendicular to the plate. Instead of balancing the plate upon two points, an operation which may require repeated trials, it is more expeditious, when practicable, to suspend it freely in a vertical position by a point near its circumference, and to suspend a plumb-line from the same point. The course of this line must be marked on the plate, and the operation must then be repeated, using a different point of suspension. The intersection of the two lines thus obtained will, as before, be opposite to the centre of gravity of the plate. Both the methods described in this paragraph are applicable even to plates which are not homogeneous.

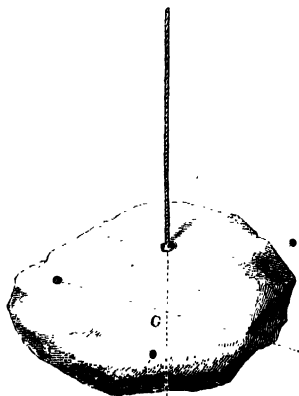


Fig. 26.—Experimental Determination of Centre of Gravity.

CHAPTER V.

LAWS OF FALLING BODIES.

32. In air, bodies fall with unequal velocities; a sovereign or a ball of lead falls rapidly, a piece of down or thin paper slowly. It was formerly thought that this difference was inherent in the nature of the materials; but it is easy to show that this is not the case, for if we compress a mass of down or a piece of paper by rolling it into a ball, and compare it with a piece of gold-leaf, we shall find that the latter body falls more slowly than the former. The inequality of the velocities which we observe is due to the resistance of the air, which increases with the extent of surface exposed by the body.

It was Galileo who first discovered the cause of the unequal rapidity of fall of different bodies. To put the matter to the test, he prepared small balls of different substances, and let them fall at the same time from the top of the tower of Pisa; they struck the ground almost at the same instant. On changing their forms, so as to give them very different extents of surface, he observed that they fell with very unequal velocities. He was thus led to the conclusion that gravity acts on all substances with the same intensity, and that in a vacuum all bodies would fall with the same velocity.

This last proposition could not be put to the test of experiment in the time of Galileo, the air-pump not having yet been invented. The experiment was performed by Newton, and is now commonly exhibited in courses of experimental physics. For this purpose a tube from a yard and a half to two yards long is used, which can be exhausted of air, and which contains bodies of various densities, such as grains of lead, pieces of paper, and feathers. When the tube is full of air and is inverted, these different bodies are seen to fall with very unequal velocities; but if the experiment is repeated after the

tube has been exhausted of air, no difference can be perceived between the times of their descent.

33. Laws of Falling Bodies.—Having found that the effect of gravity is the same on all bodies, Galileo proposed to himself the problem of determining, by experiments on one body, the law which regulates their descent; and, inasmuch as the observation of a body falling freely is very difficult, on account of the rapidity of its motion, he adopted a method of diminishing this rapidity without in other respects altering the law of motion. This method consisted in the use of the inclined plane.

Consider, in fact, a heavy body M , free to move along the inclined plane ABC . The weight of the body M being represented by MP , it can, (by § 16), be decomposed into two other forces, viz. MN perpendicular to the plane, which is destroyed by the resistance of the plane itself, and MT parallel to the plane, which alone produces the motion. Now this latter force is less than MP , but is a constant fraction of it, for at all points in the plane the parallelogram of forces will have the same form, and the ratio of MT to MP will be constant. This ratio is in fact the same as that of the height AC of the plane to its length AB , or in other words is the sine of the inclination of the plane to the horizon. The motion will therefore be less rapid, but will follow the same law as that of a body falling freely, and will be much easier to observe. The diminution of velocity has the further advantage of diminishing the relative importance of the resistance of the air,

which increases very rapidly with every augmentation of velocity.

The inclined plane employed by Galileo consisted of a long ruler, with a longitudinal groove, along

which he caused a small heavy ball to roll. Having thus observed the spaces traversed in 1, 2, and 3 units of time, he found that these

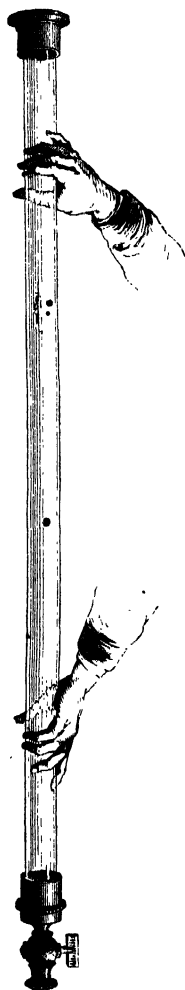


Fig. 27.—Fall of Bodies in Vacuo.

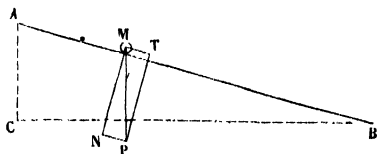


Fig. 28.—Inclined Plane.

spaces were in the ratio of the numbers 1, 4, and 9; that is to say, when the time of descent was doubled or tripled the space traversed became 4 or 9 times greater. This law can be expressed by saying that *the spaces traversed are proportional to the squares of the times of descent.*

34. Attwood's Machine.

—Attwood, a fellow and tutor of Trinity College, Cambridge, invented, towards the end of last century, a machine which affords great facilities for verifying the laws of falling bodies. It involves, like Galileo's inclined plane, a method of diminishing the velocity of descent; but this result is obtained by very different means.

The machine consists of a column, having at its top a very freely moving pulley, which forms the essential part of the apparatus. In order to obtain great freedom for the movements of the pulley, the ends of its axis are made to rest, not on fixed supports, but on the

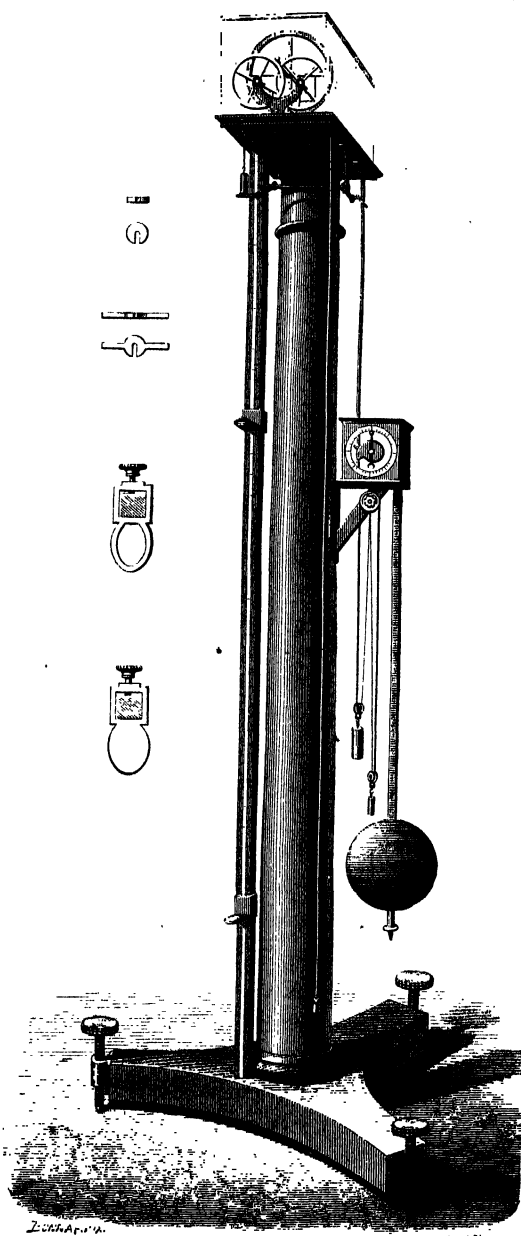


Fig. 29.—Attwood's Machine.

circumferences of wheels (two at each end of the axis) called friction-

wheels, because this arrangement produces a great diminution of friction. Over the pulley passes a fine thread, carrying at its extremities two equal weights P . Neglecting the weight of the thread, it is obvious that these weights will be in equilibrium in every position. If however one of them be loaded with an additional weight p , the system will be put in motion, and all parts of it will move with the same velocity. We may therefore regard the moving force as distributed uniformly through it. But this force is simply the weight of p . If then, for example, the movable system $2P + p$ has 20 times the weight of p , each portion of the system is urged with a force equal to $\frac{1}{21}$ of its own weight. The force which produces motion is in general diminished, as compared with a body falling freely, in the ratio expressed by the fraction $\frac{p}{2P + p}$; and as

this ratio continues constant through the whole motion, the law of the motion will be the same as that for free descent.

- The following are the arrangements for observing the motion:—One of the weights moves in front of a graduated scale, and a plane stop for intercepting the descending weight can be fixed at pleasure at any part of this scale. A clock with a pendulum beating seconds serves for the measurement of time. To measure the space traversed in a second, the weight is raised to the commencement of the graduation, is then loaded with the additional weight, and is dropped precisely at one of the beats of the pendulum. The stop is placed by trial at such a point of the scale that the blow of the weight against it precisely coincides with another beat of the pendulum,—a coincidence which can be obtained with great accuracy, inasmuch as the ear easily detects the smallest interval between the two sounds. In order to insure a similar coincidence at the commencement of the fall, the weight is supported by a movable platform M (Fig. 30), which is prevented from falling by the upper end of the lever aob , whose lower end is guided by a cam¹ fixed to the escapement wheel

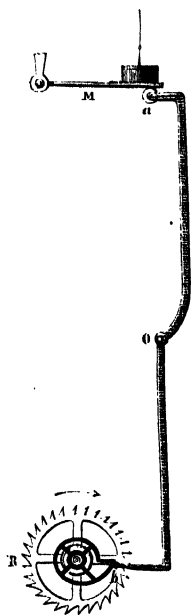


Fig. 30.—Detent in Attwood's Machine.

¹ A cam is a rotating piece which, by means of projections or indentations in its outline, guides the movements of another piece which presses against it.

of the clock, and is kept constantly pressed against the cam, by means of a spring not shown in the figure. Suppose the wheel to be turning in the direction indicated by the arrow. It is obvious that as soon as the tooth of the cam has passed the end of the lever the latter will fly to the left, and therefore the upper end will fly to the right, since the lever turns about an axis at O. The platform M is thus suddenly dropped, and the fall commences. The position of the cam must be so adjusted that this movement shall take place exactly at the instant of the escapement of one of the teeth.

It is thus easy to measure the spaces traversed by the movable system of weights in 1, 2, and 3 seconds, and the result obtained will be as follows:—Suppose that in the first second the space traversed is 11 divisions, then we shall find:

Space traversed in 2 seconds.....	= 44=11 × 2 ²
Space traversed in 3 seconds.....	= 99=11 × 3 ²
Space traversed in 4 seconds.....	= 176=11 × 4 ²

We see, then, that the spaces vary as the squares of the times employed in describing them. If we use the indefinite symbol K to denote the space described in the first unit of time, the space described in the time t will be given by the formula

$$s = Kt^2. \quad (1)$$

35. Velocities.—Attwood's machine also affords the means of studying the successive velocities which gravity imparts to the system. Before describing the means employed for attaining this end, it will be desirable to make a few remarks respecting velocity.

When a material point moves uniformly; that is to say, when it traverses equal spaces in equal times, the meaning of velocity is perfectly clear: it is the space traversed in unit time. Thus, if a point moving uniformly describes 2 feet in each second, we say that the velocity is 2 feet per second; or if it is understood that the foot and second are to be our units of space and time, we simply say that the velocity is 2. If a point moving uniformly describes 5 feet in 2 seconds, its velocity is $2\frac{1}{2}$, since the space described in one second must be half that described in two; and in general, in any case of uniform motion of a point, the velocity (in feet per second) will be obtained by dividing the whole space (in feet) by the whole time occupied in its description (in seconds).

But uniform motion is in nature the exception rather than the rule. In fact, it can only occur when the moving body is acted on

either by no forces at all, or by forces in equilibrium. This, in fact, is merely a statement of the principle of inertia. When a body is constantly acted on by a force, this force must evidently have the effect of continually modifying the motion, and consequently the above mode of computing velocity is not directly applicable. If, however, we suppose the action of the force suddenly to cease, the motion will become uniform, and the velocity during this uniform motion will serve as a measure of the velocity which existed at the instant when the action of the force ceased.

Attwood's machine contains an arrangement for thus suddenly arresting the action of gravity at a given instant. For this purpose, a ring large enough to allow either of the two equal weights to pass through it, is to be fixed at the point of the scale at which the moving weight arrives at the end of one second. The additional weight, which for this purpose is made long and flat, is intercepted by the ring, and the subsequent motion, being due merely to the momentum already acquired, will be uniform. The stop is to be placed at the point at which the weight arrives a second later. The distance between the ring and the stop will then represent the velocity acquired during the first second. Making this experiment under the same conditions as the foregoing experiments on spaces, we find that the velocity acquired during the first second is represented by 22 divisions. We then place the ring at the point at which the system arrives after 2, 3, &c., seconds, and the stop at the point at which it arrives a second later; we thus measure the velocities acquired in 2, 3, &c., seconds, and find them equal to 44, 66, &c. We see, then, that the *velocities acquired in different times are proportional to the times*. Further, the velocity acquired in one second, 22 (feet per second), is double of the space, 11 (feet), described in the first second.

In formula (1), K denotes the space described in the first unit of time; the velocity acquired is then $2K$, and consequently the velocity acquired in time t is given by the formula

$$V = 2Kt. \quad (2)$$

36. Attwood's machine, when fitted with the appliances above described, leaves little to be desired in point of accuracy, but its complication renders it expensive. We subjoin a figure representing Bourbouze's modification of Attwood's machine, which is much simpler. AB is the pulley, on the axis of which is a cylinder P ,

surrounded with smoked paper. One of the iron weights, M , is held at the bottom of the apparatus by an electro-magnet, which is magnetized by means of the galvanic cell O . The weight

M' , loaded with the additional weight N , is thus prevented from obeying the force of gravity. Again, the vibrating plate L , carrying a very light style for tracing a mark on the smoked cylinder, is held by the electro-magnet, E' , which is magnetized by the same cell. If at any moment the current is interrupted, the weight M' falls, and the plate vibrates, describing an undulated curve on the surface of the cylinder. The undulations of this curve correspond to the vibrations of an elastic body, and, consequently, to equal times; while the distance of any undulation from the beginning of the curve is equal to the distance turned by the cylinder P , and is consequently proportional to the distance travelled by the weights.

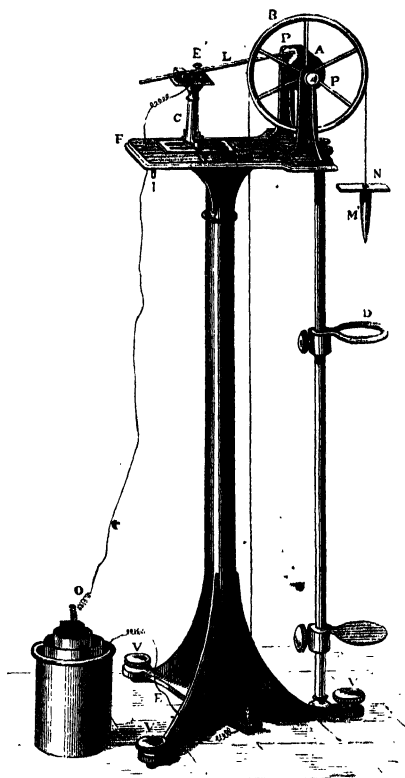


Fig. 31.—Bourbonze's Modification of Attwood's Machine.

These distances are found to be exactly proportional to the series of numbers 1, 4, 9, &c. The ring D serves to intercept, at a given instant during the descent, the additional weight N ; from this time onward the motion is uniform and the undulations of the curve are equidistant.

Attwood's machine, however modified, gives only indirect evidence regarding the motion of bodies falling freely. Although this circumstance cannot affect the legitimacy or accuracy of the conclusions to which it leads, it would be interesting, if possible, to observe the phenomenon of free fall, and show that the laws just obtained are verified. This is the object of Morin's apparatus.

37. Morin's Apparatus.—Morin's apparatus consists of a wooden cylinder covered with paper, which can be set in uniform rotation about its axis by the fall of a heavy weight. The cord which supports the weight is wound upon a drum, furnished with a toothed wheel which works on one side with an endless screw on the axis of the cylinder, and on the other drives an axis carrying fans which serve to regulate the motion.

In front of the turning cylinder is a cylindro-conical weight of cast-iron carrying a pencil whose point presses against the paper, and having ears which slide on vertical threads, serving to guide it in its fall. By pressing a lever, the weight can be made to fall at a chosen moment. The proper time for this is when the motion of the cylinder has become sensibly uniform. It follows from this arrangement that during its vertical motion the pencil will meet in succession the different generating

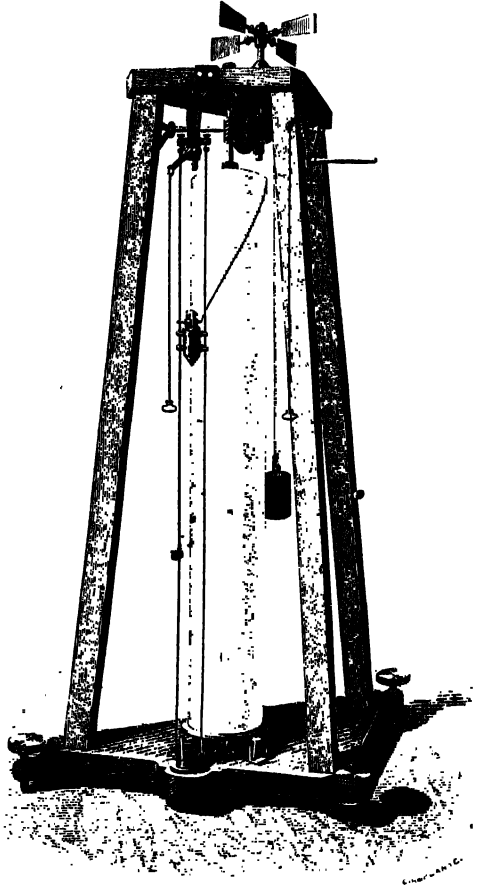


Fig. 32.—Morin's Apparatus.

lines¹ of the revolving cylinder, and will consequently describe on its surface a certain curve, from the study of which we shall be

¹That is, lines drawn on the surface of the cylinder parallel to its axis. A cylindric surface could obviously be described by the motion of a straight line in space. The line so moving is said to generate the cylindric surface, and the different positions which it successively occupies in its supposed motion are called generating lines of the cylinder. If a cylindric surface is cut open along a generating line and flattened out so as to become plane, its form will be rectangular, and its generating lines will be parallel to two sides of the rectangle.

able to gather the law of the fall of the body which has traced it. With this view, we describe (by turning the cylinder while the pencil is stationary) a circle passing through the commencement of the curve, and also draw a vertical line through this point. We cut the paper along this latter line and develop it (that is, flatten it out into a plane). It then presents the appearance shown in Fig. 33.

If we take on the horizontal line equal distances at 1, 2, 3, 4, 5 . . . ,

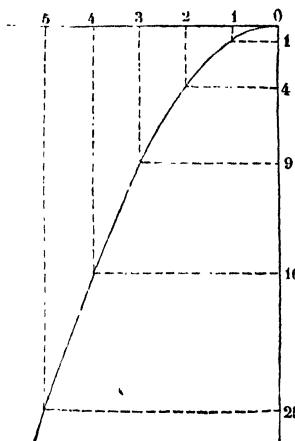


Fig. 33.—Parabolic Trace.

and draw perpendiculars at their extremities to meet the curve, it is evident that the points thus found are those which were traced by the pencil when the cylinder had turned through the distances 1, 2, 3, 4, 5. . . . The corresponding verticals represent the spaces traversed in the times 1, 2, 3, 4, 5. . . . Now we find, as the figure shows, that these spaces are represented by the numbers 1, 4, 9, 16, 25 . . . , thus verifying the principle that the spaces described are proportional to the squares of the times employed in their description.

We may remark that the proportionality of the vertical lines to the squares of the horizontal lines shows that the curve is a parabola. The parabolic trace is thus the consequence of the law of fall, and from the fact of the trace being parabolic we can infer the proportionality of the spaces to the squares of the times.

The law of velocities might also be verified separately by Morin's apparatus; we shall not describe the method which it would be necessary to employ, but shall content ourselves with remarking that the law of velocities is a logical consequence of the law of spaces.¹

38. Formulæ relating to Falling Bodies.—The formulæ (1) and (2), §§ 34, 35, will enable us to obtain numerical solutions of questions relating to the fall of bodies, if we can ascertain the space traversed

¹ Consider, in fact, the space traversed in any time t , this space is given by the formula $s = Kt^2$; during the time $t + \theta$ the space traversed will be $K(t + \theta)^2 = Kt^2 + 2Kt\theta + K\theta^2$, whence it follows that the space traversed during the time θ after the time t is $2Kt\theta + K\theta^2$. The average velocity during this time θ is obtained by dividing the space by θ , and is $2Kt + K\theta$, which, by making θ very small, can be made to agree as accurately as we please with the value $2Kt$. This limiting value $2Kt$ must therefore be the velocity at the end of time t .—*D.*

by a falling body in one second, or (which will be numerically double of this) the velocity acquired in one second.

The several forms of apparatus which we have been describing furnish the means of making this determination, but not with much precision. A much better method of determination is furnished by the pendulum, and will be described in the next chapter.

The result obtained is that in Great Britain the velocity acquired by a body falling in vacuo for one second is 32·2 feet per second, or 9·81 metres per second. The velocity in question is usually denoted by the letter g , and is sometimes called the intensity of gravity, because its value for different localities varies in the same ratio as the force exerted by gravity on one and the same mass, and may therefore be taken as the numerical representative of the force of gravity on unit mass.¹

The space traversed during the first second of fall is equal to $\frac{1}{2} g$, that is, to 16·1 feet.

Introducing g into the formulæ (1), (2), they become:

$$s = \frac{1}{2} g t^2 \quad (a)$$

$$v = g t \quad (b)$$

Eliminating t from the equations (a), (b), we obtain a third formula, which gives the velocity acquired in falling through a given space:

$$v = \sqrt{2 g s}. \quad (c)$$

39. Applications.—I. To calculate the space traversed by a body which falls during a given number of seconds. This will be found from formula (a) by putting for t the given number of seconds. Performing the calculation as far as 10 seconds, we obtain the following table:—

TIME OF FALL, in Seconds.	SPACE FALLEN, in Feet.	TIME OF FALL, in Seconds.	SPACE FALLEN, in Feet.
1	16·1	6	579·6
2	64·4	7	788·9
3	144·9	8	1030·4
4	257·6	9	1304·1
5	402·5	10	1610·0

II. What is the time occupied by a body in falling from a height of 750 feet, and what is the velocity with which it strikes the ground?

¹ If we adopt the "absolute" unit of force defined in § 42, the force of gravity on unit mass will be numerically equal to g .

Formulæ (a) and (c) give:

$$t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{1500}{32 \cdot 2}} = 6 \cdot 8 \text{ seconds.}$$

$$v = \sqrt{2gs} = \sqrt{64 \cdot 4 \times 750} = 219 \cdot 8 \text{ feet per second.}$$

III. From what height must a body fall to acquire a velocity of 1500 feet per second?

Formula (c) gives:

$$s = \frac{v^2}{2g} = \frac{2250000}{64 \cdot 4} = 34938 \text{ feet.}$$

The velocity of 1500 feet per second is about that of a cannon-ball on leaving the muzzle of the gun. We see that it would be obtained by falling from a height of about $6\frac{1}{2}$ miles. The duration of the fall would be about 47 seconds. It must be borne in mind that the formulæ (a), (b), (c) are only strictly applicable to fall in vacuo. In air they furnish results more and more remote from the truth, as the velocity increases.

In vacuo, a body thrown upwards from the earth would, on its return, strike the ground with a velocity equal to that with which it was thrown, and the velocity at any given point which it traverses both in its upward and downward course, would be the same in the descent as in the ascent. We see then from above that a cannon-ball would ascend to a height of about $6\frac{1}{2}$ miles. In air, the velocity is slower in the descent than in the ascent, both because the height attained by the projectile is less than it would be in vacuo, and because the velocity acquired in falling from this diminished height is still further diminished by friction in the descent.

One notable difference between fall in vacuo and in air is, that in the latter case the velocity, instead of increasing indefinitely, only increases towards a certain limit which it can never exceed; and if a body be projected downwards with a velocity greater than this, its motion will be retarded. The resistance of the air, in fact, increases with the velocity, and the limit in question is that velocity at which the resistance encountered from the air is exactly equal to the weight of the body. The limiting velocity is not the same for all bodies, but depends on their sizes, densities, and forms.

39A. Motion of Projectiles.—When a body is projected in any direction, its subsequent motion (neglecting the resistance of the air) can be determined by means of the following principles:—

1. The horizontal component of motion will remain unchanged.

2. The vertical component of motion will be the sum or difference (according as the body was projected below or above the horizontal direction) of the motion of a body falling freely, and the vertical component of the initial motion.

These principles are sufficient to determine the position, the velocity, and the direction of motion of the body, after the lapse of any given interval, until it strikes an obstacle.

By reference to Fig. 33, it will easily be understood that, if the direction of projection be horizontal, the curve described will be a parabola; for, in virtue of the first principle, the body describes equal horizontal distances in equal times; and in virtue of the second principle (since the vertical component of initial motion is in this case zero), the vertical spaces described are those of a body falling freely; hence the construction of Fig. 33 is precisely applicable to this case.

If the direction of projection be oblique (that is, neither horizontal nor vertical), the path will still be parabolic. For example, if the body be projected obliquely upwards, we may divide its path into two parts, one described in its ascent, the other in its descent. These two parts will be precisely similar, and at the highest point of the path, where they join, the motion is horizontal. We may regard the curve in Fig. 33 as representing one of the two parts, say the descending part; for the motion, after passing the highest point, must evidently be the same as if the body had been projected horizontally from the highest point with the velocity which it actually has at that point. This velocity is, in fact, the horizontal component of the actual velocity of projection.

If a body be projected vertically downwards, its motion will be the same as if it had fallen from a certain height above. If it be projected vertically upwards, the times of ascent and descent will be equal, and the velocity at any one point will be the same in the descent as in the ascent. At the highest point the body is for an instant stationary; its descent is therefore the motion of a body falling freely. Formulæ (a), (b), (c) of § 38 will apply to the ascent as well as the descent, if in the former case we understand that the time denoted by t is the time reckoned backwards from the instant of attaining the highest point.

Whether the motion be in a vertical line, or in a parabola, the following principle will be found to apply, being in fact the mathematical consequence of the two principles already enunciated, viz.:—

The velocity is the same in the ascent and descent at any two points which are on the same level; and the velocities at any two points, not on the same level, are connected by the law that the difference of their squares is equal to the difference of levels multiplied by $2g$.

40. Composition of Motions.—Principle 2 of last section is a particular case of the following general law:—

When a force acts upon a body already in motion, the subsequent motion will be obtained by compounding (in the same manner as forces are compounded by the parallelogram of forces) the motion which the force would have imparted to the body if initially at rest, with the motion which the body would have had in the absence of the force. The law, as thus stated, is applicable without qualification as long as the force continues to act parallel to a definite direction. In the case of forces which do not fulfil this condition, but gradually change their direction in space, the motion may be approximately determined by dividing the time into intervals so short, that the direction of the force does not change by a sensible amount during any one interval. The motion in each interval can then be determined by the above law. It is frequently possible, by the aid of the higher mathematics, to foresee the exact result to which this tedious method would only approximately lead, but the physical principles on which the investigation is conducted are in all cases those which we have above indicated.

41. Uniform Acceleration.—The general law above stated is exemplified in the case of gravity. In fact, if a denote the space traversed in the first second of a falling body's descent, the space in two seconds is $4a$, and consequently the space traversed in the second second is $3a$. Now the velocity at the end of the first second is $2a$, and this velocity, if it remained unchanged, would cause the space $2a$ to be traversed in the next second. Hence the space $3a$ actually described may be divided into two parts, $2a$ and a , of which the latter is due to the continued action of the force of gravity during the second second.

In like manner the space described in the third second is $9a - 4a$, that is, $5a$, which may be divided into two parts, $4a$ and a , of which the latter alone is due to the action of gravity during the third second, the former being due to the velocity $4a$, which the body possessed at the end of the third second.

So, again, the velocity acquired by the body in the first second

being $2a$, the velocity at the end of any subsequent second will be found to be the sum of two parts, one of which is the velocity at the beginning of the second, and the other is $2a$.

Motion possessing these properties is said to be *uniformly accelerated*; and the force which produces it is a uniformly accelerating force, that is to say, a constant force.

The force of gravity, however, is sensibly constant only within moderate limits of distance from the earth's surface; as we ascend from the earth its intensity continually diminishes, being nearly proportional inversely to the square of the distance from the earth's centre. Hence a body falling *in vacuo* from a great height towards the earth, would not be uniformly accelerated, but would experience continually greater acceleration as it descended. On the other hand, if there were a vacuous shaft down which a body could fall to the centre of the earth, it would fall with continually diminishing acceleration, because the force of gravity in the interior of a solid sphere diminishes as we approach the centre, and becomes zero at the centre itself, where the attractions, being equal in all directions, destroy one another. A body so falling would have its velocity continually increased, but the *rate of increase as measured by the difference between the velocity at the beginning of a second and that at the end of it*, would continually become less. The words italicized in last sentence constitute the definition of *acceleration*. It denotes the rate of increase of velocity, just as velocity itself denotes the rate of increase of distance measured along the path described from a fixed point.

42. Proportionality of Acceleration to Force directly and to Mass inversely.—The general law of motion enunciated in § 40 may be extended to the case of several forces acting simultaneously. The actual motion will be obtained by compounding (on the parallelogram principle) the motions due to the separate forces together with the motion (if any) due to the initial velocity. Just as two forces acting on a point in the same direction are equivalent to a single force equal to their sum; so two motions in the same direction constitute, when compounded, a motion equal to their sum; and this is true, both as regards velocity and space described. Two equal forces acting on a body on the same direction, will therefore produce in any given time double the velocity that one of them would have produced alone. We are thus led to the general principle, that the velocities produced in equal bodies by different forces are simply proportional to the forces.

The above reasoning is not offered as an *à priori* proof of the general principle in question, but as a logical deduction of it from the law of composition of motions due to several forces. The proportionality of velocity to the force which produces it can be proved experimentally by Attwood's machine, and in other ways; and the law of composition in question must be regarded as established by the experimental verification of these and other consequences to which it leads.

From the direct proportionality of velocity to the force by which it is generated, when the mass is given, we may infer the inverse proportionality of velocity to the mass which is set in motion, when the force is given. For instance, if we double the mass, leaving the force unchanged, we may resolve the force into two equal parts, acting one on each half of the mass. Doubling the mass has, therefore, the same effect on the motion as halving the force.

The velocity generated in a given time is thus proportional to the moving force divided by the mass moved; from which it follows that force is proportional to the product of mass by velocity generated in a given time.

When force is expressed in terms of the "absolute" or "invariable" unit, first proposed by Gauss, we can assert that the moving force is *equal* to the product of the mass moved, and the velocity generated in a unit of time.¹ For example, since the force of gravity on a body weighing M pounds causes it to acquire a velocity of g feet in a second, this force is numerically equal to Mg , it being understood that the pound is the unit of mass, and the foot and second the units of space and time.

That the pound is really and strictly a standard of mass is obvious from the consideration that the standard pound is a certain piece of platinum, preserved at the office of the Exchequer in London, and that this piece of platinum would remain a true pound if carried to any part of the earth.

At any one place, since g has a given value, the masses of bodies are proportional simply to their weights, but this proportion obviously does not hold in the comparison of masses at places where the values of g are different. When a body is carried about to different parts of space, its mass, or quantity of matter, remains of course the same,

¹ The absolute unit of force may be defined as that force which, acting on unit mass for unit time, would generate unit velocity. The force of gravity on unit mass contains g such units.

but its weight alters in proportion to the greater or less intensity of gravity;—for instance, at the centre of the earth, regarded as a uniform sphere, its weight would be nothing. This annihilation of its weight would in no way affect its resistance to acceleration. The difference between the mass of a ball of cork, and that of a ball of lead of the same diameter, could in such circumstances be readily detected by the different resistances which they would oppose to attempts to set them in rapid motion, or to check their motion when commenced.

It must be regarded as a remarkable fact, and one which could only have been established by experiment, that the two modes of comparing masses perfectly coincide; that is to say, two bodies, even though composed of different materials, if their sizes are so proportioned that they oppose equal resistances to acceleration, will also gravitate with equal forces, as tested by their balancing each other in a pair of scales. This principle is established experimentally by the equal velocities of fall of all bodies in vacuo, and, with much greater accuracy, by the equality of the number of vibrations made in the same time by pendulums of the same size and form, but of different materials.

CHAPTER VI.

THE PENDULUM.

43. **The Pendulum.**—When a body is suspended so that it can turn about a horizontal axis which does not pass through its centre of gravity, its only position of stable equilibrium is that in which its centre of gravity is in the same vertical plane with the axis and below it (§ 28). If the body be turned into any other position, and left to itself, it will oscillate from one side to the other of the position of equilibrium, until the resistance of the air and the friction of the axis gradually bring it to rest. A body thus suspended, whatever be its form, is called a pendulum. It frequently consists of a rod which can turn about an axis *O* at its upper end, and which carries at its lower end a heavy lens-shaped piece of metal *M* called the bob; this latter can be raised or lowered by means of the screw *V*. The applications of the pendulum are very important: it regulates our clocks, and it has enabled us to measure the intensity of gravity and ascertain the differences in its amount at different parts of the world; it is important then to know at least the fundamental points in its theory. For explaining these we shall begin with the consideration of an ideal body called the *simple pendulum*.

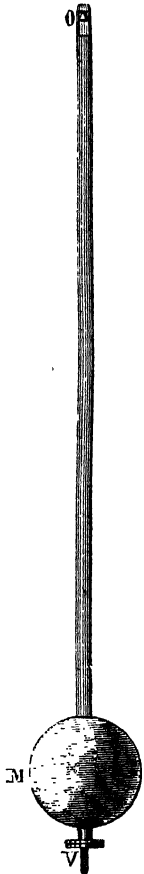


Fig. 34.—Pendulum.

44. **Simple Pendulum.**—This is the name given to a pendulum consisting of a heavy particle *M* attached to one end of an inextensible thread without weight, the other end of the thread being fixed at *A*. When the thread is vertical the weight of the particle acts in

the direction of its length, and there is equilibrium. But suppose it is drawn aside into another position, as AM . In this case the weight MG of the particle can be resolved into two forces MC and MH . The former, acting along the prolongation of the thread, is destroyed by the resistance of the thread; the other, acting along the tangent MH , produces the motion of the particle. This effective component is evidently so much the greater as the angle of displacement from the vertical position is greater. The particle will therefore move along an arc of a circle described from A as centre, and the force which urges it forward will continually diminish till it arrives at the lowest point M' . At M' this force is zero, but, in virtue of the velocity acquired, the particle will ascend on the opposite side, the effective component of gravity being now opposed to the direction of its motion; and, inasmuch as the magnitude of this component goes through the same series of values in this part of the motion as in the former part, but in reversed order, the velocity will, in like manner, retrace its former values, and will become zero when the particle has risen to a point M'' at the same height as M . It then descends again and performs an oscillation from M'' to M precisely similar to the first, but in the reverse direction. It will thus continue to vibrate between the two points M, M'' (friction being supposed excluded), for an indefinite number of times, all the vibrations being of equal extent and performed in equal periods.

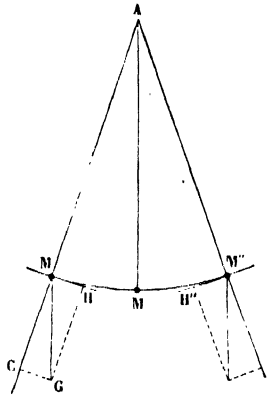


Fig. 35.—Motion of Simple Pendulum.

The distance through which a simple pendulum travels in moving from its lowest position to its furthest position on either side, is called its *amplitude*. It is evidently equal to half the complete arc of vibration, and is commonly expressed, not in linear measure, but in degrees of arc. Its numerical value is of course equal to that of the angle MAM' , which it subtends at the centre of the circle.

The *complete period* of the pendulum's motion is the time which it occupies in moving from M to M'' and back to M , or more generally, is the time from its passing through any given position to its next passing through the same position *in the same direction*.

What is commonly called the time of vibration, or the time of

a single vibration, is the half of a complete period, being the time of passing from one of the two extreme positions to the other. Hence what we have above defined as a complete period is often called a double vibration.

When the amplitude changes, the time of vibration changes also, being greater as the amplitude is greater; but the connection between the two elements is very far from being one of simple proportion. The change of time (as measured by a ratio) is much less than the change of amplitude, especially when the amplitude is small; and when the amplitude is less than about 5° , any further diminution of it has little or no sensible effect in diminishing the time. *For small vibrations, then, the time of vibration is independent of the amplitude.* This is called the law of *isochronism*.

The time of a single vibration when the amplitude is small is expressed by the formula:

$$T = \pi \sqrt{\frac{l}{g}},$$

l denoting the length of the pendulum, g the intensity of gravity, and π the ratio of the circumference of a circle to the diameter. As regards the units in which T , l , and g are expressed, it must be remarked that if g is expressed in the usual way, as in § 38, l must be expressed in feet, and the value obtained for T will be in seconds.

The formula shows that the time of vibration is proportional to the square root of the length of the pendulum, so that if the pendulum be lengthened four, nine, or sixteen fold, the time will be doubled, trebled, or quadrupled.

45. Experimental Laws of the Motion of the Pendulum.—The preceding laws apply to the simple pendulum; that is to say, to a purely imaginary existence; but they are approximately true for ordinary pendulums, which in contradistinction to the simple pendulum are called compound pendulums. The discovery of the experimental laws of the motion of pendulums was in fact long anterior to the theoretical investigation. It was the earliest and one of the most important discoveries of Galileo, and dates from the year 1582, when he was about twenty years of age. It is related that on one occasion, when in the cathedral of Pisa, he was struck with the regularity of the oscillations of a lamp suspended from the roof, and it appeared to him that these oscillations, though diminishing in extent, preserved the same duration. He tested the fact by

repeated trials, which confirmed him in the belief of its perfect exactness. This law of isochronism can be easily verified. It is only necessary to count the vibrations which take place in a given time with different amplitudes. The numbers will be found to be exactly the same. This will be found to hold good even when some of the vibrations compared are so small that they can only be observed with a telescope.

The time of vibration, then, does not depend on the amplitude, and neither does it depend on the material of which the pendulum is composed. From this last fact it follows that gravity acts in precisely the same manner on all substances. It is found, in fact, that balls of the same size, of lead, copper, ivory, &c., suspended by threads of equal length, vibrate in the same time, provided they are large enough to escape sensible retardation from the resistance of the air. This result is virtually identical with that of Galileo's experiment on the fall of bodies (§ 32), and enables us to conclude with certainty that in a vacuum these different pendulums would vibrate in rigorously equal times.

By employing balls suspended by threads of different lengths, Galileo discovered the influence of length on the time of vibration. He ascertained that when the length of the thread increases, the time of vibration increases also; not, however, in proportion to the length simply, but to its square root.

Knowing, then, that the length of the pendulum which beats seconds at Paris is about one metre ($0^m.994$), we see that a pendulum 64 metres long would make a single vibration in eight seconds. This is about the length of the pendulum employed by Foucault at the Pantheon in his celebrated experiments on the rotation of the earth.

This law of lengths experimentally discovered by Galileo, is precisely that which the formula for the simple pendulum gives; an agreement which it was natural to expect, seeing that a small ball suspended by a long string is a practical approximation to the idea of a simple pendulum. When, however, the form of the pendulum departs widely from this, the meaning to be attached to the word *length* ceases to be obvious. It becomes necessary to resort to theoretical investigations, and we shall briefly indicate the results thus obtained.

46. Equivalent Simple Pendulum.—It is demonstrated in treatises on dynamics that a compound pendulum, whatever be its form,

always keeps time with a simple pendulum of some determinate length—called the *equivalent simple pendulum*;¹ and whenever the length of a pendulum is mentioned, it is the length of the equivalent simple pendulum that is to be understood.

Any rigid body, oscillating about a fixed horizontal axis, may be regarded as a compound pendulum. That point of the axis which, in the position of equilibrium, is vertically over the centre of gravity of the body, is called the *centre of suspension*; and if we join this point to the centre of gravity, and produce the joining line downwards till its whole length is equal to that of the equivalent simple pendulum, its lower extremity is called the *centre of oscillation*. The body therefore vibrates in the same manner as if its whole mass were collected at the centre of oscillation. This point is always further from the axis than the centre of gravity, and it possesses the following remarkable property:—that if the body were made to vibrate about an axis passing through the centre of oscillation and parallel to the original axis, the time of vibration would be the same as in vibrating about the original axis. In this inverted position, the original centre of suspension becomes the new centre of oscillation, and the original centre of oscillation becomes the new centre of suspension; hence the property in question is commonly called the *convertibility of the centres of oscillation and suspension*.²

This important property, which was discovered by Huyghens, furnishes an accurate method of determining the length of the simple pendulum equivalent to a given compound pendulum. This is the principle of Kater's pendulum, which can be made to vibrate about either of two parallel knife-edges, one of which can be adjusted to any distance from the other. The pendulum is swung first upon one of these edges and then upon the other, and, if any difference is detected in the times of vibration, it is corrected by moving the adjustable edge. When the difference has been completely destroyed,

¹ Sometimes the *isochronous* simple pendulum; but it seems better to reserve this adjective and the corresponding noun *isochronism* for the use in which the latter has been employed in § 44.

² The names *centre of suspension* and *centre of oscillation* are not very appropriate, inasmuch as the properties mentioned in the text are properties of two lines rather than of two points: one of the lines being the axis about which the pendulum swings, and the other being a parallel to this axis in the plane containing it and the centre of gravity, and at a distance from it equal to the length of the equivalent simple pendulum. If we call the former the *axis of suspension* and the latter the *axis of oscillation*, we may assert that the axes of suspension and oscillation are convertible.

the distance between the two edges is the length of the equivalent simple pendulum. It is necessary, in any arrangement of this kind, that the two knife-edges should be in a plane passing through the centre of gravity; also that they should be on opposite sides of the centre of gravity and at unequal distances from it.

We see, by what precedes, that the laws of the simple pendulum are applicable to any pendulum, if we understand by its length the length of the equivalent simple pendulum, or, in other words, the distance between the axis and the centre of oscillation.

47. Determination of the Value of g .—Returning to the formula for the simple pendulum $T = \pi \sqrt{\frac{l}{g}}$, we easily deduce from it $g = \frac{\pi^2 l}{T^2}$, whence it follows that the value of g can be determined by making a pendulum vibrate and measuring T and l . T is determined by counting the number of vibrations that take place in a given time; l can be calculated, when the pendulum is of regular form, by the aid of formulæ which are given in treatises on rigid dynamics, but its value is more easily obtained by Kater's method, described above, founded on the principle of the convertibility of the centres of suspension and oscillation.

48. Variations in the Intensity of Gravity.—Pendulum observations, which have been taken in great numbers in various places, have established the result that the intensity of gravity varies over the surface of the earth. At London the value of g is 32·182; it increases in approaching the pole, and diminishes in going towards the equator. These variations, however, are not very considerable, as the following table of the values of g shows:—

Value at the equator.....	32·088.
Value in latitude 45°.....	32·171.
Value at the poles.....	32·253.

Its value for any place is approximately given by the formula:—

$$g = 32\cdot088 \left(1 + \cdot005133 \sin^2 \lambda \right) \left(1 - \frac{2h}{R} \right),$$

λ denoting the latitude of the place, h the height above the level of the sea,¹ and R the earth's radius, which is 20,900,000 feet. Local

¹ The correcting factor for elevation $\left(1 - \frac{2h}{R} \right)$ is proper to be used in determining the value of g in mid-air; for example, in the positions reached by balloons. On the summit of a mountain, or of an elevated plateau in the interior of a continent, the value of g is greater than at the same level in mid-air above the ocean, owing to the attraction of the excess of land which projects above sea-level. See a paper by Dr. Young in the *Phil. Trans.* for 1819.

peculiarities prevent the possibility of laying down any general formula with precision, and the exact value of g for any place can only be ascertained by observations on the spot.

49. Centrifugal Force.—There are two distinct causes of these differences in the intensity of gravity: the first is what is called *centrifugal force*. Consider a material point M attached to the extremity of a thread OM , and suppose an impulse to be given it in a direction perpendicular to the thread. At each instant, in virtue of inertia, the point tends to move along a tangent to the circle; but the thread prevents this movement from taking place, by drawing the point, which in its turn reacts on the thread and stretches it with a certain force which has received the name of centrifugal force. It is clear

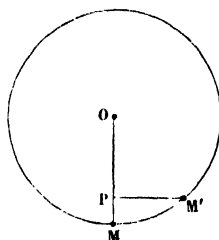


Fig. 36.—Centrifugal Force.

that we may dispense with the thread, if we suppose an attractive force equal to that which the thread would exert directed towards O . This force, which, by combining its effect with that of the initial impulse, would produce circular motion, is called *centripetal force*. It is evidently equal and opposite to centrifugal force.

The amount of the centripetal force in any given case of circular motion can be easily calculated. Let MP represent the space which the material point would describe in a certain small time t under the action of the centripetal force alone. This motion, combined with the motion due to the velocity which the particle possessed at M , gives the actual motion in the arc MM' , provided that the time considered and the distance moved in that time are so small that the directions of the centripetal forces at M and M' are sensibly parallel—in other words, provided the arc MM' is very small in comparison with the radius OM . In this case, by § 40, PM' , or rather a line equal and parallel to it drawn through M , represents the space due to the velocity which the point had at M , and is therefore equal to vt , v denoting the velocity. Now if we denote by ϕ the acceleration due to the centripetal force, this acceleration may be regarded as uniform during the time t , and we have by § 38, $MP = \frac{1}{2}\phi t^2$, whence $\phi = \frac{2MP}{t^2}$. But PM'^2 , that is $v^2 t^2$, is (by *Euclid*, iii. 35) equal to $2r \cdot MP$, r denoting the radius of the circle; therefore $\frac{2MP}{t^2}$ is equal to $\frac{v^2}{r}$. Consequently $\phi = \frac{v^2}{r}$; that is, the centripetal force upon a

particle revolving with velocity v (feet per second) in a circle of radius r (feet) is equal to a force which, acting continuously upon the same particle initially at rest, would in one second give it a velocity of $\frac{v^2}{r}$. The centripetal force upon a mass of m pounds is $\frac{mv^2}{r}$ Gaussian pound units (42), and is equal to the weight of $\frac{mv^2}{gr}$ pounds.

When the circular movement is uniform, we can give the formula for ϕ a more convenient form. In fact if we denote by T the time of revolution, and by π the ratio of circumference to diameter, we have $v = \frac{2\pi r}{T}$, and $\phi = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$.

Suppose, for example, a weight of 50 pounds attached to one end of a string 3 feet long, and revolving uniformly in a circle about the other end of the string, at the rate of 40 revolutions per minute. The time of revolution is here $\frac{3}{4}$ of a second, and the force required to be exerted by the string is equal to the weight of

$$\frac{50}{32.2} \times 4\pi^2 \times \frac{3}{4} \times 3 = 81.7 \text{ pounds};$$

and if the string be not strong enough to bear this weight it will break, and the body will fly off at a tangent.

50. Different Effects of Centrifugal Force.—Several experiments on centrifugal force are exhibited in courses of physics. For example, a rod AB (Fig. 37), passing through two ivory balls M, M', is set in rotation in a horizontal plane by means of the mechanism shown in the figure. The balls are then seen to move towards the extremities of the rod. If a spring is placed beside one of the balls M', as shown in the figure, it will be pressed with a force which is precisely equal to the centrifugal force of the ball.

The centrifugal railway (Fig. 38) shows a curious effect of this force. A carriage, starting from A, descends the inclined rails, and by following the course of the rails, which here forms a spiral convolution, to B, rises to C, and descending again on the side next A, passes on the further side of B and finally arrives at D. There is therefore an instant in the motion when the carriage is bottom upwards at the top of the convolution, and remains in contact with the rails in opposition to the force of gravity. The explanation is easy. The carriage attains a certain velocity in descending the incline which forms the first portion of its path. In virtue of its inertia it tends to move with this velocity in a tangential direc-

tion, but being compelled to follow the curved rails which lie in its way, it reacts upon them with a force whose amount is given

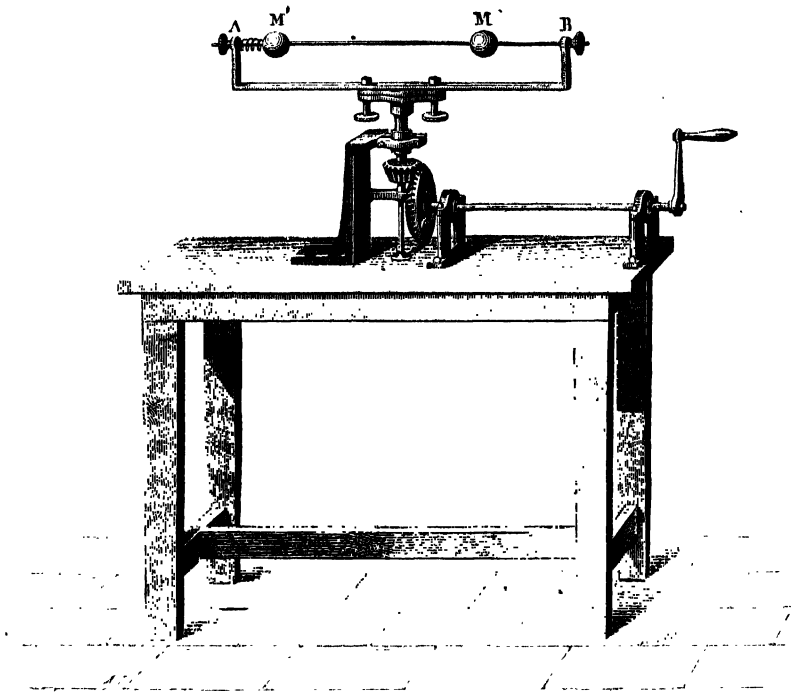


Fig. 37.—Centrifugal-force Apparatus.

by the formulæ of last section. If the point A is sufficiently elevated above C, this force will prevail over the weight of the carriage

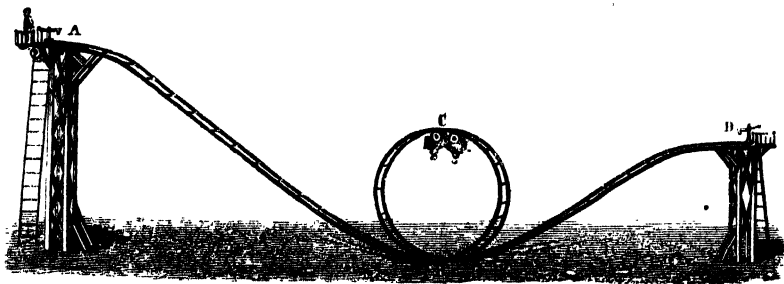
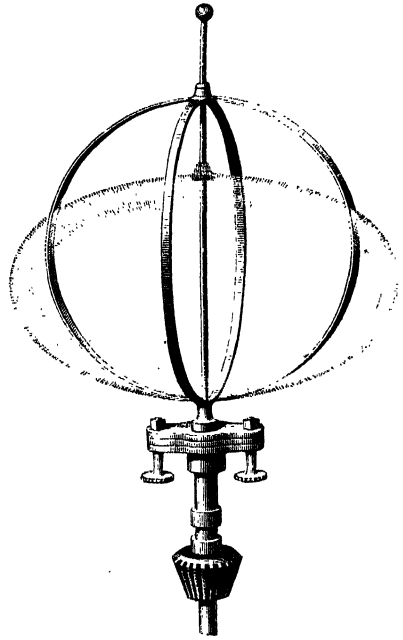


Fig. 38.—Centrifugal Railway.

and keep it pressed against the rails. It may be easily shown that to secure this result the height of A above B must be to the height of C above B in a greater ratio than 5 to 4.

If the apparatus shown in Fig. 39, consisting of a kind of sphere formed of four flexible springs, be mounted on the whirling table and made to revolve, it will be seen to become flattened, the effect being more decided as the rotation is more rapid. This result is due to centrifugal force, which gives all parts of the springs a tendency to recede from the axis of rotation, and especially those parts which are already most distant from it. This experiment may be regarded as illustrating the manner in which the earth, when in a fluid state, acquired its present form, bulging at the equator and flattened at the poles.



39. Oblateness of the Earth.

51. The influence of centrifugal force in modifying the effect of gravity is easily deduced from what precedes. The various bodies which are on the surface of the earth are retained upon it by gravity, and a certain portion of the force of gravity is expended in constraining them to move in circular paths. The modification thus produced in the apparent force and direction of gravity is the same as if a force equal and opposite to the force thus expended were compounded with the force of gravity proper; that is to say, apparent gravity is the resultant of gravity proper and centrifugal force.

At the equator, centrifugal force is directly opposed to gravity proper, and is therefore to be simply subtracted from it. Let r be the earth's radius, T the length of a sidereal day (or the time of the earth's revolution), then the intensity of centrifugal force at the equator is $\frac{4\pi^2 r}{T^2} = \frac{2\pi r \cdot 2\pi}{T^2}$. Putting for $2\pi r$, or the earth's circumference, its value in metres, viz. 40,000,000, and for T its value in seconds, 86,164, we obtain for the intensity in question the value .033. Now the intensity of gravity, expressed with reference to the metre and second as units, is about 9.8. Hence centrifugal force at

the equator is $\frac{33}{8000}$ or about $\frac{1}{250}$ of the force of gravity, and apparent gravity is less than gravity proper at the equator by this amount. The exact amount of diminution is more nearly $\frac{1}{257}$ of gravity proper; and since 289 is the square of 17, and centrifugal force varies as the square of the velocity, we see that if the rate of the earth's revolution were increased seventeenfold, bodies at the equator would lose their weight.

As we recede from the equator, the intensity of centrifugal force diminishes because the distance from the earth's axis diminishes; and, at the same time, the direction of centrifugal force, being always perpendicular to this axis, becomes less directly opposed to gravity, so that only one of its two components is subtractive. For this double reason, the effect of centrifugal force in diminishing the apparent force of gravity becomes continually less as we recede from the equator. As far then as the disturbing effect of centrifugal force is concerned, the apparent intensity of gravity should be least at the equator, and should continually increase towards the poles.

We may add that centrifugal force (except at the equator) affects not only the amount, but also the direction, of apparent gravity, since the resultant of two forces which are not directly opposed does not coincide in direction with either of them. The angle which the actual vertical (indicating the direction of apparent gravity), makes with the direction which the vertical would have if the earth were at rest, varies with the latitude; at Paris, where its value is nearly a maximum, it is between 5 and 6 minutes.

52. Universal Gravitation, Earth's Mean Density.—Terrestrial gravitation is only a particular case of universal gravitation. Newton established, by researches extending over more than twenty-five years, that the movements of the planets around the sun, and of the satellites around the planets, could be explained by assuming the existence of a mutual attraction, which, taken in conjunction with an initial impulse, determines the paths which these bodies describe. This attraction is jointly proportional to the masses of the mutually attracting bodies, and varies inversely as the square of their mutual distance. By the aid of this assumption, astronomers have succeeded not only in explaining all the diversities of motion which the solar system exhibits, but in calculating the positions of the celestial bodies at distant times, both past and future, with marvellous accuracy. The study of the movements of the heavenly

bodies, which has thus been reduced to a branch of applied mathematics, is called Physical Astronomy.

It is therefore natural to look upon terrestrial gravity as a particular case of universal gravitation, and to regard the fall of a body as a consequence of the attraction exerted on it by the different parts of the terrestrial globe. This identity is in fact established by numerous proofs. Now it is obvious from the symmetry of the component attractions, that the resultant attraction of a sphere exerted at any point of its surface must be in the direction of the radius, and must therefore be perpendicular to the surface—a result which our every-day experience confirms in the case of the earth's gravitation. Theory also shows that the attraction of an oblate spheroid is greater at the poles than at the equator. In fact, taking into account both the oblateness of the earth and centrifugal force, theory agrees with observation in indicating that apparent gravity increases in going from the equator towards the poles by an amount proportional to the square of the sine of the latitude.

The hypothesis of attraction then explains all the phenomena of gravity: we may add that the existence of attraction at the surface of the earth has been experimentally demonstrated. Maskelyne, Hutton, and Playfair, in the celebrated Schiehallien experiment, proved that the plumb-line experienced, on either side of the mountain of that name, a deviation towards the mountain; and a similar result was established by Sir Henry James at Arthur's Seat, near Edinburgh. Cavendish, by means of a very sensitive torsion-balance, showed that two large spheres of lead exercised a sensible attraction upon two small spheres, and was able to measure the amount of this attraction so precisely as to deduce, from the comparison of it with the earth's attraction, that the mean specific gravity of the earth is 5.5. Cavendish's experiment has been repeated by Reich in Germany and by Baily in this country, with nearly coincident results. The Schiehallien experiment, which was the earliest attempt to determine the earth's mean density, gave 5 for the result, and Mr. Airy's experiment at Harton Colliery gave a result exceeding 6. Baily's result, obtained by Cavendish's method, with some improvements in the details, is generally accepted as the best determination; it is 5.67.

53. Variation of Gravity with Height.—It follows from the identity of attraction and gravity that this force diminishes as we rise above

the earth's surface, but the heights with which we commonly have to do in our experiments are so small in comparison with the earth's radius that the resulting differences of force are quite inappreciable. In the case, however, of large differences of height, the variation of force can be detected; for instance, it is easy to establish experimentally that the intensity of gravity is less on the summit of a mountain than at its base. Theory shows that the attraction of a sphere upon external points is the same as if its mass were collected at the centre. Presuming this to be true in the case of the earth, it may be shown that, in ascending to any height above the earth's surface which is small in comparison with the earth's radius, the diminution in the force of gravity is twice as great in comparison with the whole force of gravity as that height is in comparison with the earth's radius.

This explains the origin of the factor $1 - \frac{2h}{R}$ in the formula of § 48.

In penetrating into the interior of the earth, the law of the variation of gravity is more complex. If the earth were homogeneous, its attraction would continually diminish in penetrating towards the centre, and would, if the earth were also truly spherical, be simply proportional to the distance from the centre. But in fact the density is greater in the central than in the superficial parts of the earth, the mean density being, as we have seen, about 5·7, while the density of the superficial beds is only about 3. This augmentation of density tends to increase the attractive force as we descend; and it will depend upon the law according to which the density varies which of these two opposite tendencies will prevail. Mr. Airy, in his experiment at Harton Colliery, found the intensity of gravity at the bottom of the mine, 1256 feet below the surface, to be greater than at the surface by about one part in 19,000. Supposing that similar results would be obtained in other places, it follows that, in penetrating the earth, gravity must go on increasing from the surface down to a certain depth, where it becomes a maximum, and from which it decreases down to the earth's centre, where it becomes zero, the equal and opposite attractions there destroying one another.

53A. Simple Vibrations.—The motion of a pendulum vibrating in a small arc, may be taken as the type of a class of motions which very extensively prevail in nature, and are of great importance in many departments of physical science. They have been called by different writers *simple vibrations*, *pendulum-like vibrations*, *sine-like vibrations*, and *simple harmonic motions*. We shall

employ the first of these designations, and define a simple vibration to be the movement of a point to and fro along one line (not necessarily straight), with acceleration simply proportional to the distance of the point from the bisection of this line. If the line of motion be curved, the distance is to be measured along the curve, and in this case the acceleration referred to is the tangential acceleration along this curve. In every case, and in all parts of the motion, the acceleration of the moving point urges it *towards* the middle point of its path, becoming zero only at the instant of passing this middle point. In the majority of cases that require to be considered, the line of motion is straight, or approximately straight.

As examples of simple vibrations, we may instance, in addition to that of a pendulum above quoted, the motion of a point on either prong of a tuning-fork, or of a point in a musical string, when vibrating so as to produce a pure note. In these cases, the force urging any point of the fork or string towards the position of equilibrium, varies directly as the distance of the point at any time from this position, and by the second law of motion acceleration is proportional to force.

In the case of the simple pendulum (§ 43), the acceleration of the heavy particle when the thread makes an angle θ with the vertical is $g \sin \theta$ (see § 53D), which when the arc of vibration is small may be taken as equal to $g\theta$, that is, $g \frac{x}{l}$, x denoting the length of arc measured from lowest point, and l the length of the string. It is therefore proportional to x , the distance of the particle from the position of equilibrium.

53B. When simple vibration is executed in a straight line, it corresponds to the projection of uniform circular motion (Fig. 39A); that is to say, if a point P move with uniform velocity round the circumference of a circle, and if Pp be the perpendicular let fall from P upon a fixed straight line in the plane of the circle, then p , the foot of this perpendicular, will execute simple vibration.

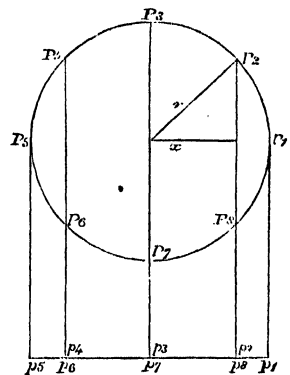


Fig. 39A. Projection of Circular Motion.

To prove this proposition, remark that by § 49 the point P is to be regarded as constantly falling away from a tangent in obedience to

acceleration directed towards the centre of the circle, the amount of the acceleration being $\frac{4\pi^2}{T^2} \cdot r$, where r denotes the radius of the circle. This acceleration, being directed from P towards the centre of the circle, may be resolved into two components, one parallel to the line of motion of p , and the other perpendicular to it. The former of these, which is obviously the acceleration of p , is $\frac{x}{r}$ of the whole acceleration, x denoting the distance of p from its mean position, that is, from the foot of a perpendicular let fall from the centre of the circle. The acceleration of p is therefore $\frac{4\pi^2}{T^2} x$, and is proportional to x .

53c. We may hence prove that simple vibrations are isochronous. For if the acceleration be expressed by μx , x denoting displacement from mean position, and μ any constant, we see, by the preceding paragraph, that the vibration keeps time with an imaginary point moving uniformly round a circle, a complete vibration being performed in the same period as a complete revolution; and if T denote this period, we have $\mu = \frac{4\pi^2}{T^2}$; whence $T = \frac{2\pi}{\sqrt{\mu}}$, an expression which is independent of the amplitude of vibration.

Applying this result to a pendulum vibrating in a small arc, we have $\mu = \frac{g}{l}$ (see § 53A); hence $T = \frac{2\pi}{\sqrt{\mu}} = 2\pi \sqrt{\frac{l}{g}}$, which is the time of a complete (or double) vibration.¹

To understand the reason of the isochronism of simple vibrations, we have only to remark that, if the amplitude be changed, the velocity at corresponding points (that is, points whose distances from the middle point are the same fractions of the amplitudes) will be changed in the same ratio. For example, compare two simple vibrations in which the values of μ are the same, but let the amplitude of

¹ The mathematical reader will remark that our definition of simple vibration corresponds to the differential equation $\frac{d^2x}{dt^2} = -\mu x$, and that the property proved in § 53B corresponds to the solution of this equation, which is $x = a \cos (t\sqrt{\mu} - c)$, a and c being arbitrary constants, the former called the *amplitude* and the latter the *epoch* of the motion. The quantity $(t\sqrt{\mu} - c)$ is called the argument, and it is obvious that x and its successive differential coefficients will all remain unaltered if the argument be increased by 2π , that is, if t be increased by any multiple of $\frac{2\pi}{\sqrt{\mu}}$. Hence the motion always repeats itself after the interval $\frac{2\pi}{\sqrt{\mu}}$, which is therefore the *period* of a complete vibration.

one be double that of the other. Then if we divide the paths of both into the same number of small equal parts, these parts will be twice as great for the one as for the other; but if we suppose the two points to start simultaneously from their extreme positions, the one will constantly be moving twice as fast as the other. The number of parts described in any given time will therefore be the same for both.

In the case of vibrations which are not simple, it is easy to see (from comparison with simple vibration) that if the acceleration increases in a greater ratio than the distance from the mean position, the period of vibration will be shortened by increasing the amplitude; but if the acceleration increases in a less ratio than the distance, as in the case of the common pendulum vibrating in an arc of moderate extent, the period is increased by increasing the amplitude.

53 D. Cycloidal Pendulum.—We saw in § 33 that the effective component of gravity upon a particle resting on a smooth inclined plane was proportional to the sine of the inclination. The acceleration of a particle so situated is in fact $g \sin \alpha$, if α denote the inclination of the plane. When a particle is guided along a smooth curve its acceleration is expressed by the same formula, α now denoting the inclination of the curve at any point to the horizon. This inclination varies from point to point of the curve, so that the acceleration $g \sin \alpha$ is no longer a constant quantity. The motion of a common pendulum corresponds to the motion of a particle which is guided to move in a circular arc; and if x denote distance from the lowest point, measured along the arc, and r the radius of the circle (or the length of the pendulum), the acceleration at any point is $g \sin \frac{x}{r}$.

This is sensibly proportional to x so long as x is a small fraction of r ; but in general it is not proportional to x , and hence the vibrations are not in general isochronous.

To obtain strictly isochronous vibrations we must substitute for the circular arc a curve which possesses the property of having an inclination whose sine is simply proportional to distance measured along the curve from the lowest point. The curve which possesses this property is the cycloid. It is the curve which is traced by a point in the circumference of a circle which rolls along a straight line. The cycloidal pendulum is constructed by suspending an ivory ball or some other small heavy body by a thread between two cheeks (Fig. 39B), on which the thread winds as the ball swings to

either side. The cheeks must themselves be the two halves of a cycloid whose length is double that of the thread, so that each

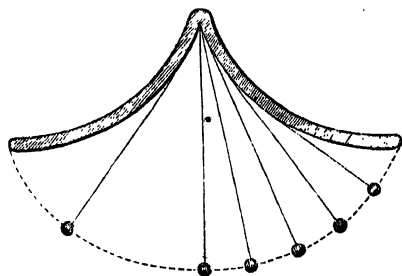


Fig. 39b.—Cycloidal Pendulum.

cheek has the same length as the thread. It can be demonstrated¹ that under these circumstances the path of the ball will be a cycloid identical with that to which the cheeks belong. Neglecting friction and the rigidity of the thread, the acceleration in this case is proportional to distance measured along the cycloid from its lowest point, and hence, by last section, the time of vibra-

tion will be strictly the same for large as for small amplitudes. It will, in fact, be the same as that of a simple pendulum having the same length as the cycloidal pendulum and vibrating in a small arc.

Attempts have been made to adapt the cycloidal pendulum to clocks, but it has been found that, owing to the greater amount of friction, its rate was less regular than that of the common pendulum. It may be remarked, that the spring by which pendulums are often suspended has the effect of guiding the pendulum bob in a curve which is approximately cycloidal, and thus of diminishing the irregularity of rate resulting from differences of amplitude.

53 E. Centre of Mass, or Centre of Inertia.—The point which has been mentioned in Chapter iv., under the name of centre of gravity, possesses important properties besides those which depend upon its being the point through which the resultant force of gravity passes.

It may be demonstrated by geometry that if a body be divided into a number of equal elements (equal in mass, not necessarily in size), each of them being so small in all its dimensions that it may be treated as a material point, there is a certain point in the body such that its distance from any plane whatever is equal to the mean distance of all the elements from the same plane—that is, to the sum of all the distances divided by the number of elements.² This point is called the *centre of mass*.

¹ Since the evolute of the cycloid is an equal cycloid.

² If the plane cuts the body, distances on one side of the plane must be reckoned positive and on the other negative, and the sum in question must be the algebraical sum.

If a set of equal and parallel forces act one on each element in the same direction, their resultant will pass through the centre of mass. Inasmuch then as the force of gravity upon a body is made up of such a set of equal and parallel forces, the centre of mass is also the *centre of gravity*.

Again, if a body at rest be set in motion in such a manner that all its points move in parallel straight lines with equal velocities, the resistance which inertia opposes is composed of a set of such equal and parallel resistances; its resultant therefore passes through the centre of mass, which is hence called the *centre of inertia*. It is evident that the force requisite for producing such motion in a body must be equivalent to a single force applied at this point, and the same remark applies to the force necessary for destroying such motion and bringing the body to rest.

Conversely, if a force be applied to a body at its centre of mass, or in a line passing through the centre of mass, the body will be set in motion in such a way that all its points will have equal and parallel velocities, their common direction being parallel to the line of action of the force.

A force applied to a free body in a direction not passing through the centre of mass will produce movement of the centre of mass combined with rotation of the body about the centre of mass. Of these two components of motion, the former will be the same as would be produced by the given force if it acted in a direction passing through the centre of mass; and the latter—the rotation—will be the same as if the centre of mass were fixed.

A couple applied to a free body will produce rotation of the body about the centre of mass, but will not produce any motion of the centre of mass.

When a body moves so that all its points are at every instant travelling in the same direction (that is in parallel directions and towards the same parts) and with equal velocities, it is said to have a movement of translation. All straight lines in a body so moving remain always parallel to their original positions, and conversely; hence this property may be taken as the definition of movement of translation. Every possible motion of a rigid body can be resolved into motion of translation accompanied by motion of rotation, and the resolution can always be so effected that the axis of rotation at any instant shall be parallel to the direction of the movement of translation. It is always possible, and is generally convenient, to

regard the motion of a rigid body under the action of any forces as compounded of a motion of translation of the body as a whole, and a rotation of the body about an axis passing through its centre of mass.

53 F. When two or more bodies, or parts of the same body, are free to move, it is impossible for any action exerted between them to alter the motion of their common centre of mass. It is also impossible for such action to alter the total angular momentum about the centre of mass. For example, when an animal is either jumping or falling, no movement that it can make in mid-air without touching other bodies can either alter the motion of its centre of gravity, or cause part of its body to rotate in one direction without causing the remainder to rotate in the opposite direction.

The recoil of fire-arms depends on the same principle. Whatever force the gases which are produced by the explosion of the powder exert in propelling themselves and the ball forwards, they must always exert the same force for the same time in urging the gun backwards. If a shell explodes at an elevation in the air, then, neglecting the effect of the wind, the common centre of gravity of the fragments of the shell and the products of explosion will describe the same path and with the same velocity which the centre of gravity of the shell would have had if there had been no explosion.

This principle is of great importance in the movement of the heavenly bodies. For example, neglecting any general movement which the solar system as a whole may have in space, we are entitled to assert that in whatever direction the common centre of gravity of the planets may be moving at any time, the centre of gravity of the sun must be moving in a parallel and opposite direction; inasmuch as the centre of gravity of the whole system, consisting of sun and planets, remains always at rest.

53 G. Moment of Inertia.—When a body is capable of turning about a definite axis, its inertia opposes resistance to any force which may be applied to set it in rotation, and, if it has once been set in rotation, its inertia gives it a tendency to continue rotating with constant velocity, so that it can only be brought to rest by the action of opposing force.

The power of a force as regards its tendency to produce rotation about an axis is called the *moment of the force about the axis*, and is measured by the *product of the force and the arm at which it acts*. If the body is acted on by more forces than one, the sum of

the moments of the several forces about the axis is the measure of the total tendency to produce rotation, and is called the total moment of all the forces. It is to be understood that if some of the forces tend to make the body turn in one direction and others in the opposite direction, the moments of the one set must be reckoned positive and of the other negative, and the sum in question must be the algebraical sum.

On the other hand, the resistance which the inertia of the rotating body opposes to the action of forces tending to accelerate or retard its rotation is called its *moment of inertia*. The rate at which the angular velocity changes is equal to the total moment of the forces divided by the moment of inertia of the body.

The moment of inertia of a body about an axis is the sum of all the terms which are obtained by multiplying each element by the square of its distance from the axis.

The *angular momentum* of a rotating body is a name given to the product of the moment of inertia and the angular velocity. Equal forces acting at equal arms for the same time upon different bodies produce equal angular momenta.

The *energy of rotation* of a rotating body is half the product of its moment of inertia and the square of its angular velocity. Equal amounts of work spent upon different bodies in producing rotation yield equal amounts of energy of rotation.

These ideas may be illustrated by a reference to the use of fly-wheels in machinery. A fly-wheel is a wheel which, by means of its inertia, acts as an equalizer of the motion of the machine to which it is attached, resisting, to an extent measured by its moment of inertia, all sudden changes of velocity. It is chiefly employed in cases where either the driving power or the resistance to be overcome is liable to rapid alternations of magnitude. When the power is in excess of the resistance the motion of the fly-wheel is accelerated, and the energy thus accumulated is given out again when the resistance is in excess of the power, the inertia of the fly-wheel then assisting to overcome the resistance, while at the same time the velocity of the wheel is diminished.

Fly-wheels are always made with heavy rims, the rest of the wheel being usually as light as is compatible with the requisite strength. This arrangement is adopted with the view of obtaining the greatest possible moment of inertia; for if all the matter of the wheel were collected at its rim, the moment of inertia would be equal to the mass multiplied by the square of the radius.

53 H. Centre of Percussion.—We have already seen that when a force acts upon a rigid body in a direction not passing through the centre of mass, it tends to produce a motion consisting partly of translation and partly of rotation of the body about the centre of mass. This principle remains true when the force is applied in the shape of a blow, and may easily be tested experimentally in a rough way by suspending a straight rod by a long string attached to one end and striking it with a hammer in different points. If the rod be struck in a horizontal direction near its top, its bottom will at the instant of the blow move in the opposite direction, and if it be struck near the bottom the top will fly back. In each case there is some intermediate line at right angles to the direction of the blow, which neither moves forwards nor backwards at the instant of the blow, while points on opposite sides of it move in opposite directions. With reference to this line, regarded as an instantaneous axis of rotation, the point at which the body was struck is called the centre of percussion. It admits of proof that the centre of percussion with respect to any axis is the same as the centre of oscillation.

When a body is suspended so that it can rotate about an axis, if we desire to strike it without jarring the axis, it is necessary that the blow should be administered at the centre of percussion, and this remark is equally true if the body in question be the striking instead of the struck body. For example, the proper point of a bat for striking a ball so as not to jar the hands is the centre of percussion of the bat with respect to an axis passing through the hands.

53 I. Momentum, Energy of Motion.—The product of the mass and velocity of a body is called the momentum of the body. If equal forces act upon unequal masses *for the same time*, the momenta generated are equal. This principle applies to the recoil of fire-arms, supposing the gun to be free to move.

On the other hand, if equal forces act upon unequal masses originally at rest, *through equal distances* (and therefore do equal amounts of work upon them), the momenta generated will be unequal; the greater mass will receive the greater momentum. Equal products will however be obtained in this case, if we multiply each mass by the square of its velocity. In the case of a falling body, we have seen that the velocity acquired in falling through a height s is $v = \sqrt{2gs}$, whence $\frac{1}{2}v^2 = gs$, and if the mass of the body (in lbs.) is m , we have $\frac{1}{2}mv^2 = gms$. Now, the force which produces the descent is the weight of m lbs., which is equivalent to gm Gaussian

units of force, and as the space through which the force works is s , the work done is gms , which is the second member of the above equation, and is equal to $\frac{1}{2}mv^2$. We see, then, that in this case the work done, expressed in Gaussian units of work (of which a foot-pound contains g), is equal to half the product of the mass (in pounds) and the square of the velocity. This principle is perfectly general, and may be extended to bodies already in motion as well as to bodies initially at rest by substituting for " $\frac{1}{2}mv^2$," "the change produced in the value of $\frac{1}{2}mv^2$." Conversely, since the height to which a body will rise when thrown upwards with a given velocity is the same as the height from which it must fall to acquire that velocity, it follows from the foregoing equations that the value of $\frac{1}{2}mv^2$ at the commencement of the ascent is equal to the work which gravity would do upon the body during its descent from the height to which it rises to the point from which its ascent commenced; and if we denote the product of force and distance moved in the case when the direction of the motion is opposite to that of the force, by the name *negative work*, we may assert that the diminution which occurs in the value of $\frac{1}{2}mv^2$ during the whole ascent or during any part of it is equal to the negative work done upon the body by gravity during that part of the motion.

It is in this sense that work and motion are said to be convertible, and the product $\frac{1}{2}mv^2$, whose changes of value are always equal to the work done upon the body, is called the *energy of motion*, or the *kinetic energy* of the body. This equality subsists not only for the case of gravity, but for all forces whatever: we may assert universally (neglecting for the present the effects of friction and molecular changes), that when a body of mass m moves at one time with a velocity v_1 , and at a subsequent time with velocity v_2 , the whole amount of work done upon the body during the interval (the algebraic sum being taken if any of the work is negative) is equal to $\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$.

The product $\frac{1}{2}mv^2$ has sometimes been called the *accumulated work* in a body, or the work stored up in the body, inasmuch as a moving body is able in virtue of its motion to overcome resistance through such a distance that the work done (or product of resistance and distance through which it is overcome) will be equal to $\frac{1}{2}mv^2$. We have seen one example of this in the case of a body thrown upwards, which overcomes the resistance mg of gravity through a height s such that $mgs = \frac{1}{2}mv^2$.

53J. Energy of Position, or Potential Energy.—We have now to introduce a new idea, which is of comparatively recent origin, and plays an important part in modern dynamics. When a body of mass m is at the height s above the ground, which we will suppose level, we can cause it to acquire a certain velocity v such that $\frac{1}{2}mv^2 = mgs$ by simply allowing it to fall to the earth. The position of a body in this instance confers the power to obtain motion, and therefore kinetic energy; and as we have just seen, kinetic energy can be made to yield work. A body in an elevated position may therefore be regarded as a reservoir of work: the water in a mill-dam is, in fact, a case in point; and for this reason such a body is said to possess *energy of position*, or, as it is more commonly called, *potential energy*. In contradistinction from this latter name, the energy which a moving body possesses in virtue of its motion is sometimes called *actual energy*.

It should be remarked that energy of position is essentially relative, depending on the position of one body with reference to one or more others. In the case just considered the other body is the earth. In order to be philosophically correct in our language, we should speak not of the potential energy of a body, but rather of the potential energy of two or more bodies with reference to each other in a given relative position; or more briefly, of the potential energy of a certain relative position of the bodies.

It must also be remarked that while we can speak with precision of the *difference* between the potential energies of two specified positions, we cannot in strictness assign a definite value to the potential energy of one specified position unless we know the limits to the possible motion of the bodies in obedience to their mutual forces. For example, in the case just considered—that of a body at a certain height above level ground—the present position of the body is compared with that which it will occupy when it lies upon the ground. But a shaft might be sunk in the ground, and with reference to the bottom of this shaft a body lying on the surface of the ground would possess a certain amount of potential energy, which must be added to that above considered to obtain the potential energy of the body in its first position as compared with the position which it would occupy when lying at the bottom of the shaft.

Whenever motion takes place in obedience to natural forces, the increase or diminution of potential energy which takes place in passing from one position to another is always exactly compensated

by an opposite change in the total amount of kinetic energy; from which it follows that the sum of potential and kinetic energies remains unchanged. Whenever kinetic energy is increased at the expense of potential energy, the forces concerned do an amount of positive work equal to the amount by which the former is increased or the latter diminished. On the other hand, whenever potential energy is increased at the expense of kinetic energy, the forces do negative work equal in absolute value to the energy thus transferred. Instances of the former kind of transfer are furnished by the motion of a falling body and the motion of a planet from aphelion to perihelion; instances of the latter kind are furnished by the motion of a body thrown upwards, and the motion of a planet from perihelion to aphelion.

53 K. Effect of Friction upon Transformation of Energy.—Thus far we have been supposing that frictional resistances are neglected. Friction, in fact, causes an apparent loss of energy, but this loss is accompanied by a generation of heat which is itself a form of energy, and a definite amount of heat is produced by each unit of work thus apparently wasted. Conversely, whenever heat is employed as a motive power (in the steam-engine, for example), a quantity of heat is destroyed equivalent, on the same scale, to the work produced.

Another kind of energy is developed when friction is employed as a means of generating electricity. In this case the potential energy of electrical attraction which is called into existence is the precise equivalent of the work spent in producing it.

Similar principles apply to all other cases in which energy is apparently destroyed. *Any particular form of energy may be destroyed, but only on condition of an equivalent amount of energy in some other shape coming into existence.* The whole amount of energy in the universe cannot undergo either increase or diminution. This great natural law is called the *principle of the conservation of energy*.

The exact nature of the various forms of molecular energy, such as heat, light, electricity, magnetism, and chemical affinity, is not at present known, but we run little risk of error in affirming that they all consist either of peculiar kinds of molecular motion or of peculiar arrangements of molecules as regards relative position. They must therefore fall under one or other of the two heads "energy of position" and "energy of motion."

CHAPTER VII.

THE BALANCE.

54. The object of the balance is the measurement of the weights of bodies. It consists essentially of a rigid lever AB called the beam, movable about an axis O at the centre of its length. This axis rests

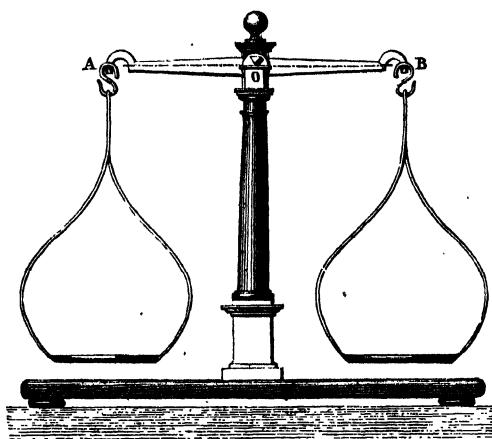


Fig. 40.—Balance.

upon two planes, and as it is a little above the centre of gravity, the beam takes a position of stable equilibrium. An index needle attached to the beam traverses a graduated arc, and indicates the position of equilibrium of the beam by pointing to zero.

This equilibrium will not be disturbed if we suspend from the extremities of the beam two scale-pans of the same substance, form, and dimensions. Neither will it be disturbed if in these scale-pans we place bodies of equal weight. And conversely, if two bodies placed in the two scale-pans equilibrate each other, their weights are

equal. This, then, is the principle of the well-known use of the balance.

55. Correctness of the Balance.—It is necessary to the validity of the preceding reasoning that the scale-pans should be suspended at exactly the same distance from the axis, or, in other words, that the arms of the balance should be rigorously equal in length. This is known to be the case if the needle points to zero both when the scale-pans are empty and when they are loaded with two bodies of equal weight. If we have not two weights exactly equal, it is sufficient to place any body whatever in one of the scale-pans, and equilibrate it by placing so much matter in the other scale as will bring the index to zero; if we then interchange the contents of the two scale-pans, the needle should still point to zero. If it does not, the reason is that the arms are not of equal length. Easy as it is, however, to make the arms of approximately equal length, it is exceedingly difficult to make them rigorously equal; and accordingly, whenever great accuracy is required, the method of double weighing is employed, which enables us to obtain the exact weight, even when the arms of the balance are slightly unequal. This method consists in first counterpoising the body to be weighed with any substance—as, for example, shot or sand—and then replacing the body by weights sufficient to produce equilibrium. It is evident that these latter, as they produce the same effect as the body under the same circumstances, must have the same weight.

56. Sensibility of the Balance.—A balance is said to be more or less sensitive when the beam, supposed to be originally horizontal, is more or less inclined for a given difference of weights. The sensibility depends, in the first place, on the friction of the axis against its supports. In carefully constructed balances this axis is formed by the edge of a triangular prism of very hard steel, called a knife-edge, which rests upon a plane of steel or agate. In this way, as rotation takes place about a very fine axis, and as, besides, the materials employed are very hard, the friction is rendered exceedingly small.

Supposing friction to be eliminated, the sensibility of the balance depends upon the weight of the beam, its length, and the distance between its centre of gravity and the axis of suspension. We shall proceed to investigate the influence of these different elements.

Let *A* and *B* be the points from which the scale-pans are suspended, *O* the axis about which the beam turns, and *G* the centre of

gravity of the beam. If when the scale-pans are loaded with equal weights, we put into one of them an excess of weight p , the beam

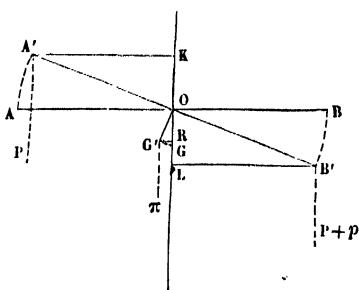


Fig. 41.

will become inclined, and will take a position such as $A'B'$, turning through an angle which we will call α , and which is easily calculated.

In fact, let the two forces P and $P + p$ act at A' and B' respectively, where P denotes the less of the two weights, including the weight of the pan. Then the two forces P destroy each other in consequence of the resistance of the axis O ; there is left

only the force p applied at B' , and the weight π of the beam applied at G' , the new position of the centre of gravity. These two forces are parallel, and are in equilibrium about the axis O , that is, their resultant passes through the point O . The distances of the points of application of the forces from a vertical through O are therefore inversely, proportional to the forces themselves, which gives the relation

$$\pi \cdot G'R = p \cdot B'L.$$

But if we call half the length of the beam l , and the distance OG r , we have

$$G'R = r \sin \alpha, \quad B'L = l \cos \alpha,$$

whence $\pi r \sin \alpha = pl \cos \alpha$, and consequently

$$\tan \alpha = \frac{pl}{\pi r}. \quad (a)$$

The formula (a) contains the entire theory of the sensibility of the balance when properly constructed. We see, in the first place, that $\tan \alpha$ increases with the excess of weight p , which was evident beforehand. We see also that the sensibility increases as l increases and as π diminishes, or, in other words, as the beam becomes longer and lighter. At the same time it is obviously desirable that, under the action of the weights employed, the beam should be stiff enough to undergo no sensible change of shape. The problem of the balance then consists in constructing a beam of the greatest possible length and lightness, which shall be capable of supporting the action of given forces without bending.

Förtin, whose balances are justly esteemed, employed for his beams bars of steel placed edgewise; he thus obtained great rigidity, but certainly not all the lightness possible. At present the makers of balances employ in preference beams of copper or steel made in the form of a frame, as shown in Fig. 42. They generally give them the shape of a very elongated lozenge, the sides of which are connected by bars variously arranged. The determination of the best shape is, in fact, a special problem, and is an application on a small scale of that principle of applied mechanics which teaches us that hollow pieces have greater resisting power in proportion to their weight than solid pieces, and consequently, for equal resisting power, the former are lighter than the latter. Aluminium, which with a rigidity nearly equal to that of copper, has less than one-fourth of its density, seems naturally marked out as adapted to the construction of beams. It has as yet, however, been little used.

The formula (a) shows us, in the second place, that the sensibility increases as r diminishes; that is, as the centre of gravity approaches the centre of suspension. These two points, however, must not coincide, for in that case for any excess of weight, however small, the beam would deviate from the horizontal as far as the mechanism would permit, and would afford no indication of approach to equality in the weights. With equal weights it would remain in equilibrium in any position. In virtue of possessing this last property, such a balance is called *indifferent*. Practically the distance between the centre of gravity and the point of suspension must not be less than a certain amount depending on the use for which the balance is designed. The proper distance is determined by observing what difference of weights corresponds to a division of the graduated arc along which the needle moves. If, for example, there are 20 divisions on each side of zero, and if 2 milligrammes are necessary for the total displacement of the needle, each division will correspond to an excess of weight of $\frac{2}{20}$ or $\frac{1}{10}$ of a milligramme. That this may be the case we must evidently have a suitable value of r , and the maker is enabled to regulate this value with precision by means of the screw which is shown in the figure above the beam, and which enables him slightly to vary the position of the centre of gravity.

In the above analysis we have supposed that the three points of suspension of the beam and of the two scale-pans are in one straight line; in which case the value of $\tan \alpha$ does not include P , that is, the sensibility is independent of the weight in the pans. This follows

from the fact that the resultant of the two forces P passes through O , and is thus destroyed, because the axis is fixed. This would not be the case if, for example, the points of suspension of the pans were above that of the beam; in this case the point of application of the common load is above the point O , and, when the beam is inclined, acts in the same direction as the excess of weight; whence the sensibility increases with the load up to a certain limit, beyond which the equilibrium becomes unstable.¹ On the other hand, when the points of suspension of the pans are below that of the beam, the sensibility increases as the load diminishes, and, as the centre of gravity of the beam may in this case be above the axis, equilibrium may become unstable when the load is less than a certain amount. This variation of the sensibility with the load is a serious disadvantage; for, as we have just shown, the displacement of the needle is used as the means of estimating weights, and for this purpose we must have the same displacement corresponding to the same excess of weight. If we wish to employ either of the two above arrangements, we should weigh with a constant load. The method of doing so, which constitutes a kind of double weighing, consists in retaining in one of the pans a weight equal to the maximum load. In the other pan is placed the same load subdivided into a number of marked weights. When the body to be weighed is placed in this latter pan, we must, in order to maintain equilibrium, remove a certain number of weights, which evidently represent the weight of the body.

We may also remark that, strictly speaking, the sensibility always depends upon the load, which necessarily produces a variation in the friction of the axis of suspension. Besides, it follows from the nature of bodies that there is no system that does not yield somewhat even to the most feeble action. For these reasons, there is a decided advantage in operating with constant load.

57. Suspension of the Scale-pans.—A fundamental condition of the correctness of the balance is, that the weight of each pan and of the load which it contains should always act exactly at the same point, and therefore at the same distance from the axis of suspension. This important result is attained by different methods. The arrange-

¹ This is an illustration of the general principle, applicable to a great variety of philosophical apparatus, that a maximum of sensibility involves a minimum of stability; that is, a very near approach to instability. This near approach is usually indicated by excessive slowness in the oscillations which take place about the position of equilibrium.

ment represented in Fig. 42 is one of the most effectual. At the extremities of the beam are two knife-edges, parallel to the axis of rotation, and facing upwards. On these knife-edges rests, by a hard plane surface of agate or steel, a stirrup, the front of which has been taken away in the figure. On the lower part of the stirrup rests another knife-edge, at right angles to the former, the

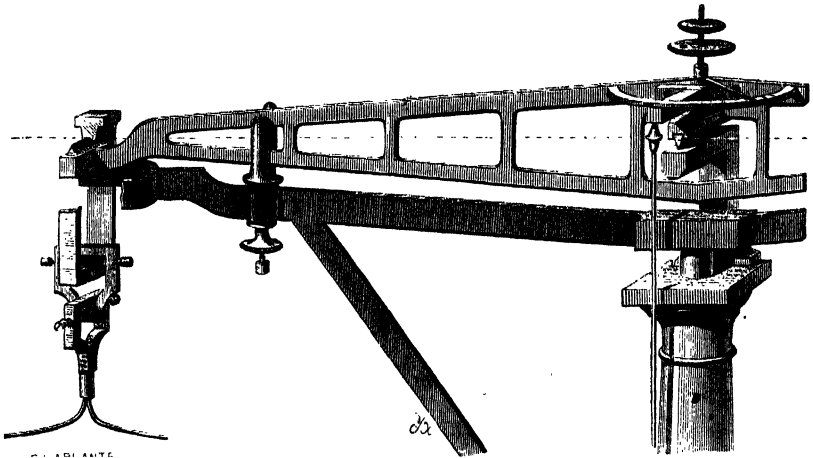


Fig. 42.—Beam of Balance.

two being together equivalent to a universal joint supporting the scale-pan and its contents. By this arrangement, whatever may be the position of the weights, their action is always reduced to a vertical force acting on the upper knife-edge.

Fig. 43 represents a balance of great delicacy, with the glass case that contains it. At the bottom is seen the extremity of a lever, which enables us to raise the beam, and thus avoid wearing the knife-edge when not in use. At the top may be remarked an arrangement employed by some makers, consisting of a horizontal graduated circle, on which a small metallic index can be made to travel; its different displacements, whose value can be determined once for all, are used for the final adjustment to produce exact equilibrium.

58. Densities.—If we weigh equal volumes of different bodies in nature, we find that they have different weights. Thus, a litre of water weighs 1 kilogramme, a litre of mercury weighs .136 kilos, a litre of alcohol 0.79 kilos. This we express by saying that different bodies have different densities. It is evidently important to

know the density of the different substances with which we have to deal; for this is, in fact, a fundamental element of their physical constitution. Attempts have therefore been made to form a list containing the weight of a given volume of each of the substances

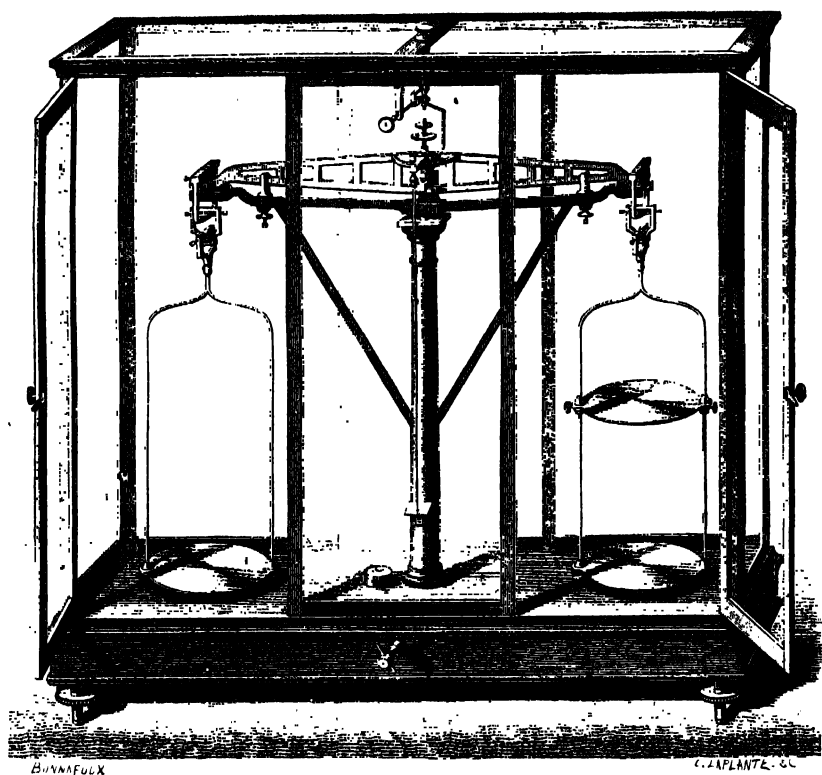


Fig. 43.—Balance for Purposes of Accuracy.

known in nature. The simplest way of determining the density of a substance is to weigh a certain known volume of it, and to divide the weight obtained by the volume; we shall thus obtain the weight of unit volume.

The same object is attained indirectly by determining the *specific gravity* of the substance; that is to say, the ratio of its density to that of some standard substance whose density is known. The standard substance commonly employed for this comparison is distilled water at the temperature of maximum density (about 39° F. Fahrenheit). The weight of a cubic foot of such water is 62.425 lbs. avoirdupois; hence the specific gravity of a substance multiplied

by $\cdot 62\cdot 425$ is the weight of a cubic foot of the substance in lbs., which may be called the *density of the substance in lbs. per cubic foot*.

In the metrical system the conversion of specific gravities into densities is much simpler; for since the gramme, kilogramme, and tonne are the weights respectively of a cubic centimetre, cubic decimetre, and cubic metre of distilled water at the temperature of maximum density, it follows that the same number which denotes the specific gravity of a substance, also denotes the weight of a cubic centimetre of the substance in grammes, the weight of a cubic decimetre in kilogrammes, or the weight of a cubic metre in tonnes. In other words, the specific gravity is equal to the density, whether the latter be expressed in grammes per cubic centimetre, in kilogrammes per cubic decimetre, or in tonnes per cubic metre.

If V denote the volume of a body, P its weight, and D its density, or weight per unit volume, we have

$$D = \frac{P}{V}, \quad P = VD, \quad V = \frac{P}{D};$$

so that if any two of these three elements are given, the third can be computed.

EXAMPLE I.—What is the weight of a mass of granite of 84 cubic metres, the density of granite being $2\cdot 75$? The formula gives $P = 84 \times 2\cdot 75 = 231$ tonnes.

EXAMPLE II.—What is the volume of 1000 kilos. of mercury, the density of mercury being $13\cdot 6$?

$V = \frac{1000}{13\cdot 6} = 73\cdot 5$ litres, since the litre is equal to the cubic decimetre.

59. Experimental Determination of Densities.—The following is one of the simplest methods for the practical determination of densities. We begin by weighing the body. Suppose, for example, its weight to be 10 grammes. It is then placed upon one of the scale-pans of a balance, along with a flask with a wide neck, of a form such as is shown in Fig. 44, and exactly full of water; these are balanced in the other scale by weights of any kind. The body is then introduced into the flask, and evidently displaces a volume of water equal to its own volume. If now we close the flask, taking care that it is filled to the same level as before, wipe it, and put it back on the scale-pan, there will not be equilibrium. In order to re-establish equilibrium we must add, suppose, $2\cdot 5$ grammes; this is the weight

of a volume of water equal to that of the body; the specific gravity of the latter then is $\frac{10}{2.5} = 4$. When we wish to determine the density of a liquid, we employ a flask (Fig. 45), the upper extremity of which terminates in a narrow tube on which there is a mark. After

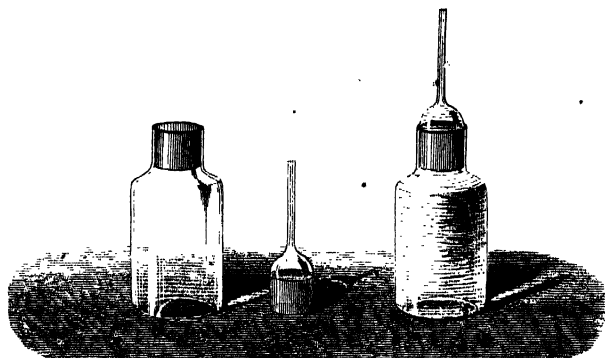


Fig. 44.—Specific Gravity Flask for Solids.



Fig. 45.—Specific Gravity Flask for Liquids.

having weighed the flask empty, we fill it successively with the liquid and with water, as far as the mark. This gives us the weights of equal volumes of water and of the liquid whose density is to be determined; and the quotient obtained by dividing the former by the latter is the specific gravity of the liquid.

There is generally some difficulty in filling the flask, on account of the very small diameter of the tube. The usual way is to put a little liquid into the cup at the upper extremity of the tube, and press it with the stopper; this pressure is generally sufficient to make the liquid pass into the flask. These two methods for determining densities are susceptible of great precision.

Other methods will be described in Chapter x.

In the following table we give the specific gravities of some liquids and solids.

LIQUIDS, at Temperature of Freezing Water.

Water, sea, ordinary,	1.026	Oil, linseed,940
Alcohol, pure,791	„ olive,915
„ proof spirit,916	„ whale,923
Ether,716	„ of turpentine,870
Mercury,	13.596	Blood, human,	1.055
Naphtha,848	Milk, of cow,	1.03

SOLIDS.

Brass, cast,	7·8 to 8·4	Ice,	·92
„ wire,	8·54	Basalt,	3·00
Bronze,	8·4	Brick,	·2 to 2·17
Copper, cast,	8·6	Brickwork,	1·8
„ sheet,	8·8	Chalk,	1·8 to 2·8
• „ hammered,	8·9	Clay,	1·92
Gold,	19 to 19·6	Glass, crown,	2·5
Iron, cast,	6·95 to 7·3	„ flint,	3·0
„ wrought,	7·6 to 7·8	Quartz (rock-crystal),	2·65
Lead,	11·4	Sand,	1·42
Platinum,	21 to 22	Fir, spruce,	·48 to ·7
Silver,	10·5	Oak, European,	·69 to ·99
Steel,	7·8 to 7·9	Lignum-vitæ,	·65 to 1·33
Tin,	7·3 to 7·5	Sulphur, octahedral,	2·05
Zinc,	6·8 to 7·2	„ prismatic,	1·98

The unit of specific gravity is the specific gravity of pure water at the temperature of maximum density (39°·1 Fahr.)

The weight of a cubic foot of any substance is equal to 62·425 lbs. avoirdupois, multiplied by its specific gravity.

The weight of a cubic centimetre of any substance, in grammes, is equal to its specific gravity.

The weight of a litre (or cubic decimetre) of any substance, in kilogrammes, is equal to its specific gravity.

The weight of a gallon of any liquid, in lbs. avoirdupois, is equal to its specific gravity multiplied by 10.

CHAPTER VIII.

HYDROSTATICS.

60. Transmission of Pressure.—The peculiar constitution of liquids (§ 19) involves some important properties as regards pressure and the transmission of pressure. If we suppose that in a vessel A (Fig. 46), full of liquid, an opening is made at P, and a certain pressure applied

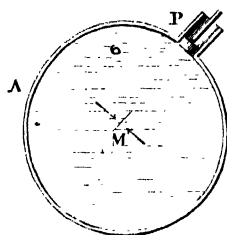


Fig. 46.—Transmission of Pressures.

there by means of a piston, the effect of this pressure will be to bring the molecules closer together, and, consequently, to create a repulsive action between them. As this result takes place throughout the whole extent of the mass, it is evident that each point in the sides of the vessel will be pressed, and that thus the effect of the single pressure will be transmitted in an infinite number of different directions. This kind of irradiation of pressure in fluids is a distinctive characteristic,

and has most important applications.

The pressure exerted at P takes effect not only upon the sides of the vessel, but also at every point in the liquid. Thus a small plane lamina, which we will suppose placed at M, will be subjected to two equal and opposite pressures upon its two faces. It is also very important to remark, that on account of the uniform nature of the liquid, these pressures will not change in magnitude if we suppose the lamina turned round so as to take different directions in the liquid mass, for there is evidently no reason why the pressure should be greater in one direction than in another.

61. Direction of Pressure.—The same reason of symmetry shows us that, at each of their points of application, these pressures are normal or perpendicular to the surface; for if any reason were assigned for

their inclination in a certain direction, a similar reason could also be assigned for their inclination in any other direction. This important truth may also be inferred from observing that if at any point *M* in the side of a vessel (Fig. 47) the pressure *PM* was not normal, it could be decomposed into two: one, *MN*, along the normal to the surface, which would be destroyed by the resistance of the surface; the other, *MA*, along the surface itself, which latter force would cause a sliding motion in the liquid molecule at *M*, which serves as the medium of transmitting the pressure.

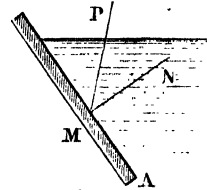


Fig. 47.

Experiment enables us, if not rigorously to demonstrate this principle, at least to show that the direction of transmitted pressure is sensibly normal. (For example, if a sphere is taken, pierced with several holes, and containing a liquid, which is compressed by means of a piston in a tube communicating with the sphere, the liquid is seen to spout out in jets which take a curvilinear form under the action of gravity, but which at their origin appear perpendicular to the spherical surface. The effect is the more striking the greater the pressure exerted on the piston.)

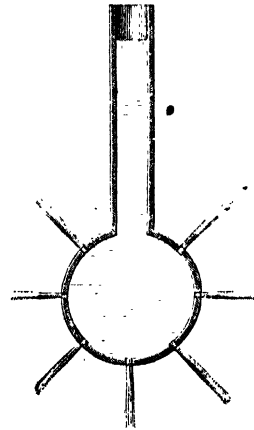


Fig. 48.

62. Pascal's Principle, or the Equal Transmission of Pressure in all Directions.—If we have a vessel *A* full of a liquid (Fig. 49), and if at a certain point *P* a certain pressure be exerted by means of a piston of the area of a square inch, suppose, each square inch of the sides of the vessel will be subjected to an equal pressure. If then at any point an opening is made of the area of a square inch, and closed with a piston, it will be necessary, in order to prevent the piston from moving, to apply to it from outside a pressure equal to that which is directly applied to the piston *P*. A lamina of the same area placed in any direction, in the liquid will also be subjected to an equal pressure on each of its two faces.

Hence it follows that if we suppose a piston of the area of two square inches closing a corresponding opening, since each of these two inches of area receives a pressure equal to that which acts upon

P, the whole piston will receive a double pressure; whence we see that in general the transmitted pressure should vary as the area of the surface pressed.

This is the form in which Pascal enunciated the principle in his celebrated treatise on the *Equilibrium of Liquids*. ("If a vessel full of water, closed on all sides, has two openings, the one a hundred times as large as the other, and if each be supplied with a piston which fits exactly, a man pushing the small piston will exert a force which will equilibrate that of a hundred men pushing the piston which is a hundred times as large, and will overcome that of ninety-nine.

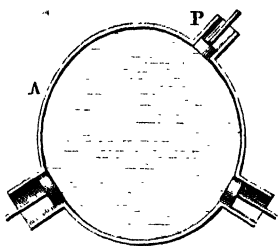


Fig. 49.—Pascal's Principle.

And whatever may be the proportion of these openings, if the forces applied to the pistons are to each other as the openings, they will be in equilibrium.")

In general, let P be the pressure exerted upon a liquid by the aid of a piston of superficial extent S, each unit of surface of this piston will be subjected to a pressure $\frac{P}{S}$, and, as a consequence, on each unit of surface of the sides of the vessel a similar pressure will be produced. If then openings of areas S', S'' . . . are made at different points, and closed with pistons, we must, in order to prevent the pistons from moving, apply to them forces P', P'' . . . equal respectively to S' $\frac{P}{S}$, S'' $\frac{P}{S}$. . . which gives the following equations:—

$$\left| \begin{array}{l} P' = S' \frac{P}{S}, \quad P'' = S'' \frac{P}{S}, \quad \text{or} \quad \frac{P}{S} = \frac{P'}{S'} = \frac{P''}{S''} \end{array} \right.$$

63. Pascal's principle leads to a consequence which we may verify by experiment. If into a system of two tubes in communication with each other, and of unequal sectional area, we introduce a liquid, it will stand at the same height in both branches. If then we place a piston on the liquid in the narrow tube, and subject it to a certain pressure P, this pressure will be transmitted to the liquid, which will be forced back into the large tube; to hinder this motion we must place a piston in the large tube, and apply to it a force which has the same ratio to the force P as the area of the larger piston to that of the smaller. If, for example, the former has an area 16 times that of the latter, a pressure of 1 pound exerted at one of the

extremities of the liquid column will produce a pressure of 16 pounds at the other extremity. We thus see that a small force may be made to produce a very great one. This is the principle of the hydraulic press, a machine which we shall describe further on.

We must remark, however, that if work is to be done, the one piston must displace the other; and it is very evident that, on account of the difference of section, if the small piston moves through a certain length, the large piston will move through one-sixteenth of that length; so that in this apparatus we have a direct verification of this general principle of mechanics, that *what is gained in force is lost in velocity*.

This twofold observation has been clearly enunciated by Pascal, who expresses himself in the following manner at the end of the passage which we have already quoted:—"Whence it appears that a vessel full of water is a new principle of mechanics, and a new machine for the multiplication of force to any required degree, since one man will by this means be able to raise any given weight.

"It is, besides, worthy of admiration that in this new machine we find that constant rule which is met with in all the old ones, such as the lever, wheel and axle, screw, &c., which is that the distance is increased in proportion to the force; for it is evident that as one of these openings is a hundred times as large as the other, if the man who pushes the small piston drives it forward one inch, he will drive the large piston backward only one-hundredth part of that length."

If we endeavoured to perform the preceding experiment in order to demonstrate experimentally the principle of Pascal, we should arrive at only an approximate verification; for to obtain an accurate experiment it would be necessary that the pistons should fit their openings with great exactness, and this would involve a large amount of friction.

The verification would be still more difficult if we endeavoured to perform the experiment described in § 62, for in this case, besides the cause of error which we have just mentioned, the phenomena would be complicated by the action of gravity, which of itself pro-

1 Kil.

16 K



Fig. 50.—Principle of the Hydraulic Press.

duces variable pressures on the different openings, according to their depth below the surface of the liquid.¹ In fact, the principle of Pascal is an abstract principle, a kind of general synthesis of phenomena which cannot be made the subject of direct demonstration. It is by the constant agreement of its consequences with our observation that the authority and legitimacy of the principle are established; we shall see from all that follows how complete and invariable is that agreement.

64. Fundamental Principle of Equilibrium in Heavy Liquids. Surfaces of Equal Pressure.—Pascal's principle is a general consequence of the constitution of liquids, and is independent of the action of gravity. By introducing this latter force we arrive at special results, which we shall describe in succession. The most important, which may be considered as the fundamental rule in hydrostatics, consists in the fact that the different points in a horizontal layer of a heavy liquid are subject to the same pressure. Let us con-

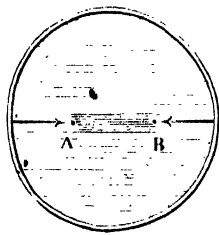


Fig. 51.

sider two points, A and B (Fig. 51), situated in the same horizontal plane in a heavy liquid. If we suppose that A and B are the centres of two small plane surfaces which are vertical and parallel, we may consider these surfaces as the bases of a very narrow horizontal cylinder of liquid. As this cylinder is in separate equilibrium in the general mass, we may conclude that its bases A and B are subject to equal and contrary pressures in the direction of the arrows shown in the figure; for the remaining pressures due to the surrounding liquid act in directions perpendicular to the axis of the cylinder, and thus cannot influence the equilibrium in the direction of the axis. The two elements A and B are therefore subject to the same pressure in one direction; but the pressure at a point is equal in all directions (§ 60); and as A and B are any points in the same horizontal layer, it follows that the pressures at all points in the same horizontal layer are equal. We may add, as a consequence of this, that the density is also the same at all points of a horizontal layer. On account of the slight compressibility of liquids, the variation of their density with the depth

¹ The pressures on the different pistons (supposed equal in area) would differ from each other by constant amounts, depending on their differences of level; but, on account of this constancy, any increase of pressure on one piston would require an equal increase of pressure on every other in order to the maintenance of equilibrium.

is scarcely sensible; but the above result is true for all heavy fluids, whether compressible or incompressible.

In proceeding from one surface of equal pressure to another, the pressure increases or diminishes according as the depth increases or diminishes. Thus, for example, if we consider a small horizontal element m of a horizontal layer AB , and conceive the vertical cylinder mm' reaching to the horizontal layer CD ; it is quite clear that, independently of the pressure at m' , which is transmitted undiminished to m , this latter element is subject to a pressure equal to the weight of the liquid contained in the cylinder mm' .

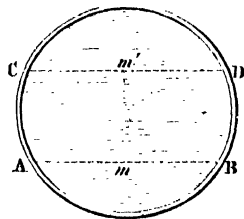


Fig. 52.

If we call s the area of the element m , h the distance between the two horizontal layers, and d the weight of unit volume of the liquid, the volume of the cylinder is expressed by sh , and its weight by shd . This last expression, therefore, represents the variation of pressure for an element of area s , when its depth below the surface varies by a quantity equal to h ; and dividing by the area s we see that hd is the expression for the difference of pressure per unit area, corresponding to a difference of depth h .

65. Free Surface.—It follows from the foregoing principles that the free surface of a heavy liquid must be horizontal. We have already given an experimental demonstration of this important fact.



Fig. 53.

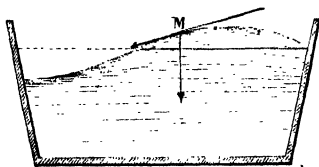


Fig. 54.

It could also have been predicted from *a priori* considerations. Let CD (Fig. 53) be the free surface, and m, m' two small equal elements of surface, in the horizontal layer AB . These two elements must be subject to equal pressures, which are evidently represented by the weights of the cylinders $mn, m'n'$; these cylinders must, therefore, have the same height, or the points n and n' must be in the same horizontal plane.

The same conclusion is arrived at by observing that if at any point whatever of the surface M (Fig. 54) the liquid did not assume a horizontal position, the weight of the liquid molecule at M could be decomposed into two forces, one perpendicular to and one along the surface of the liquid. The effect of the first would merely be to compress the liquid, and it would be destroyed by the reaction of the liquid; but the second would produce a displacement of the molecule. Equilibrium, therefore, can only exist on condition of the annihilation of this second component; that is, the surface must be horizontal at all points.

This mode of reasoning shows us that, in general, when the liquid mass is subjected to the action of any number of forces, it is necessary for equilibrium that the free surface be, at all points, perpendicular to the resultant of the acting forces. If, for instance, we place upon the whirling table a glass containing a liquid (Fig. 55), and give it a rotatory motion, we shall see the surface become hollowed, and assume a curvilinear form. In fact, each of the molecules is subjected simultaneously to the action of gravity and of centrifugal force; and it is the resultant of these two forces which must be everywhere perpendicular to the free surface. It is easily shown that this surface must be a paraboloid of revolution, so that the section represented in the figure is a parabola.

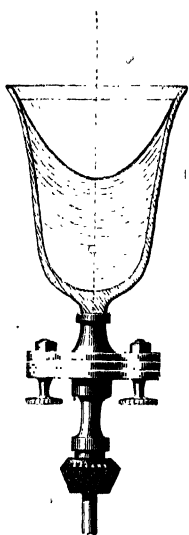


Fig. 55.—Rotating Vessel of Liquid.

66. Pressure upon the Bottom of Vessels.—If we consider a heavy liquid placed in a vessel the bottom of which is formed by a *plane horizontal surface*, it is easy to determine the pressure exerted by the liquid upon this plane. Let $ABMN$ (Fig. 56) be a vessel filled with liquid to the level MN , and m an element of surface on the bottom AB . On m suppose a small vertical cylinder to stand, meeting the horizontal layer LL' in m' . On the element n , equal to m , and in the horizontal layer LL' , suppose a vertical cylinder to stand, cutting the horizontal layer RR' in n' . Suppose another similar vertical cylinder on r , an element equal to m , and let this cylinder meet SS' in r' ; it is evident that, by continuing this construction, we shall finally arrive, whatever be the shape of the vessel, at a cylinder SS' , which will extend upwards to the free surface

MN. Now, the element m is subject to a pressure greater than that at m' by a quantity equal to the weight of the cylinder mm' . Similarly m' is subject to a pressure which is equal to that at n , and greater than that at n' by a quantity equal to the weight of the cylinder nn' ; whence it is evident, by pursuing this reasoning, that the element n supports a pressure equal to the sum of the weights of the cylinders mm' , nn' , rr' , ss' ; that is, to the weight of a cylinder of liquid standing on the base m , and extending vertically upwards to the free surface. As all points in the bottom AB are subject to the same pressure, it follows that the entire pressure on the bottom of the vessel is equal to the weight of a liquid column whose base is the bottom of the vessel, and height the vertical distance between the bottom and the free surface, or what is called the height of the liquid in the vessel.¹

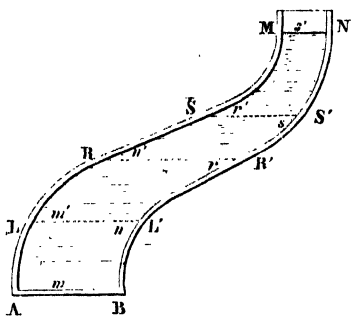


Fig. 56.—Pressure on the Bottom of Vessels.

Let B be the area of the bottom of the vessel, H the height of the liquid, and D its weight per unit volume, then the pressure is expressed by the formula BHD .

If, for instance, in a vessel whose bottom is two square decimetres in area, there is a column of mercury of the height of $5\frac{1}{2}$ decimetres, the volume of this column, which measures the pressure, is $2 \times 5\frac{1}{2} = 11$ cubic decimetres, and its weight² is $11 \times 13.59 = 149.49$ grammes.

67. Experiment of Pascal's Vases.—The preceding proposition shows that the pressure on the bottom of a vessel depends only upon the area of the bottom and the height of the liquid, the form of the vessel being quite immaterial. In order to verify this fact, Pascal contrived an experiment which, with some modifications, is now commonly introduced in courses of physics. The apparatus employed is a tripod (Fig. 57) supporting a ring, into which can successively

¹ In these remarks we neglect the atmospheric pressure which is exerted upon the free surface and transmitted to the bottom of the vessel.

² This example illustrates the convenience of the metrical system. The weight of a cubic decim. of water is 1 kilo., and the sp. gr. of mercury is 13.59; hence the weight of a cubic decim. of mercury is 13.59 kilos.

be screwed three vessels of different shapes, one widened upwards, another cylindrical, and the third tapering upwards. At the lower part of the ring is a disc, supported by a thread fixed to one of the scales of a balance. Weights placed in the other scale keep the disc pressed against the ring with a certain force. Let the cylindrical vase be placed upon the tripod, and filled with water until the

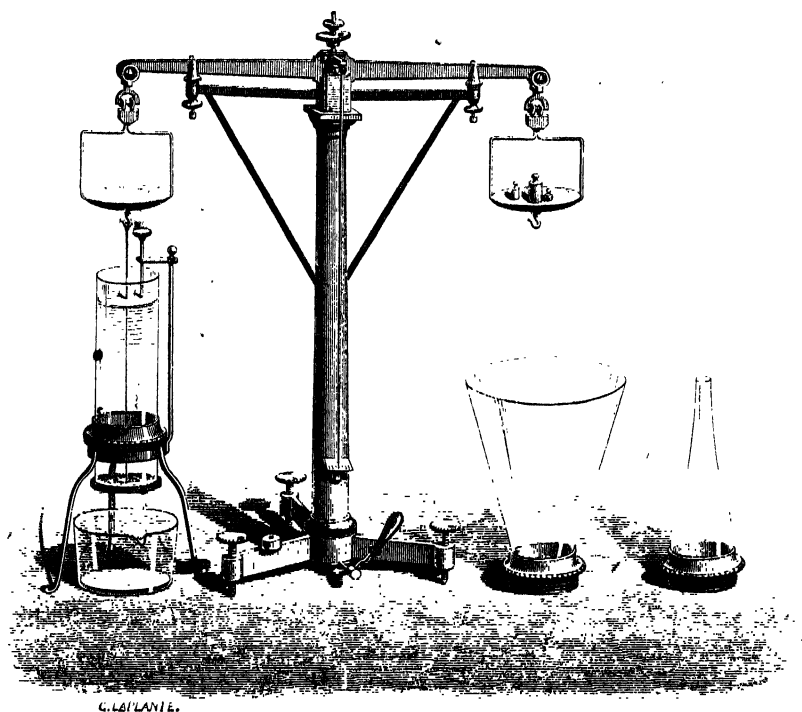


Fig. 57.—Experiment of Pascal's Vases.

pressure exerted on the disc detaches it from the ring. An indicator marks the level of the liquid when this takes place. Let the experiment be repeated with the two other vases, and the disc will be detached when the water has reached the same height; showing plainly that the pressure on the bottom of a vessel is independent of the shape of the vessel.

But we may go further; for in the case of the cylindrical vessel it is evident that the pressure on the bottom is equal to the weight of the contained liquid. Now this weight is necessarily equal to that which counterpoised it in the other scale of the balance; hence in all three cases the pressure on the bottom of the vessel is equal to

the weight of a liquid column with the bottom as base and of the same height as the liquid in the vessel.

68. Upward Pressure.—The pressure exerted at any point of a liquid mass being the same in all directions, a horizontal surface facing downwards should be subjected to an upward pressure equal to the downward pressure which would be exerted if the liquid were acting in the other direction. Suppose we take a tube open at both ends (Fig. 58), and apply to the lower end a flat cover. If we plunge the tube into a liquid, the liquid will press up the cover against the bottom of the tube with a force which increases as the tube is plunged deeper in the liquid. If we then pour liquid into the tube, this will produce a downward pressure upon the cover, and when the level of the liquid is the same inside the tube as outside the cover will be detached, the upward pressure being destroyed by the pressure exerted by the liquid inside the tube in the contrary direction.

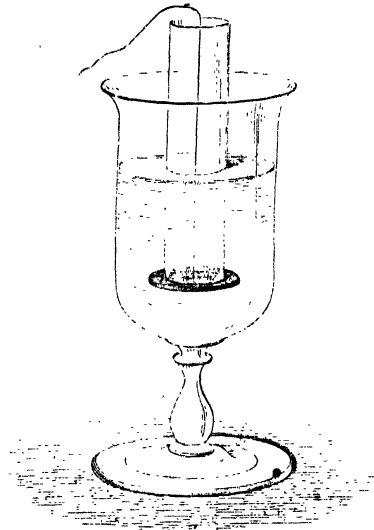


Fig. 58.—Upward Pressure.

69. Total Pressure. Resultant Pressure on Vessel. Hydrostatic Paradox.—When a vessel of any shape is filled with a liquid, normal pressures are exerted against all points of its sides, increasing with the depth, and equal in each case to the pressure in the corresponding horizontal layer of the liquid. We may suppose a summation of all the pressures thus exerted upon the different superficial elements of the sides; this gives what is called the *total pressure* exerted by the liquid (see § 72).

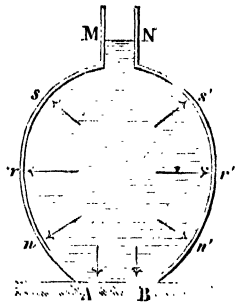


Fig. 59.—Total Pressure.

This total pressure must not be confounded with the resultant pressure which is transmitted to the stand on which the vessel rests. In fact we see that, of the elements of pressure, some are transmitted *entire* to the stand; these are the vertical pressures exerted upon the bottom AB; others,

those for instance at n and n' , are only transmitted in part, because their direction is oblique; the horizontal pressures at r and r' are evidently without influence; and the pressures exerted at s and s' tend to raise the vessel. It is by the combination and *composition* of these pressures of different intensity and direction that the resultant pressure upon the stand on which the vessel rests is obtained.

Hydrostatic Paradox.—It has been thought paradoxical that vessels whose bottoms were subjected to equal pressures did not transmit equal pressures to the stand on which they were placed. In reality, nothing is less paradoxical; the pressure on the bottom of the vessel is only one of the elements which combine to produce the resultant pressure transmitted to the stand.

70. Composition of Pressures.—It may be regarded as quite evident that this latter pressure is in all cases equal to the weight of the liquid; which amounts to saying that if we place a vessel containing liquid in one scale of a balance, we must, in order to equilibrate it, place in the other scale weights equal to the sum of the weights of the liquid and the vessel. This result is also easily shown from *a priori* considerations in some simple cases.

Thus in the case of a cylindrical vessel ABDC (Fig. 60) it is evi-

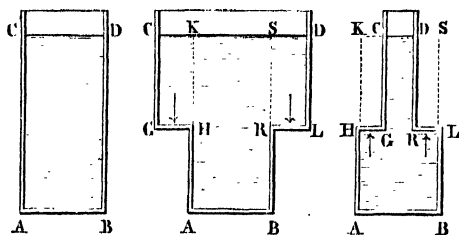


Fig. 60.—Hydrostatic Paradox.

dent that the only pressure transmitted to the stand is that exerted upon the bottom, which is equal to the weight of the liquid. In the case of the vessel which is wider at the top, the stand is subjected to the weight

of the liquid column ABSK, which presses on the bottom AB, together with the columns GHKC, RLDS, pressing on GH and RL; the sum of which weights composes the total weight of liquid contained in the vessel. Finally, in the third case, the pressure on the bottom AB, which is equal to the weight of a liquid column ABSK, must be diminished by the pressures in the opposite direction on HG and RL. These last being represented by liquid columns HGCK, RLSD, there is only left to be transmitted to the stand a pressure equal to the weight of the water in the vessel. By the application of the ordinary rules for the composition of forces, it can easily be shown that this result is perfectly general. Here, then, we have Pascal's principle leading to a conclu-

sion which is obviously true, and which therefore furnishes a confirmation of the principle itself.

71. Motion produced by the Flowing of a Liquid.—The demonstration to which we have just referred amounts to showing, by the analysis of the different pressures, that the horizontal components of these pressures equilibrate each other, and that the vertical components are equivalent to a single force equal to the weight of the liquid. The evidence of experiment is as strong for the first part of this proposition as for the second. If we place a vessel (Fig. 61) in such a position as to be very susceptible of motion in a horizontal direction, whether by suspending it by a thread, or by floating it in water, and under these circumstances fill it with liquid, however great may be the mobility of the apparatus, not the slightest displacement is observed. This proves that the horizontal components of pressure equilibrate one another. This equilibrium is effected through the medium of the vessel; but if we suppose an opening made at any point of the vessel, the liquid will run out, and the pressure exerted at the point diametrically opposite will propel the vessel in the opposite direction to that in which the liquid is escaping. •

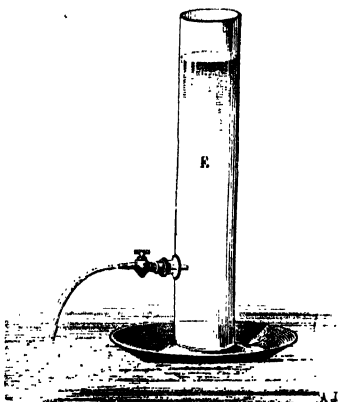


Fig. 61.—Backward Movement of Discharging Vessel.

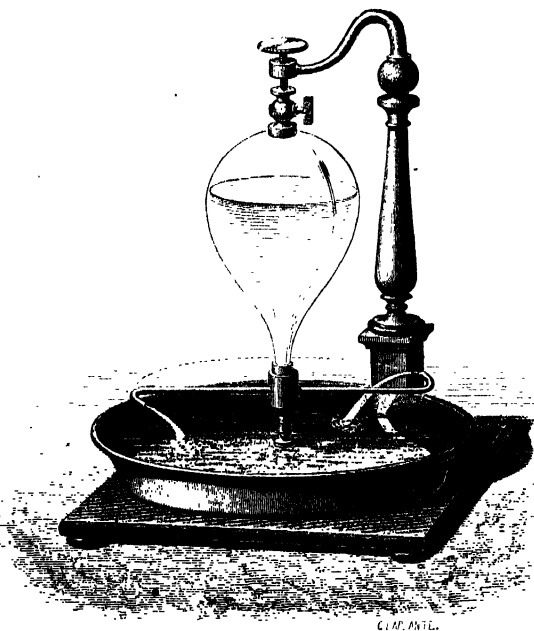


Fig. 62.—Hydraulic Tourniquet.

This observation explains the motion of the apparatus called the *hydraulic tourniquet*. It consists (Fig. 62) of a vessel capable of rotation about a vertical axis, having at its lower extremity a tube, the two portions of which are bent in opposite directions, and left open at the ends to allow the liquid to escape. The reaction at the points opposite to the openings causes the rotatory motion of the apparatus.

When the velocity of efflux is sufficiently great, the motion can be employed for practical purposes, and hydraulic engines based on this principle have frequently been proposed and tried; Barker's mill is one of the best known.

72. Centre of Pressure.—When we consider the particular case of the pressure exerted by a liquid upon a *plane* surface, the different elements of pressure being all parallel, it may be required to determine the point of application of the resultant pressure. This point is called the centre of pressure. The centre of pressure does not coincide with the centre of gravity; it is always situated below this latter, since the elementary forces which must be combined in order to obtain it, instead of being uniformly distributed over the surface, increase with the depth.

The investigation of the centre of pressure constitutes a separate chapter of mathematical physics, and we shall not enter upon it here; we shall simply examine a particular case, fitted to give accurate ideas with regard to this point.

Let a rectangular surface RB be immersed in a liquid, which extends

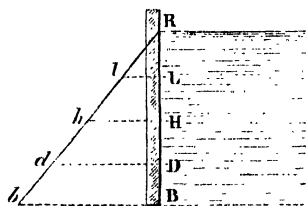


Fig. 63.—Centre of Pressure.

as far as R; we may suppose it, for instance, a flood-gate or the side of a dam for holding water. The pressure goes on increasing from the point R, where it is zero, to the point B, where it attains its maximum value; it has the same value for all points in the same horizontal line, and is at each point proportional to the distance from the surface of the water.

If, then, at the point B we draw a perpendicular Bb equal to RB, and join Rb, the different parallel lines Dd, Hh, Ll, in the triangle RBb will be proportional to the pressures at the points D, H, L. The composition of these pressures, then, amounts to finding the centre of gravity of the triangle RbB; but the height of this from the base is one-third of the height of the triangle; the centre of pressure thus

lies at one-third of the height RB. It is further evident by symmetry that it will lie on the line joining the middle points of the upper and lower sides of the rectangle.

As for the total pressure on RB, it may be obtained in the special case under consideration by observing, that since the pressure increases uniformly from R to B, its average intensity is equal to the pressure at the middle point. The total pressure is therefore the same in amount (though not in distribution) as if the surface were pressed at all points by a body of water of half the height of RB.

Suppose the height of RB = 3 metres, and its breadth = 5 metres, the total pressure will be equal to the weight of $5 \times 1.5 = 7.5$ cubic metres of water, that is, to 7500 kilos., and will have for its resultant a single force of this amount applied not at the centre of gravity, but at the centre of pressure.

We may remark that the middle point of the height of the rectangle exactly corresponds to the centre of gravity of the figure, and it may be demonstrated in general that *the total pressure on any surface; whether plane or curved, is equal to the weight of a liquid column having that surface for base, and for height the distance of the centre of gravity from the surface of the water.*

CHAPTER IX.

PRINCIPLE OF ARCHIMEDES.

73. Pressure of Liquids on Bodies immersed.—When a body is immersed in a liquid, the different points of its surface are subjected to pressures which obey the rules laid down in the preceding chapter. As these pressures increase with the depth, it is evident that those which tend to raise the body overcome those which tend to sink it, so that the resultant effect is a force in the direction opposite to that of gravity.

By means of an analysis, similar to that in § 70, it may be shown that this resultant upward force is exactly equal to the weight of the liquid displaced by the body.

◀ This conclusion can very readily be verified in some simple cases: suppose, for example (Fig. 64), a right cylinder plunged vertically in a liquid, and let us examine the effect of the different pressures exerted by the liquid upon its surface. It is evident, in the first place, that if we consider any point on the sides of the cylinder, the normal and horizontal pressure on that point is destroyed by the equal and contrary pressure at the point diametrically opposite; and, as the same is the case for all similar points, we see that the horizontal pressures destroy each other. As regards the vertical pressures on the ends, one of them, that on the upper end AB, is in a downward direction, and equal to the weight of the liquid column ABNN; the other, that on the lower end CD, is in an upward direction, and equal to the weight of the liquid column CNND; this latter pressure exceeds the former by the weight of the liquid cylinder ABCD, so that the resultant effect of the pressure is to raise the body with a force equal to the weight of the liquid displaced.

By a synthetic process of reasoning, we may, without having recourse to the analysis of the different pressures, show that this

conclusion is perfectly general. Suppose we have a liquid mass in equilibrium, and that we consider specially the portion M (Fig. 65); this portion is likewise in equilibrium. If we suppose it to become solid, without any change in its weight or volume, equilibrium will

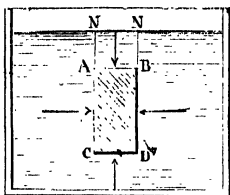


Fig. 64.

Principle of Archimedes.

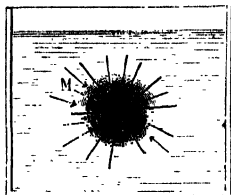


Fig. 65.

still subsist. Now this is a heavy mass, and as it does not fall, we must conclude that the effect of the pressures on its surface is to produce a resultant upward pressure exactly equal to its weight, and acting in a line which passes through its centre of gravity. If we now suppose M replaced by a body exactly occupying its place, the exterior pressures remaining the same their resultant effect will also be the same.

The name *centre of buoyancy*, or *centre of displacement*, is given to the centre of gravity of the liquid displaced by a body immersed, and we see that we may always suppose that it is in this point that the upward pressure of the liquid is applied. The results of the above explanations may thus be included in the following proposition: *Every body immersed in a liquid is subjected to an upward vertical pressure equal to the weight of the liquid displaced, and applied at the centre of displacement.*

This proposition constitutes the celebrated principle of Archimedes. It is often enunciated in the following form: *Every body immersed in a liquid loses a portion of its weight equal to the weight of the liquid displaced.* This enunciation, though perhaps less correct than the former, is fundamentally identical with it; for if we weigh a body immersed in a liquid, the weight will evidently be diminished by a quantity equal to the upward pressure.

74. Experimental Demonstration of the Principle of Archimedes.—The following experimental demonstration of the principle of Archimedes is commonly exhibited in courses of physics:—

From one of the scales of a hydrostatic balance is suspended a hollow cylinder of copper, and below this a solid cylinder, whose

volume is equal to the interior volume of the hollow cylinder; these are balanced by weights in the other scale. A vessel of water is then placed below the cylinders, in such a position that the lower cylinder shall be immersed in it. The equilibrium is immediately destroyed, and the upward pressure of the water causes the scale with the weights to descend. If we now pour water into the hollow cylinder, equilibrium will gradually be re-established; and the beam

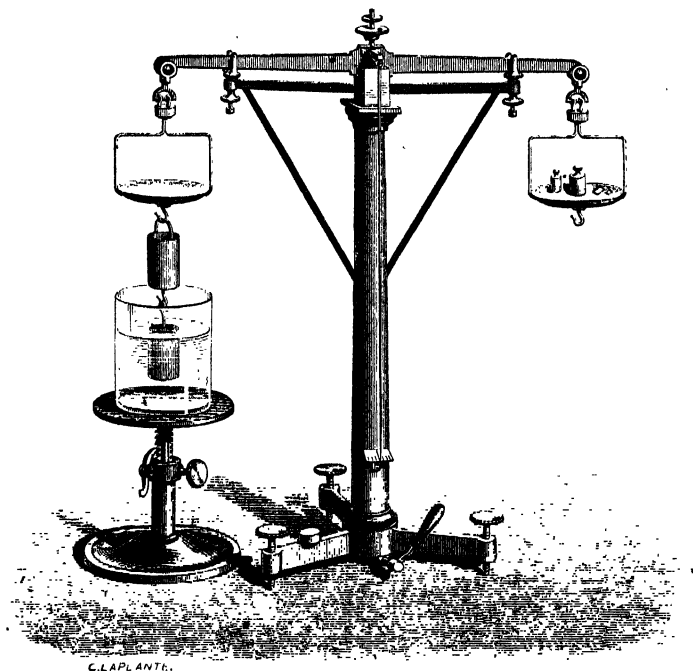


Fig. 66.— Experimental Verification of Principle of Archimedes.

will be observed to resume its horizontal position when the hollow cylinder is full of water, the other cylinder being at the same time completely immersed. The upward pressure upon this latter is thus equal to the weight of the water added, that is, to the weight of the liquid displaced.

75. Body immersed in a Liquid.—It follows from the principle of Archimedes that when a body is immersed in a liquid, it is subjected to two forces: one equal to its weight and applied at its centre of gravity, tending to make the body descend; the other equal to the weight of the displaced liquid, applied at the centre of buoyancy, and

tending to make it rise. There are thus three different cases to be considered:

(1.) The weight of the body may exceed the weight of the liquid displaced, or, in other words, the mean density of the body may be greater than that of the liquid; in this case, the body sinks in the liquid, as, for instance, a piece of lead dropped into water.

(2.) The weight of the body may be less than that of the liquid displaced; in this case the body rises partly out of the liquid, until the weight of the liquid displaced is equal to its own weight. This is what happens, for instance, if we immerse a piece of cork in water and leave it to itself.

(3.) The weight of the body may be equal to the weight of the liquid displaced; in this case, the two opposite forces being equal, the body takes a suitable position (§ 77) and remains in equilibrium.

These three cases are exemplified in the three following experiments (Fig. 67):—

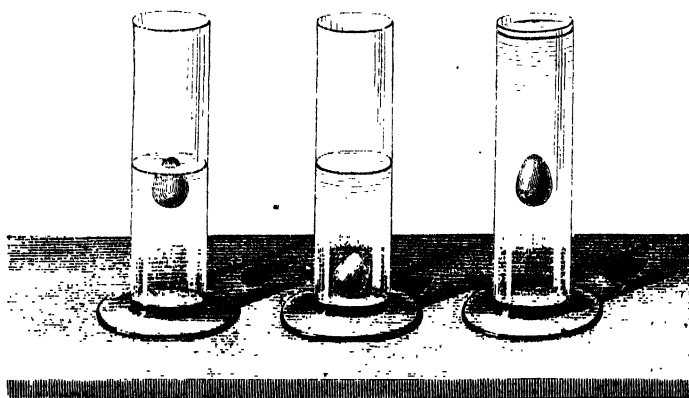


Fig. 67.—Egg Plunged in Fresh and Salt Water.

(1.) An egg is placed in a vessel of water; it sinks to the bottom of the vessel, its mean density being a little greater than that of the liquid.

(2.) Instead of fresh water, salt water is employed; the egg floats at the surface of the liquid, which is a little denser than it.

(3.) Fresh water is carefully poured on the salt water; a mixture of the two liquids takes place where they are in contact; and if the egg is put in the upper part, it will be seen to descend, and, after a few oscillations, remain at rest in a layer of liquid of which it dis-

places a volume whose weight is equal to its own. We may remark that, in this position the egg is in stable equilibrium; for, if it rises, the upward pressure diminishing, its weight tends to make it descend again; if, on the contrary, it sinks, the pressure increases and tends to make it reascend.

76. Cartesian Diver.—The experiment of the *Cartesian diver*, which is described in old treatises on physics, shows each of the different cases that can present themselves when a body is immersed. The diver (Fig. 68) consists of a hollow ball, at the bottom of which is a

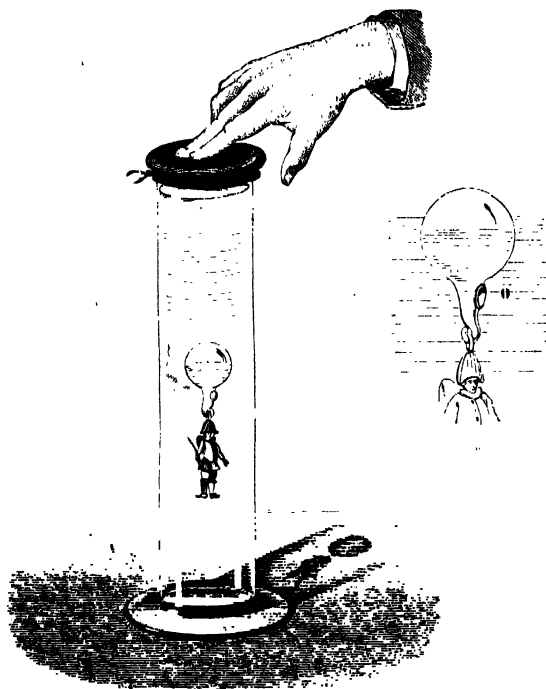


Fig. 68.—Cartesian Diver.

small opening O; a little porcelain figure is attached to the ball, and the whole floats upon water contained in a glass vessel, the mouth of which is closed by a strip of caoutchouc or a bladder. If we press with the hand on the bladder, the air is compressed, and the pressure, transmitted through the different horizontal layers, condenses the air in the ball, and causes the entrance of a portion of the liquid by the opening O; the floating body becomes heavier, and in

consequence of this increase of weight the diver descends. When we cease to press upon the bladder, the pressure becomes what it was before, some water flows out and the diver ascends. It must be observed, however, that as the diver continues to descend more and more water enters the ball, in consequence of the increase of pressure, so that if the depth of the water exceeded a certain limit, the diver would not be able to rise again from the bottom.

If we suppose that at a certain moment the weight of the diver

becomes exactly equal to the weight of an equal volume of the liquid, there will be equilibrium; but, unlike the equilibrium in the experiment in § 75, this will evidently be *unstable*, for a slight movement either upwards or downwards will alter the resultant force so as to produce further movement in the same direction.

77. Relative Positions of the Centre of Gravity and Centre of Buoyancy.

—In order that a floating body, wholly or partially immersed in a liquid, may be in equilibrium, it is evidently necessary that its weight be equal to the weight of the liquid displaced.

This condition, which is absolutely necessary, is, however, not sufficient; we require, in addition, that the action of the upward pressure should be exactly opposite to that of the weight; that is, that the centre of gravity and the centre of buoyancy be in the same vertical line; for if this were not the case, the two contrary forces would compose a couple, the effect of which would evidently be to cause the body to turn.

In the case of a body completely immersed, it is further necessary for stable equilibrium that *the centre of gravity should be below the centre of buoyancy*; in fact we see, by Fig. 69, that in any other

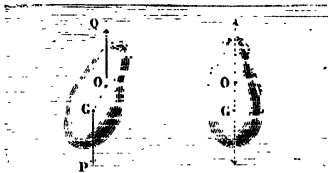


Fig. 69.



Fig. 70.

Relative Positions of Centre of Gravity and Centre of Pressure.

position than that of equilibrium, the effect of the two forces applied at the two points G and O would be to turn the body, so as to bring the centre of gravity lower. But this is not the case when the body is only partially immersed, as most frequently happens. In this case it may indeed happen that, with stable equilibrium, the centre of gravity is below the centre of pressure; but this is not necessary, and in the majority of instances is not the case. Let Fig. 70 represent the lower part of a floating body—a boat, for instance. The centre of pressure is at O, the centre of gravity at G, considerably above; if the body is displaced, and takes the position shown in the figure, it will be seen that the effect of the two forces acting at O and at G is to restore the body to its former position. This difference from

what takes place when the body is completely immersed, depends upon the fact that, in the case of the floating body, the figure of the liquid displaced changes with the motions of the body, and the centre of buoyancy moves towards the side on which the body is more deeply immersed. It will depend upon the form of the body whether this lateral movement of the centre of buoyancy is sufficient to carry it beyond the vertical through the centre of gravity. The two equal forces which act on the body will evidently turn it to or from the original position of equilibrium, according as the new centre of buoyancy lies beyond or falls short of this vertical.¹

78. Advantage of Lowering the Centre of Gravity.—Although stable equilibrium may subsist with the centre of gravity above the centre of buoyancy, yet for a body of given form the stability is always increased by lowering the centre of gravity; as we thus lengthen the arm of the couple which tends to right the body when displaced. It is on this principle that the use of ballast depends.

79. Phenomena in apparent Contradiction to the Principle of Archi-

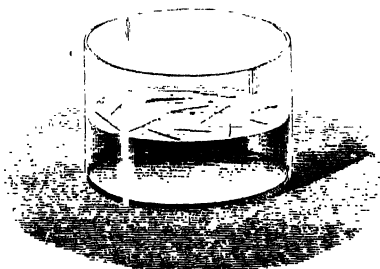


Fig 71.—Steel Needles Floating on Water.

medes.—A body cannot float in a liquid unless it have a density less than that of the liquid. This natural consequence of the principle of Archimedes seems at first sight to be contradicted by some well-known facts. Thus, for instance, if small needles are placed carefully on the surface of water, they will remain there in equilibrium (Fig. 71). It is on a similar principle that several insects *walk* on

water (Fig. 72), that a great number of bodies of various natures, provided they be *very minute*, can, if we may so say, be placed on the surface of a liquid without penetrating into its interior. These curious facts depend on the circumstance that the small bodies in

¹ If a vertical through the new centre of buoyancy be drawn upwards to meet that line in the body which in the position of equilibrium was a vertical through the centre of gravity, the point of intersection is called the *metacentre*. Evidently when the forces tend to restore the body to the position of equilibrium, the metacentre is above the centre of gravity; when they tend to increase the displacement, it is below. In ships the distance between these two points is usually nearly the same for all amounts of heeling, and this distance is a measure of the stability of the ship.

We have defined the metacentre as the intersection of two lines. When these lines lie

question are not wetted by the liquid, and hence, in virtue of principles which will be explained in connection with capillarity (Chap. xi.), depressions are formed around them on the liquid surface, as represented in Fig. 73. The curvature of the liquid surface in the neighbourhood of the body is very distinctly shown by observing the shadow cast by the floating body, when it is illumined by the sun; it is seen to be bordered by luminous bands, which are owing to the refraction of the rays of light in the portion of the liquid bounded by a curvilinear surface.



Fig. 72.—Insect Walking on Water.

The existence of the depression about the floating body enables us to bring the condition of equilibrium in this special case under the general enunciation of the principle of Archimedes.

Let M be a section of the body, CD the distance to which the depression extends, and AB the corresponding portion of any horizontal layer; since the pressure at each of the points of AB must be the same as in the other parts of the layer, the liquid acts in exactly the same way as if M did not exist, and the cavity were filled by the liquid itself.

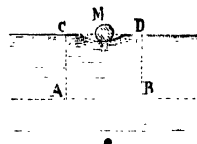


Fig. 73.

We may thus say in this case also that the weight of the floating body is equal to the weight of the *liquid displaced*, understanding by these words the liquid which would occupy the whole extent of the depression due to the presence of the body.

80. Liquids in Superposition.—When liquids of different densities, which do not readily mix, are placed in the same vessel, the particles of the denser liquids unite and fall to the bottom, just as a solid body sinks in a liquid of less density; finally, the liquids arrange themselves in the order of their respective densities, the surfaces of separation being horizontal. This fact is verified by means of the phial called the *phial of the four elements*. It is a flask (Fig. 74) containing mercury, water, and oil. In the state of equilibrium the mercury is at the bottom, the oil at the top, and the water in the middle; if the

in different planes, and do not intersect each other, there is no metacentre. This indeed is the case for most of the displacements to which a floating body of irregular shape can be subjected. There are in general only two directions of heeling to which metacentres correspond, and these two directions are at right angles to each other. For an investigation of the conditions of stability in floating bodies, see Thomson and Tait's *Natural Philosophy*, §§ 763–768.

flask is shaken, the liquids are for the moment mixed, but in returning to repose do not fail to resume their former positions.

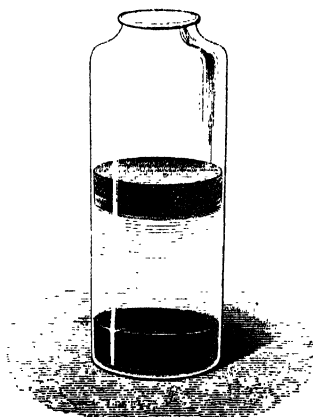


Fig. 74.
Phial of the Four Elements.

It is easily seen from the ordinary rules of hydrostatics, that the surface of separation of two different liquids must be horizontal. Let there be two liquids in a vessel (Fig. 75); the free surface is necessarily horizontal. If now we take two equal superficial elements n and n' in a horizontal layer of the lower liquid, they must be subjected to equal pressures; these pressures are measured by the weights of the liquid cylinders nrs and $n'tl$; and these latter cannot be equal unless there be the same height of the lower liquid above the elements n and n' . This reasoning holds for all points in the horizontal layer, which must therefore be at

a constant distance from the surface of separation; in other words, this surface must be horizontal.

This property is liable to considerable modification in the case of liquids which can dissolve each other or act chemically upon each other. Thus, if alcohol be carefully poured upon water in a glass, the two liquids will be seen to have for their surface of junction a horizontal plane; but on agitation a single liquid will be formed by their mutual action, and the separation will not again take place.

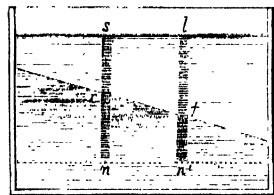


Fig. 75.

If the agitation is not sufficiently great, this intimate mixture will only partially ensue, and will be confined to the neighbourhood of the surface of contact. Two uniform layers of liquid will thus be formed, separated by an intermediate zone of unequal density. This is the case at the mouth of a river, where the fresh water forms on the surface of the sea a layer, the base of which is a compound of fresh and salt water.

CHAPTER X.

APPLICATION OF THE PRINCIPLE OF ARCHIMEDES TO THE DETERMINATION OF DENSITIES.—HYDROMETERS.

81. Determination of Densities.—We have seen in Chap. vii. that in order to determine the density of a body it is only necessary to measure the ratio existing between the weight of a certain volume of the body and the weight of an equal volume of water. The principle of Archimedes enables us to effect this measurement very easily, and the process which it suggests is sometimes more convenient than that which has been described in the chapter mentioned above.

(1.) *Solid bodies.*—Suppose that the object whose density we wish to determine is a piece of copper. It is suspended by a very fine thread to one of the scales of a balance (Fig. 76), its weight is determined, and found to be, say $125\cdot35^{\text{gr}}$. The body is then immersed in water; the equilibrium is destroyed on account of the upward pressure of the water, and in order to re-establish it, we must add a weight of $14\cdot24^{\text{gr}}$ to the scale supporting the body. This additional weight, according to the principle of Archimedes, represents the weight of a volume of water equal to the volume of the body. The density of copper is thus $\frac{125\cdot35}{14\cdot24} = 8\cdot8$.

(2.) *Liquid bodies.*—From one of the scales of the balance is suspended (Fig. 77) any body whatever, which must, however, not be capable of being attacked by the liquids in which it is to be immersed; a ball of glass weighted inside with mercury is very well adapted to this purpose. The exact weight of this is obtained; it is then immersed in the liquid whose density is sought—alcohol, for example; an upward pressure is thus produced, and in order to re-establish equilibrium, a weight of $35\cdot43^{\text{gr}}$ must be added to the scale.

The experiment is repeated by immersing the ball in water, in which case the upward pressure is stronger, and a weight of $44.28^{\text{gr.}}$ is

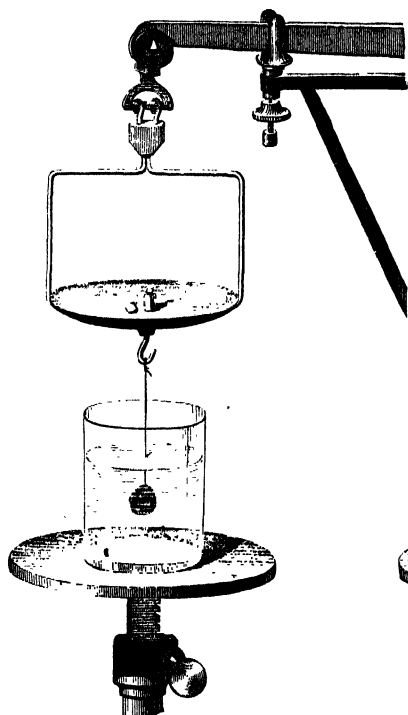


Fig. 76.—Specific Gravity of Solids.

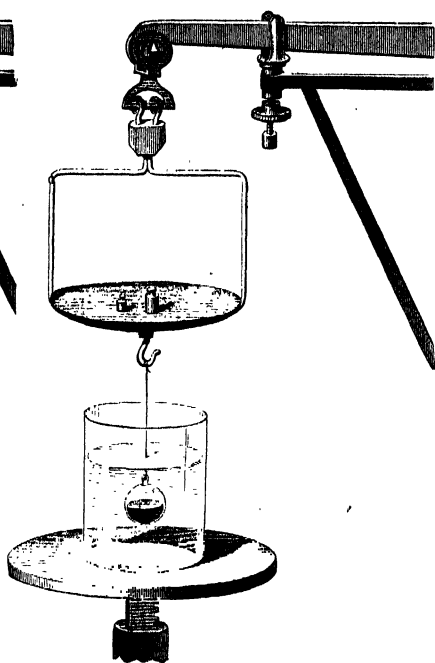


Fig. 77.—Specific Gravity of Liquids.

necessary to re-establish equilibrium. The weights $44.28^{\text{gr.}}$ and $35.43^{\text{gr.}}$ are the weights of equal volumes of water and alcohol; the density of the latter liquid is therefore $\frac{35.43}{44.28} = 0.8$.

82. Hydrometers.—The name hydrometer is given to a class of instruments used for determining the densities of liquids by observing either the depths to which they sink in the liquids or the weights required to be attached to them to make them sink to a given depth. According as they are to be used in the latter or the former of these two ways, they are called hydrometers of constant or of variable immersion. The name areometer (from *ἀραιός*, rare) is used as synonymous with hydrometer, being probably borrowed from the French name of these instruments, *aréomètre*. The

hydrometers of constant immersion most generally known are those of Nicholson and Fahrenheit.

83. Nicholson's Hydrometer.—This instrument, which is represented in Fig. 78, consists of a hollow cylinder of metal with conical ends, terminated above by a very thin rod bearing a small dish, and

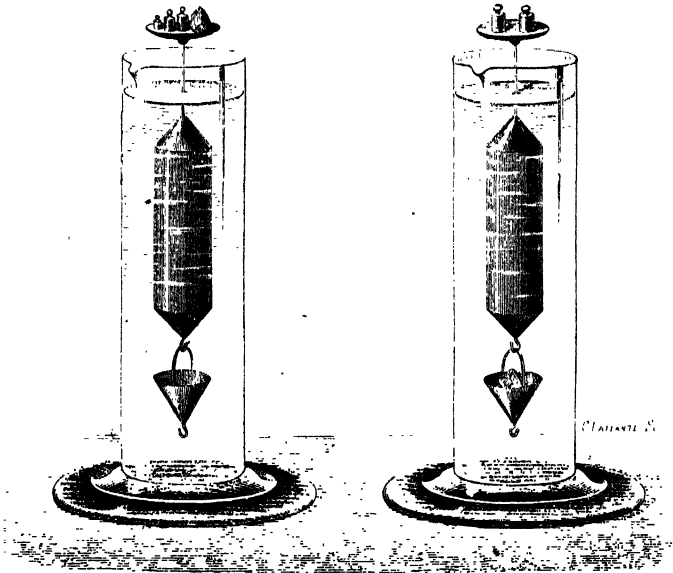


Fig. 78.—Nicholson's Hydrometer.

carrying at its lower end a kind of basket. This latter is of such weight that when the instrument is immersed in water a weight of 1000 grains must be placed in the dish above in order to sink the apparatus as far as a certain mark on the rod. By the principle of Archimedes the weight of the instrument, together with the 1000 grains which it carries, is equal to the weight of the water displaced. Now, let the instrument be placed in another liquid, and the weights in the dish above be altered until they are just sufficient to make the instrument sink to the mark on the rod. If the weights in the dish be called w , and the weight of the instrument itself W , the weight of liquid displaced is now $W + w$, whereas the weight of the same volume of water was $W + 1000$; hence the specific gravity of

the liquid is $\frac{W + w}{W + 1000}$.

This instrument can also be used either for weighing small solid

bodies or for finding their specific gravities. To find the weight of a body (which we shall suppose to weigh less than 1000 grains), it must be placed in the dish at the top, together with weights just sufficient to make the instrument sink in water as far as the mark. Obviously these weights are the difference between the weight of the body and 1000 grains.

To find the specific gravity of a solid, we first ascertain its weight by the method just described; we then transfer it from the dish above to the basket below, so that it shall be under water during the observation, and observe what additional weights must now be placed in the dish. These additional weights represent the weight of the water displaced by the solid; and the weight of the solid itself divided by this weight is the specific gravity required.

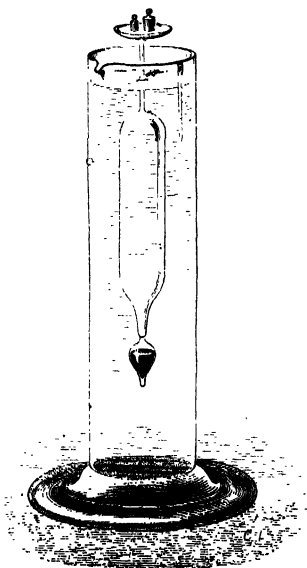


Fig. 79.—Fahrenheit's Hydrometer.

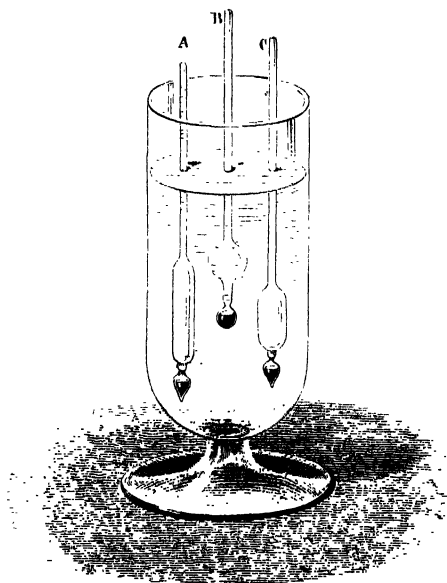


Fig. 80.—Forms of Hydrometers.

84. Fahrenheit's Hydrometer.—This instrument, which is represented in Fig. 79, is generally constructed of glass, and differs from Nicholson's in having at its lower extremity a ball weighted with mercury instead of the basket. It resembles it in having a dish at the top, in which weights are to be placed sufficient to sink the instrument to a definite mark on the stem.

85. Hydrometers of Variable Immersion.—These instruments are usu-

ally of the forms represented at A, B, C, Fig. 80. The lower end is weighted with mercury in order to make the instrument sink to a convenient depth and preserve an upright position. The stem is cylindrical, and is graduated, the divisions being frequently marked upon a piece of paper inclosed within the stem, which must in this case be of glass. It is evident that the instrument will sink the deeper the less is the specific gravity of the liquid, since the weight of the liquid displaced must be equal to that of the instrument. Hence if any uniform system of graduation be adopted, so that all the instruments give the same readings in liquids of the same densities, the density of a liquid can be obtained by a mere immersion of the hydrometer—an operation not indeed very precise, but very easy of execution. These instruments have thus come into general use for commercial purposes.

86. General Theory of Hydrometers of Variable Immersion.—Let V be the volume of a hydrometer which is immersed when the instrument floats freely in a liquid whose density (that is, weight per unit volume) is d , then Vd represents the weight of liquid displaced, which by the principle of Archimedes is the same as the weight of the hydrometer itself. If V', d' be the corresponding values for another liquid, we have therefore

$$Vd = V'd', \text{ or } d : d' :: V' : V,$$

that is, the density varies inversely as the volume immersed. Let $d_1, d_2, d_3 \dots$ be a series of densities in diminishing order, and $V_1, V_2, V_3 \dots$ the corresponding volumes immersed, which will be in ascending order; then we have

$$d_1, d_2, d_3 \dots \text{ proportional to } \frac{1}{V_1}, \frac{1}{V_2}, \frac{1}{V_3} \dots$$

$$\text{and } V_1, V_2, V_3 \dots \text{ proportional to } \frac{1}{d_1}, \frac{1}{d_2}, \frac{1}{d_3} \dots$$

Hence, if we wish the divisions to indicate equal differences of density, we must place them so that the corresponding volumes immersed form a harmonical progression. This implies that the divisions must approach nearer together for increasing densities. This is of course on the assumption that the stem is of equal sectional area in all parts as far as the divisions extend.

The following investigation shows how the density of a liquid may be computed from observations made with a hydrometer graduated with equal divisions. It is necessary first to know the divisions to which the instrument sinks in two liquids of known density. Let

these divisions be numbered n_1, n_2 , reckoning from the top downwards, and let the corresponding densities be d_1, d_2 . Now if we take for our unit of volume one of the equal parts on the stem, and if we take c to denote the volume which is immersed when the instrument sinks to the division marked zero, it is obvious that, when the instrument sinks to the n th division (reckoned downwards on the stem from zero) the volume immersed is $c - n$, and if the corresponding density be called d , then $(c - n) d$ is the weight of the hydrometer. We have therefore

$$(c - n_1) d_1 = (c - n_2) d_2, \text{ whence } c = \frac{n_1 d_1 - n_2 d_2}{d_1 - d_2}.$$

This value of c can be computed once for all.

Then the density D corresponding to any other division N can be found from the equation

$$(c - N) D = (c - n_1) d_1 \text{ which gives } D = \frac{c - n_1}{c - N},$$

if d_1 , which we may suppose to be the density of water, be called unity.

87. Beaumé's Hydrometers.—In these instruments the divisions



Fig. 81.
Beaumé's Salinometer.

are equidistant. There are two distinct modes of graduation, according as the instrument is to be used for determining densities greater or less than that of water. In the former case the instrument is called a salimeter, and is so constructed that when immersed in pure water of the temperature 12° Cent. it sinks nearly to the top of the stem, and the point thus determined is the zero of the scale. It is then immersed in a solution of 15 parts of salt to 85 of water, the density of which is about 1.116, and the point to which it sinks is marked 15. The interval is divided into 15 equal parts, and the graduation is continued to the bottom of the stem, the length of which

varies according to circumstances; it generally terminates at the degree 66, which corresponds to sulphuric acid, whose density is commonly the greatest that it is required to determine. Referring to the formulæ of last section, we have here

$$\therefore n_1 = 0, d_1 = 1, n_2 = 15, d_2 = 1.116;$$

whence

$$c = \frac{15 \times 1.116}{1.116} = 144, D = \frac{144}{144 - N}.$$

When the instrument is intended for liquids lighter than water, it is called an alcoholimeter. In this case the point to which it sinks in water is near the bottom of the stem, and is marked 10; the zero of the scale is the point to which it sinks in a solution of 10 parts of salt to 90 of water, the density of which is about 1.085, the divisions in this case being numbered upward from zero.

In order to adapt the formulæ of last section to the case of graduations numbered upwards, it is merely necessary to reverse the signs of n_1 , n_2 , and N ; that is we must put

$$c = \frac{n_2 d_2 - n_1 d_1}{d_1 - d_2}, D = \frac{c + n_1}{c + N};$$

and as we have now $n_1=10$, $d_1=1$, $n_2=0$, $d_2=1.085$ the formulæ give¹

$$c = \frac{10}{.085} = 118, D = \frac{128}{118 + N}.$$

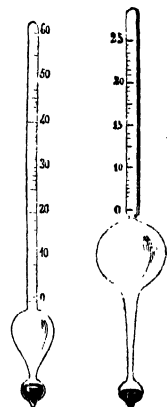


Fig. 82. Fig. 83.
Beaumé's Alcoholi-
meters.

87A. Twaddell's Hydrometer.—In this instrument the divisions are placed not as in Beaumé's, at equal distances, but at distances corresponding to equal differences of density. In fact the specific gravity of a liquid is found by multiplying the reading by 5, cutting off three decimal places, and prefixing unity. Thus the degree 1 indicates specific gravity 1.005, 2 indicates 1.010, &c.

88. Gay Lussac's Centesimal Alcoholimeter.—When a hydrometer is to be used for a special purpose it may be convenient to adopt a mode of graduation different in principle from any that we have described above, and adapted to give a direct indication of the proportion in which two ingredients are mixed in the fluid to be examined. It may indicate, for example, the quantity of salt in sea-water, or the quantity of alcohol in a spirit consisting of alcohol and water. Where there are three or more ingredients of different specific gravities the method fails. Gay-Lussac's alcoholimeter is graduated to indicate, at the temperature of 15° Cent.,

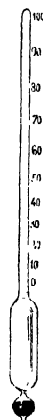


Fig. 87.
Centesimal Alco-
holimeter.

¹ On comparing the two formulæ for D in this section with the tables in the Appendix to Miller's *Chemical Physics*, I find that as regards the salimeter they agree to two places of decimals and very nearly to three. As regards the alcoholimeter, the table in *Miller* implies that c is about 136, which would make the density corresponding to the zero of the scale about 1.074.

the percentage of pure alcohol in a specimen of spirit. At the top of the stem is 100, the point to which the instrument sinks in pure alcohol, and at the bottom is 0, to which it sinks in water. The position of the intermediate degrees must be determined empirically, by placing the instrument in mixtures of alcohol and water in known proportions, at the temperature of 15° . The law of density, as depending on the proportion of alcohol present, is complicated by the fact that, when alcohol is mixed with water, a diminution of volume (accompanied by rise of temperature) takes place.

88A. Specific Gravity of Mixtures.—When two or more substances are mixed without either shrinkage or expansion (that is, when the volume of the mixture is equal to the sum of the volumes of the components), the density of the mixture can easily be expressed in terms of the quantities and densities of the components.

First, let the volumes $v_1, v_2, v_3 \dots$ of the components be given, together with their densities $d_1, d_2, d_3 \dots$.

Then their masses (or weights) are $v_1 d_1, v_2 d_2, v_3 d_3 \dots$.

The mass of the mixture is the sum of these masses, and its volume is the sum of the volumes $v_1, v_2, v_3 \dots$; hence its density is

$$\frac{v_1 d_1 + v_2 d_2 + \dots}{v_1 + v_2 + \dots}.$$

Secondly, let the weights or masses $m_1, m_2, m_3 \dots$ of the components be given, together with their densities $d_1, d_2, d_3 \dots$.

Then their volumes are $\frac{m_1}{d_1}, \frac{m_2}{d_2}, \frac{m_3}{d_3} \dots$.

The volume of the mixture is the sum of these volumes, and its mass is $m_1 + m_2 + m_3 + \dots$; hence its density is

$$\frac{m_1 + m_2 + \dots}{\frac{m_1}{d_1} + \frac{m_2}{d_2} + \dots}.$$

88B. Graphical Method of Graduation.—When the points on the stem which correspond to some five or six known densities, nearly equidifferent, have been determined, the intermediate graduations can be inserted with tolerable accuracy by the graphical method of interpolation, a method which has many applications in physics besides that which we are now considering. Suppose A and B (Fig. 85) to represent the extreme points, and I, K, L, R intermediate points, all of which correspond to known densities. Erect ordinates (that is to say, perpendiculars) at these points, proportional to the respective densities, or (which will serve our purpose equally well)

erect ordinates II' , KK' , LL' , RR' , BC proportional to the excesses of the densities at I , K , L , R , B above the density at A . Any scale of equal parts can be employed for laying off the ordinates, but it is convenient to adopt a scale which will make the greatest ordinate BC not much greater nor much less than the base line AB . In the figure, the density at B is supposed to be 1.80, the density at A being 1. The difference of density is therefore .80, as indicated by the figures 80 on the scale of equal parts.

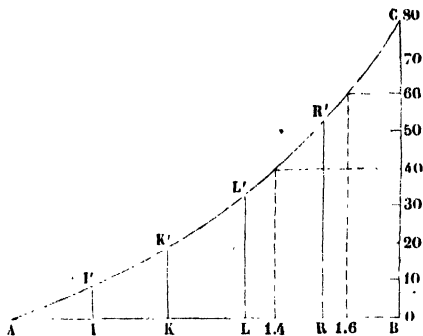


Fig. 85.—Graphical Method of Graduation.

Having erected the ordinates, we must draw through their extremities the curve $AI'K'L'R'C'$, making it as free from sudden turns as possible, as it is upon the regularity of this curve that the accuracy of the interpolation depends. Then to find the point on the stem AB at which any other density is to be marked—say 1.60, we must draw through the 60th division, on the line of equal parts, a horizontal line to meet the curve, and, through the point thus found on the curve, draw an ordinate. This ordinate will meet the base line AB in the required point, which is accordingly marked 1.6 in the figure. The curve also affords the means of solving the converse problem, that is, of finding the density corresponding to any given point on the stem. At the given point in AB , which represents the stem, we must draw an ordinate, and through the point where this meets the curve we must draw a horizontal line to meet the scale of equal parts. The point thus determined on the scale of equal parts indicates the density required, or rather the excess of this density above the density of A .

CHAPTER XI.

VESSELS IN COMMUNICATION.—CAPILLARITY.

89. Equilibrium in Vessels in Communication.—When a liquid is contained in vessels communicating with each other, and is in equilibrium, it stands at the same height in the different parts of the system, so that the free surfaces all lie in the same horizontal plane.

This is an immediate consequence of the fact that layers of equal

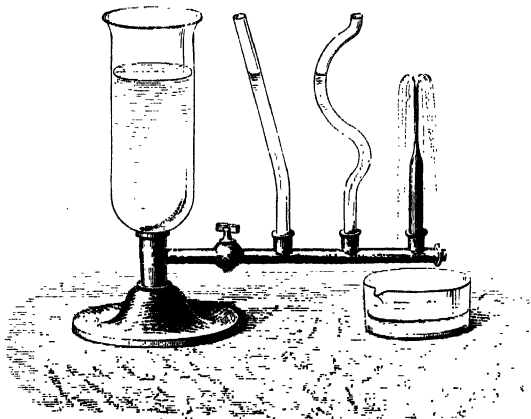


Fig. 89.—Vessels in Communication.

pressure in a liquid are always horizontal (§ 64); for if we take any such layer at the bottom of the system, we must proceed upwards through the same vertical height in all parts of the system in order to reach the free surface which corresponds to the pressure. Thus, in the system represented by Fig. 89, the liquid is seen to stand at the same height in the principal vessel and in the variously shaped tubes communicating with it. If one of these tubes is cut off at a height less than that of the liquid in the principal vessel, and if it

be made to terminate in a narrow mouth, the liquid will be seen to spout up nearly to the level of that in the principal vessel.

This tendency of liquids to find their own level is very important, and of continual application. Thus, a reservoir of water may have different pipes issuing from it and spreading out in all possible directions with any number of turns and windings; provided that the ends of these pipes lie below the level of the reservoir, the water will flow through the pipes and run out at their extremities. The velocity of exit, however, will depend on the form and arrangement of the pipes, as well as on the difference of level. This velocity must of course be taken into account in calculating the quantity of water that will flow in a given time; and in forming plans for the proper distribution of public supplies of water. It also determines the height to which a jet of water can be discharged from an opening at the end of the pipe.

90. **Water-level.**—The well-known instrument called the water-level depends upon the property just mentioned. It consists of a

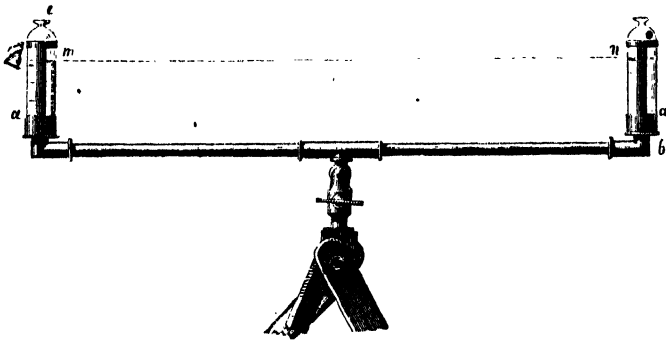


Fig. 90.---Water-level.

metal tube *bb*, bent at right angles at its extremities. These carry two glass tubes *aa*, very narrow at the top, and of the same diameter. The tube rests on a tripod stand, at the top of which is a joint that enables the observer to turn the apparatus and set it in any direction. The tube is placed in a position *nearly* horizontal, and water, generally coloured a little, is poured in until it stands at about three-fourths of the height of each of the glass tubes.

By the principle of equilibrium in vessels communicating with each other, the surfaces of the liquid in the two branches are in the same horizontal plane, so that if the line of the observer's sight just grazes the two surfaces, it will be horizontal.

This is the principle of the operation called *levelling*, the object of which is to determine the difference of vertical height, or *difference of level*, between two given points. Suppose A and B to be the two points (Fig. 91). At each of these points is fixed a levelling-staff,



Fig. 91.—Levelling.

that is, an upright rod divided into parts of equal length, on which slides a small square board whose centre serves as a mark for the observer.

The level being placed at an intermediate station, the observer directs the line of sight towards each levelling-staff, and the mark is raised or lowered till the line of sight passes through its centre. The marks on the two staves are in this way brought to the same level. The staff in the rear is then carried in advance of the other, the level is again placed between the two, and another observation taken. In this way, by noting the division of the staff at which the sliding mark stands in each case, the difference of levels of two distant stations can be deduced from observations at a number of intermediate points.

91. Spirit-level.—These observations can be made in a much more exact and convenient manner by means of the spirit-level. This

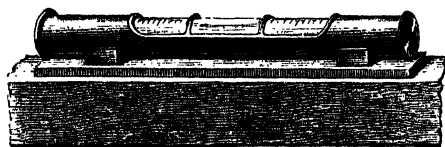


Fig. 92.—Spirit-level.

instrument is composed of a glass tube slightly curved, containing a liquid, which is generally alcohol, and which fills the whole extent of the tube, except a small space occupied by an air-bubble.

This tube is inclosed in a mounting which is firmly supported on a stand.

Suppose the tube to have been so constructed that a vertical section of its upper surface is an arc of a circle, and suppose the

instrument placed upon a horizontal plane (Fig. 93). The air-bubble will take up a position MN at the highest part of the tube, such that the arcs MA and NB are equal. Hence it follows that if the level be reversed end for end, the bubble will occupy the same position, the point N coming to M, and *vice versa*. This will not be the case

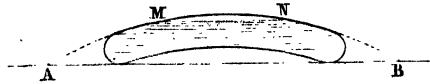


Fig. 93.

if AB is inclined to the horizon (Fig. 94), for then the distance MA being different from NB,

after the apparatus has been turned, the bubble will assume a symmetrical position at the opposite end of the tube. The condition,

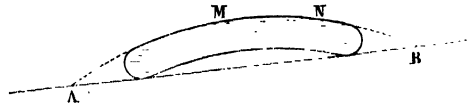


Fig. 94.

therefore, that the line on which the spirit-level rests should be horizontal is, that after this operation of reversal the bubble should remain within the same limits. In order to avoid the trouble of turning the instrument, the maker marks these limits by reference-marks on the tube or its mounting, and in order to determine that a line is horizontal it is only necessary to make sure that, when the level is placed upon it, the bubble lies exactly between these reference-marks.

In order that a plane surface may be horizontal, we must have two lines in it horizontal. This result is practically attained in the

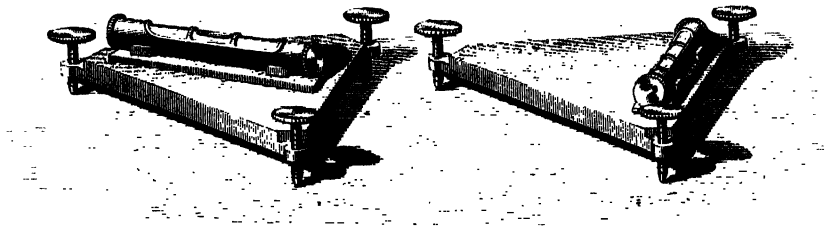


Fig. 95.—Testing the Horizontality of a Surface.

following manner:—The surface is made to rest on three levelling screws which form the three vertices of an isosceles triangle; the level is first placed parallel to the base of the triangle, and, by means of one of the screws, the bubble is brought between the reference-marks. The instrument is then placed perpendicularly to its first position, and the bubble is brought between the marks by means of

the third screw; this second operation cannot disturb the result of the first, since the plane has only been turned about a horizontal line as hinge.

92. *Level furnished with Telescope.*—In order to apply the spirit-level to land-surveying, an apparatus such as that represented in the figure is employed.

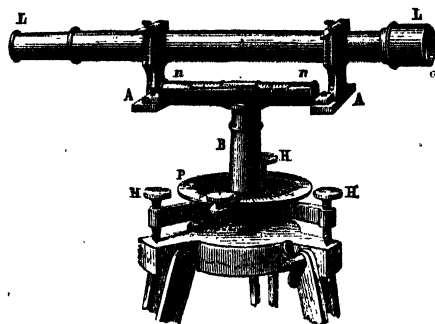


Fig. 96.—Spirit-level with Telescope.

Upon a frame AA, movable about a vertical axis B, are placed a spirit-level nn, and a telescope LL, in positions parallel to each other. The telescope is furnished at its focus with two fine wires crossing one another, whose point of intersection determines the line of sight with great precision. The appara-

tus, which is provided with levelling screws H, rests on a tripod stand, and the observer is able, by turning it about its axis, to command the different points of the horizon. By a process of adjustment which need not here be described, it is known that when the bubble is between the marks the line of sight is horizontal; so that we may proceed to find the difference of level between two points in the same way as with the water-level; but the operation is much more precise, and the range of vision much more extensive. By furnishing the instrument with a graduated horizontal circle P, we may obtain the azimuths of the points observed, and thus map out contour lines.

On each side of the reference-marks of the bubble are divisions for measuring small deviations from horizontality. It is, in fact, easy to see, by reference to Fig. 93, that by tilting the level through any small angle, the bubble is displaced by a quantity proportional to this angle, at least when the curvature of the instrument is that of a circle.

For determining the angular value corresponding to each division of the tube, it is usual to employ an apparatus opening like a pair of compasses by a hinge C, on one of the legs of which rests, by two V-shaped supports, the tube T of the level. The compass is opened by means of a micrometer screw V, of very regular action; and as the distance of the screw from the hinge is known, as well as the

distance between the threads of the screw, it is easy to calculate beforehand the value of the divisions on the micrometer head. The levelling screws of the instrument serve to bring the bubble between

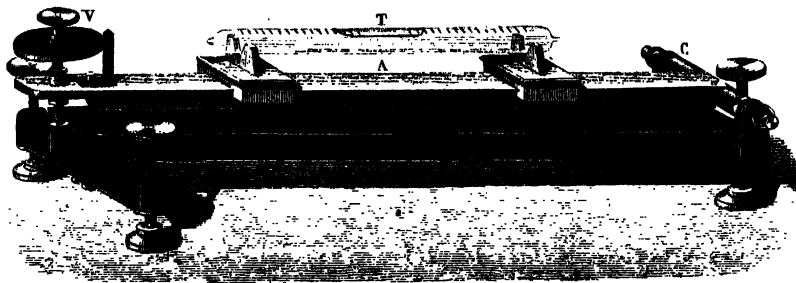


Fig. 97.—Graduation of Spirit-level.

its reference-marks, so that the micrometer screw is only used to determine the value of the divisions on the tube.

93. Equilibrium of Two Different Liquids in Communicating Vessels.

—If into one of two tubes in communication we pour a liquid, say mercury, this liquid will rise to the same height in both branches. If we now pour water into one of them, the mercury will be pushed back in the other branch; and when equilibrium has been established, the *heights of the two liquids above the surface of separation* will be very unequal, as shown in the figure. In general, these heights, since they correspond to the same pressure upon the surface of separation, *will be inversely proportional to the densities.*

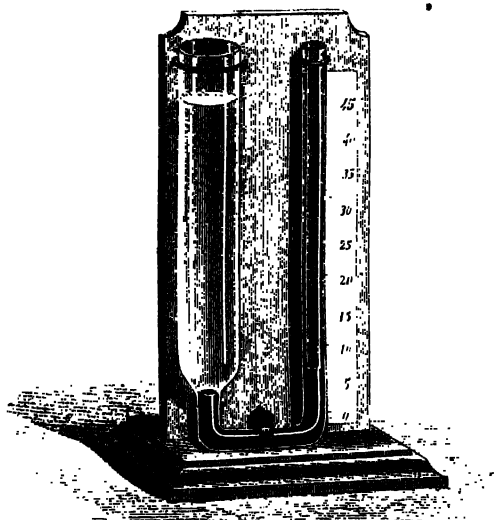


Fig. 98.—Equilibrium of Two Fluids in Communicating Vessels.

94. Capillarity—General Phenomena.—The different principles of equilibrium which have been explained in the preceding paragraphs, are subject to remarkable exceptions when the vessels in

which the liquids are contained are very narrow, or, as they are called, capillary (*capillus*, a hair); and also in the case of vessels of any size, when we consider the portion of the liquid which is in close proximity to the sides.

1. *Free Surface*.—The surface of a liquid is not horizontal in the neighbourhood of the sides of the vessel, but presents a very decided curvature. When the liquid wets the vessel, as in the case of water in a glass vessel (Fig. 99), the surface is concave; on the contrary when the liquid does not wet the vessel, as in the case of mercury in a glass vessel (Fig. 100), the surface is, generally speaking, convex.

2. *Capillary Elevation and Depression*.—If a very narrow tube of glass be plunged in water, or any other liquid that will wet it

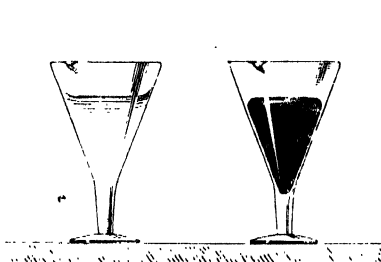


Fig. 99.

Fig. 100.

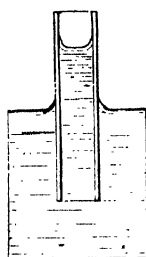


Fig. 101.



Fig. 102.

(Fig. 101), it will be observed that the level of the liquid, instead of remaining at the same height inside and outside of the tube, stands perceptibly higher in the tube; a *capillary ascension* takes place,

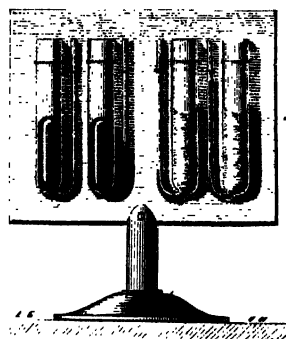


Fig. 103.

which varies in amount according to the nature of the liquid and the diameter of the tube. It will also be seen that the liquid column thus raised terminates in a concave surface. If a glass tube be dipped in mercury, which does not wet it, it will be seen, by bringing the tube to the side of the vessel, that the mercury is depressed in its interior, and that it terminates in a convex surface (Fig. 102).

3. *Capillary Vessels in Communication with Others*.—If we take two bent tubes, each having one branch of a considerable diameter and the other extremely narrow, and pour into one of them a liquid which

wets it, and into the other mercury, the liquid will be observed in the former case to stand higher in the capillary than in the principal branch, and in the latter case to stand lower; the free surfaces being at the same time concave in the case of the liquid which wets the tubes, and convex in the case of the mercury.

95. Circumstances which influence Capillary Elevation and Depression.—In wetted tubes the elevation depends upon the nature of the liquid; thus, at the temperature of 18° Cent., water rises 29.79^{mm} (1.2 inch) in a tube 1 millimetre ($\frac{1}{8}$ inch) in diameter, alcohol rises 12.18^{mm} , nitric acid 22.57^{mm} , essence of lavender 4.28^{mm} , &c. The nature of the tube is almost entirely immaterial, provided the precaution be first taken of wetting it with the liquid to be employed in the experiment, so as to leave a film of the liquid adhering to the sides of the tube.

Capillary depression, on the other hand, depends both on the nature of the liquid and on that of the tube. Both ascension and depression diminish as the temperature increases; for example, the elevation of water, which in a tube of a certain diameter is equal to 132^{mm} at 0° Cent., is only 106^{mm} at 100° .

96. Law of Diameters.—*Capillary elevations and depressions when all other circumstances are the same, are inversely proportional to the diameters of the tubes.* As this law is a consequence of the mathematical theories which are generally accepted as explaining capillary phenomena, its verification has been regarded as of great importance.

The experiments of Gay-Lussac, which confirmed this law, have been repeated, with slight modifications, by several observers. The method employed consists essentially in measuring the capillary elevation of a liquid by means of a cathetometer (Fig. 104). The telescope ll is directed first to the top n of the column in the tube, and then to the end of a pointer b , which touches the surface of the liquid at a point where it is horizontal. In observing the depression of mercury, since the opacity of the metal prevents us from seeing the tube, we must bring the tube close to the side of the vessel e .

The diameter of the tube can be measured directly by observing its section through a microscope, or we may proceed by the method employed by Gay-Lussac. He weighed the quantity of mercury which filled a known length l of the tube; this weight w is that of a cylinder of mercury whose radius x is determined by the equation $13.59 \pi x^2 l = w$.*

* This formula is only true for the metrical system, 13.59 being the specific gravity of mercury. If l and l are in centimetres, w will be in grammes.

The result of these different experiments is, that in the case of wetted tubes the law is exactly fulfilled, provided that they be previously washed with the greatest care, so as to remove all foreign matters, and that the liquid on which the experiment is to be per-

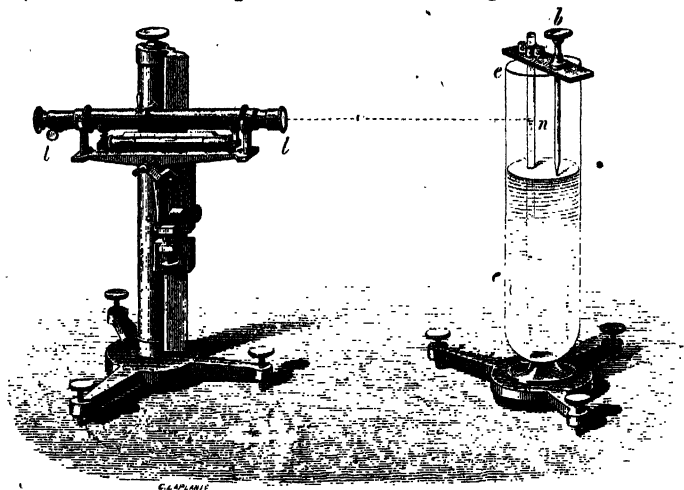


Fig. 104.—Verification of Law of Diameters.

formed be first passed through them. When the liquid does not wet the tube, various causes combine to affect the form of the surface in which the liquid column terminates; and we cannot infer the depression from knowing the diameter, unless we also take into consideration some element connected with the form of the terminal surface, such as the length of the sagitta, or the angle made with the sides of the tube by the extremities of the curved surface, which is called the *angle of contact*.

97. Cause of Capillary Phenomena.—Capillary phenomena, as they take place alike in air and in vacuo, cannot be attributed to the action of the atmosphere. They depend upon molecular actions which take place between the particles of the liquid itself, and between the liquid and the solid containing it, the actions in question being purely superficial—that is to say, being confined to an extremely thin layer forming the external boundary of the liquid, and to an extremely thin superficial layer of the solid in contact with the liquid. For example, it is found in the case of glass tubes, that the amount of capillary elevation or depression is not at all affected by the thickness of the sides of the tube. The following are some of the principles which govern capillary phenomena.

1. For a given liquid in contact with a given solid, with a definite intimateness of contact (this last element being dependent upon the cleanness of the surface, upon whether the surface of the solid has been recently washed by the liquid, and perhaps upon some other particulars), there is (at any specified temperature) a definite angle of contact, which is independent of the directions of the surfaces with regard to the vertical.

2. Every liquid behaves as if a thin film, forming its external layer, were in a state of tension, and exerting a constant effort to contract. This tension, or contractile force, is exhibited over the whole of the free surface (that is, the surface which is exposed to air), but wherever the liquid is in contact with a solid, its existence is masked by other molecular actions. It is uniform in all directions in the free surface, and at all points in this surface, being dependent only on the nature and temperature of the liquid. Its intensity for several specified liquids is given in tabular form further on (§ 97r) upon the authority of Van der Mensbrugghe. Tension of this kind must of course be stated in units of force per linear unit, because by doubling the width of a band we double the force required to keep it stretched. Mensbrugghe considers that such tension really exists in the superficial layer; but the majority of authors (and we think with more justice) regard it rather as a convenient fiction, which accurately represents the effects of the real cause. Two of the most eminent writers on the cause of capillary phenomena are Laplace and Dr. Thomas Young. The subject presents difficulties which have not yet been fully surmounted.

The law of diameters is a direct consequence of the two preceding principles; for if α denote the external angle of contact (which is acute in the case of mercury against glass), T the tension per unit length, and r the radius of the tube, then $2\pi rT$ will be the whole amount of force exerted at the margin of the surface; and as this force is exerted in a direction making an angle α with the vertical, its vertical component will be $2\pi rT \cos \alpha$, which is exerted in pulling the tube upwards and the liquid downwards.

If w be the weight of unit volume of the liquid, then $\pi r^2 w$ is the weight of as much as would occupy unit length of the tube; and if h denote the height of a column whose weight is equal to the force tending to depress the liquid, we have

$$\pi r^2 h w = 2\pi r T \cos \alpha;$$

whence $h = \frac{2T \cos \alpha}{r w}$, which, when the other elements are given, varies inversely as r , the radius of the tube.

Having regard to the fact that the surface is not of the same height in the centre as at the edges, it is obvious that h denotes the mean height.

If α be obtuse, h will be negative—that is to say, there will be elevation instead of depression. In the case of water against a tube which has been well wetted with that liquid, α is 180° —that is to say, the tube is tangential to the surface. For this case the formula for h gives

$$\text{elevation} = \frac{2T}{rw}$$

Again, for two parallel vertical plates at distance u , the vertical force of capillarity for a unit of length is $2T \cos \alpha$, which must be equal to whu , being the weight of a sheet of liquid of height h , thickness u , and length unity. We have therefore

$$h = \frac{2T \cos \alpha}{uw},$$

which agrees with the expression for the depression or elevation in a circular tube whose radius is equal to the distance between these parallel plates.

The surface tension always tends to reduce the surface to the smallest area which can be inclosed by its actual boundary; and therefore always produces a normal force directed *towards* the concave side of the superficial film. Hence, wherever there is capillary elevation the free surface must be concave; wherever there is depression it must be convex.

97A. It follows from a well-known proposition in statics (Todhunter's *Statics*, § 194), that if a *cylindrical* film be stretched with a uniform tension T (so that the force tending to pull the film asunder across any short line drawn on the film, is T times the length of the line), the resultant normal pressure (which the film exerts, for example, against the surface of a solid internal cylinder over which it is stretched) is T divided by the radius of the cylinder.

It can be proved that a film of any form, stretched with uniform tension T , exerts at each point a normal pressure equal to the sum of the pressures which would be exerted by two overlapping cylindrical films, whose axes are at right angles to one another, and whose cross sections are circles of curvature of normal sections at the

point. That is to say, if P be the normal force per unit area, and rr' the radii of curvature in two mutually perpendicular normal sections at the point, then

$$P = T \left(\frac{1}{r} + \frac{1}{r'} \right).$$

At any point on a curved surface, the normal sections of greatest and least curvature are mutually perpendicular, and are called the principal normal sections at the point. If the corresponding radii of curvature be R, R' , we have

$$P = T \left(\frac{1}{R} + \frac{1}{R'} \right); \quad (1)$$

or the normal force per unit area is equal to the tension per unit length multiplied by the sum of the principal curvatures.

In the case of capillary depressions and elevations, the superficial film at the free surface is to be regarded as pressing the liquid inwards, or pulling it outwards, according as this surface is convex or concave, with a force P given by the above formula. The value of P at any point of the free surface is equal to the pressure due to the height of a column of liquid extending from that point to the level of the general horizontal surface. It is therefore greatest at the edges of the elevated or depressed column in a tube, and least in the centre; and the curvature, as measured by $\frac{1}{R} + \frac{1}{R'}$, must vary in the same proportion. If the tube is so large that there is no sensible elevation or depression in the centre of the column, the centre of the free surface must be sensibly plane.

97B. Another consequence of the formula is, that in circumstances where there can be no normal pressure towards either side of the surface,

$$\frac{1}{R} + \frac{1}{R'} = 0; \quad (2)$$

which implies that either the surface is plane, in which case each of the two terms is separately equal to zero, or else

$$R = -R'; \quad (3)$$

that is, the principal radii of curvature are equal, and lie on opposite sides of the surface. The formulæ (2), (3) apply to a film of soapy water attached to a loop of wire. If the loop be in one plane, the film will be in the same plane. If the loop be not in one plane, the film cannot be in one plane; and will in fact assume that form which

gives the least area consistent with having the loop for its boundary. At every point it will be observed to be, if we may so say, concave towards both sides, and convex towards both sides, the concavity being precisely equal to the convexity—that is to say, equation (3) is satisfied at every point of the film.

In this case both sides of the film are exposed to atmospheric pressure. In the case of a common soap-bubble the outside is exposed to atmospheric pressure, and the inside to a pressure somewhat greater, the difference of the pressures being compensated by the tendency of the film to contract. Formula (1) becomes for either the outer or inner surface of a spherical bubble

$$P = \frac{2T}{R};$$

but this result must be doubled, because there are two free surfaces; hence the excess of pressure of the inclosed above the external air is $\frac{4T}{R}$, R denoting the radius of the bubble.

The value of T for soapy water is about 1 grain per linear inch; hence, if we divide 4 by the radius of the bubble expressed in inches, we shall obtain the excess of internal over external pressure *in grains per square inch*.

The value of T for any liquid may be obtained by observing the amount of elevation or depression in a tube of given diameter, and employing the formula

$$T = \frac{whr}{2\cos\alpha}, \quad (4)$$

which follows immediately from the formula for h in § 97.

97c. It is this uniform surface tension, of which we have been speaking, which causes a drop of a liquid falling through the air either to assume the spherical form, or to oscillate about the spherical form. The phenomena of drops can be imitated on an enlarged scale, under circumstances which permit us to observe the actual motions, by a method devised by Professor Plateau of ~~Geneva~~. Olive-oil is intermediate in density between water and alcohol. Let a mixture of alcohol and water be prepared, having precisely the density of olive-oil, and let about a cubic inch of the latter be gently introduced into it with the aid of a funnel or pipette. It will assume a spherical form, and if forced out of this form and then left free, will slowly oscillate about it; for example, if it has been compelled to assume the form of a prolate spheroid, it will pass to the

oblate form, will then become prolate again, and so on alternately, becoming however more nearly spherical every time, because its movements are hindered by friction, until at last it comes to rest as a sphere.

97D. Capillarity furnishes no exception to the principle that the pressure in a liquid is the same at all points at the same depth. When the free surface within a tube is convex, and is consequently depressed below the general level of the external surface, the pressure becomes suddenly greater on passing downwards through the superficial layer, by the amount due to the curvature. Below this it increases regularly by the amount due to the depth of liquid passed through. The pressure at any point vertically under the convex meniscus¹ may be computed, either by taking the depth of the point below the general free surface, and adding atmospheric pressure to the pressure due to this depth, according to the ordinary principles of hydrostatics, or by taking the depth of the point below that point of the meniscus which is vertically over it, adding the pressure due to the curvature at this point, and also adding atmospheric pressure.

When the free surface of the liquid within a tube is concave, the pressure suddenly diminishes on passing downwards through the superficial layer, by the amount due to the curvature as given by formula (1); that is to say, the pressure at a very small depth is less than atmospheric pressure by this amount. Below this depth it goes on increasing according to the usual law, and becomes equal to atmospheric pressure at that depth which corresponds with the level of the general external surface. The pressure at any point in the liquid within the tube can therefore be obtained either by subtracting from atmospheric pressure the pressure due to the elevation of the point above the general surface, or by adding to atmospheric pressure the pressure due to the depth below that point of the meniscus which is on the same vertical, and subtracting the pressure due to the curvature at this point.

These rules imply, as has been already remarked, that the curvature is different at different points of the meniscus, being greatest where the elevation or depression is greatest, namely at the edges of the meniscus; and least at the point of least elevation or depression, which in a cylindrical tube is the middle point.

¹ The convex or concave surface of the liquid in a tube is usually denoted by the name *meniscus* (μηνίσκος, a crescent), which denotes a form approximately resembling that of a watch-glass.

The principles just stated apply to all cases of capillary elevation and depression.

They enable us to calculate the force with which two parallel vertical plates, partially immersed in a liquid which wets them, are urged towards each other by capillary action. The portion of liquid elevated between them is at less than atmospheric pressure, and therefore is insufficient to resist the atmospheric pressure which is exerted on the outer faces of the plates. The average pressure in the elevated portion of liquid is that which exists half-way up it, and is less than atmospheric pressure by the pressure of a column of liquid whose height is half the elevation.

Even if the liquid be one which does not wet the plates, they will still be urged towards each other by capillary action; for the inner faces of the plates are exposed to merely atmospheric pressure over that portion of their areas which corresponds to the depression, while the corresponding portions of the external faces are exposed to atmospheric pressure increased by the weight of a portion of the liquid.

These principles explain the apparent attraction exhibited by bodies floating on a liquid which either wets them both or wets neither of them. When the two bodies are near each other they behave somewhat like parallel plates, the elevation or depression of the liquid between them being greater than on their remote sides.

If two floating bodies, one of which is wetted and the other unwetted by the liquid, come near together, the elevation and depression of the liquid will be less on the near than on the remote sides, and apparent repulsion will be exhibited.

In all cases of capillary elevation or depression, the solid is pulled downwards or upwards with a force equal to that by which the liquid is raised or depressed. In applying the principle of Archimedes to a solid partially immersed in a liquid, it is therefore necessary (as we have seen in § 79), when the solid produces capillary depression, to reckon the void space thus created as part of the displacement; and when the solid produces capillary elevation, the fluid raised above the general level must be reckoned as *negative* displacement, tending to *increase* the apparent weight of the solid.

97E. Thus far all the effects of capillary action which we have mentioned are connected with the curvature of the superficial film, and depend upon the principle that a convex surface increases and a concave surface diminishes the pressure in the interior of the liquid.

But there is good reason for maintaining that whatever be the form of the free surface there is always a certain amount of pressure in the interior due to the molecular action at this surface, and that the pressure due to the curvature of the surface is to be added to or subtracted from a definite amount of pressure which is independent of the curvature and depends only on the nature and condition of the liquid. This indeed follows at once from the fact that capillary elevation can take place in vacuo. As far as the principles of the preceding paragraphs are concerned, we should have, at points within the elevated column, a pressure less than that existing in the vacuum. This, however, cannot be; we cannot conceive of negative pressure existing in the interior of a liquid, and we are driven to conclude that the elevation is owing to the excess of the pressure caused by the plane surface in the containing vessel above the pressure caused by the concave surface in the capillary tube.

There are some other facts which seem only explicable on the same general principle of interior pressure due to surface action,—facts which attracted the notice of some of the earliest writers on pneumatics, namely, that siphons will work in vacuo, and that a column of mercury at least 75 inches in length can be sustained—as if by atmospheric pressure—in a barometer tube, the mercury being boiled and completely filling the tube.

97F. We have now to notice certain phenomena which depend on the difference in the surface tensions of different liquids, or of the same liquid in different states.

Let a thin layer of oil be spread over the upper surface of a thin sheet of brass, and let a lamp be placed underneath. The oil will be observed to run away from the spot directly over the flame, even though this spot be somewhat lower than the rest of the sheet. This effect is attributable to the excess of surface tension in the cold oil above the hot.

In like manner, if a drop of alcohol be introduced into a thin layer of water spread over a nearly horizontal surface, it will be drawn away in all directions by the surrounding water, leaving a nearly dry spot in the space which it occupied. In this experiment the water should be coloured in order to distinguish it from the alcohol.

Again, let a very small fragment of camphor be placed on the surface of hot water. It will be observed to rush to and fro with frequent rotations on its own axis, sometimes in one direction and

sometimes in the opposite. These effects, which have been a frequent subject of discussion, are now known to be due to the diminution of the surface tension of the water by the camphor which it takes up. Superficial currents are thus created, radiating from the fragment of camphor in all directions; and as the camphor dissolves more quickly in some parts than in others, the currents which are formed are not equal in all directions, and those which are most powerful prevail over the others and give motion to the fragment.

The values of T , the apparent surface tension, for several liquids, are given in the following table, on the authority of Van der Mensbrugghe, in milligrammes (or thousandth parts of a gramme) per millimetre of length. They can be reduced to grains per inch of length by multiplying them by $\cdot 392$; for example, the surface tension of distilled water is $7\cdot 3 \times \cdot 392 = 2\cdot 86$ grains per inch.

Distilled water at 20° Cent.,	7·3	Solution of Marseilles soap, 1 part of	
Sulphuric ether,	1·88	soap to 40 of water,	2·83
Absolute alcohol,	2·5	Solution of saponine,	4·67
Olive-oil,	3·5	Saturated solution of carbonate of	
Mercury,	49·1	soda,	4·28
Bisulphide of carbon,	3·57	Water impregnated with camphor, .	4·5

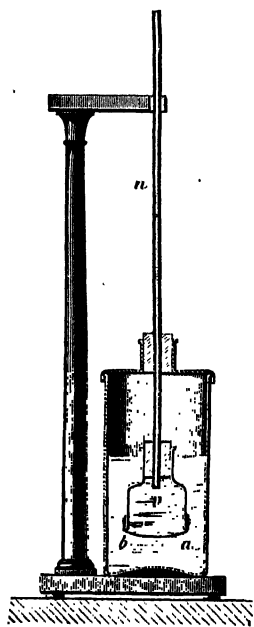


Fig. 105.—Endosmometer.

98. Endosmose.—Capillary phenomena have undoubtedly some connection with a very important property discovered by Dutrochet, and called by him *endosmose*.

The *endosmometer* invented by him to illustrate this phenomenon consists of a reservoir v closed below by a membrane ba , and terminating above in a tube of considerable length. This reservoir is filled, suppose, with a solution of gum in water, and is kept immersed in water. At the end of some time the level of the liquid in the tube will be observed to have risen to n , suppose, and at the same time traces of gum will be found in the water in which the reservoir is immersed. Hence we conclude that the two liquids have penetrated through the membrane, but in different proportions; and this is what is called *endosmose*.

If instead of a solution of gum we employed water containing albumen, sugar, or gelatine in solution, a

similar result would ensue. The membrane may be replaced by a slab of wood or of porous clay. Physiologists have justly attached very great importance to this discovery of Dutrochet. It explains, in fact, the interchange of liquids which is continually taking place in the tissues and vessels of the animal system, as well as the absorption of water by the spongioles of roots, and several similar phenomena.

As regards the power of passing through porous diaphragms, Graham has divided substances into two classes—*crystalloids* and *colloids* (κόλλη, glue). The former are susceptible of crystallization, form solutions free from viscosity, are sapid, and possess great powers of diffusion through porous septa. The latter, including gum, starch, albumen, &c., are characterized by a remarkable sluggishness and indisposition both to diffusion and to crystallization, and when pure are nearly tasteless.

CHAPTER XII.

THE BAROMETER.

99. Weight of the Air and of Gases.—Gaseous bodies possess a number of properties in common with liquids; like them, they transmit pressures entire and in all directions, according to the principle of Pascal; but they differ essentially from liquids in the permanent repulsive force exerted between their molecules, in virtue of which a mass of gas always tends to expand.

The opinion was long held that the air was without weight; or, to speak more precisely, it never occurred to any of the philosophers who preceded Galileo to attribute any influence in natural phenomena to the weight of the air. And as this influence is really of the first importance, and comes into play in many of the commonest phenomena, it very naturally happened that the discovery of the weight of air formed the commencement of the modern revival of physical science.

It appears, however, that Aristotle conceived the idea of the possibility of air having weight, and, in order to convince himself on this point, he weighed a skin inflated and collapsed. As he obtained the same weight in both cases, he relinquished the idea which he had for the moment entertained. In fact, the experiment, as he performed it, could only give a negative result; for if the weight of the skin was increased, on the one hand, by the introduction of a fresh quantity of air, it was diminished, on the other, by the corresponding increase in the upward pressure of the air displaced. In order to draw a certain conclusion, the experiment should be performed with a vessel which could receive within it air of different degrees of density, without changing its own volume.

Galileo is said to have devised the experiment of weighing a globe filled alternately with ordinary air and with compressed air.

As the weight is greater in the latter case, Galileo should have drawn the inference that air is heavy. It does not appear, however, that the importance of this conclusion made much impression on him, for he did not give it any of those developments which might have been expected to present themselves to a mind like his.

100. Experiment of Otto Guericke.—Otto Guericke, the illustrious inventor of the air-pump, in 1650 performed the following experiment, which is decisive:—

A globe of glass, furnished with a stop-cock, and of a sufficient capacity (about twelve litres), is exhausted of air. It is then suspended from one of the scales of a balance, and a weight sufficient to produce equilibrium is placed in the other scale. The stop-cock is then opened, the air rushes into the globe, and the beam is observed gradually to incline, so that an additional weight is required in the other scale, in order to re-establish equilibrium. If the capacity of the globe is 12 litres, about 15·5 grammes will be needed, which gives 1·3 gramme as the approximate weight of a litre of air.¹

If, in performing this experiment, we take particular precautions to insure its precision, as we shall explain in the book on heat, it will be found that, at the temperature of freezing water, and under the pressure of one atmosphere, a litre of air weighs 1·293 gramme.² Under these circumstances, the ratio of a volume of air to an equal volume of water is $\frac{1\cdot293}{1000} = \frac{1}{773}$. Air is thus 773 times lighter than water.

By repeating this experiment with other gases, we may determine

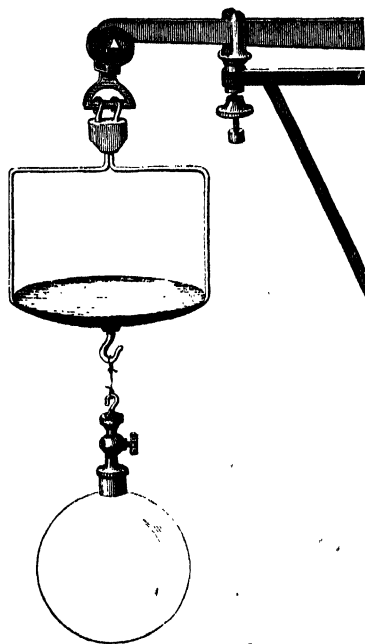


Fig. 106.—Weight of Air.

¹ A cubic foot of air in ordinary circumstances weighs about an ounce and a quarter.

² In strictness, the weight in grammes of a litre of air under the pressure of 760 millimetres of mercury is different in different localities, being proportional to the intensity of gravity—not because the force of gravity on the litre of air is different, for though this

their weight as compared with that of air, and the absolute weight of a litre of each of them. Thus it is found that a litre of oxygen weighs 1.43 gramme, a litre of carbonic acid 1.97 gramme, a litre of hydrogen 0.089 gramme, &c.

101. Atmospheric Pressure.—The atmosphere encircles the earth with a layer some 50 or 100 miles in thickness; this heavy fluid mass exerts on the surface of all bodies a pressure entirely analogous both in nature and origin to that sustained by a body wholly immersed in a liquid. It is subject to the fundamental law mentioned in § 64. The pressure should therefore diminish as we ascend from the surface of the earth, but should have the same value for all points in the same horizontal layer, provided that the air is in a state of equilibrium. On account of the great compressibility of gas, the lower layers are much more dense than the upper ones; but the density, like the pressure, is constant in value for the same horizontal layer, throughout any portion of air in a state of equilibrium. Whenever there is an inequality either of density or pressure at a given level, wind must ensue.

We owe to Torricelli an experiment which plainly shows the pressure of the atmosphere, and enables us to estimate its intensity with great precision. This experiment, which was performed in 1643, one year after the death of Galileo, at a time when the weight and pressure of the air were scarcely even suspected, has immortalized the name of its author, and has exercised a most important influence upon the progress of natural philosophy.

102. Torricelli's Experiment.—A tube of about a quarter or a third of an inch in diameter, and about a yard in length, is completely filled with mercury; the extremity is then stopped with the finger, and the tube is inverted in a vessel containing mercury. If the finger is now removed, the mercury will descend in the tube, and after a few

is true, it does not affect the numerical value of the weight when stated in grammes, but because the pressure of 760 millimetres of mercury varies as the intensity of gravity, so that more air is compressed into the space of a litre as gravity increases. (§ 107, 6.)

The *weight in grammes* is another name for the *mass*. The force of gravity on a litre of air under the pressure of 760 millimetres is proportional to the square of the intensity of gravity.

This is an excellent example of the ambiguity of the word *weight*, which sometimes denotes a mass, sometimes a force; and though the distinction is of no practical importance so long as we confine our attention to one locality, it cannot be neglected when different localities are compared.

Regnault's determination of the weight of a litre of dry air at 0° Cent. under the pressure of 760 millimetres at Paris is 1.293187 gramme. Gravity at Paris is to gravity at Greenwich as 3456 to 3457. The corresponding number for Greenwich is therefore 1.293561.

oscillations will remain stationary at a height which varies according to circumstances, but which is generally about 30 inches.

The column of mercury is maintained at this height by the pressure of the atmosphere upon the surface of the mercury in the

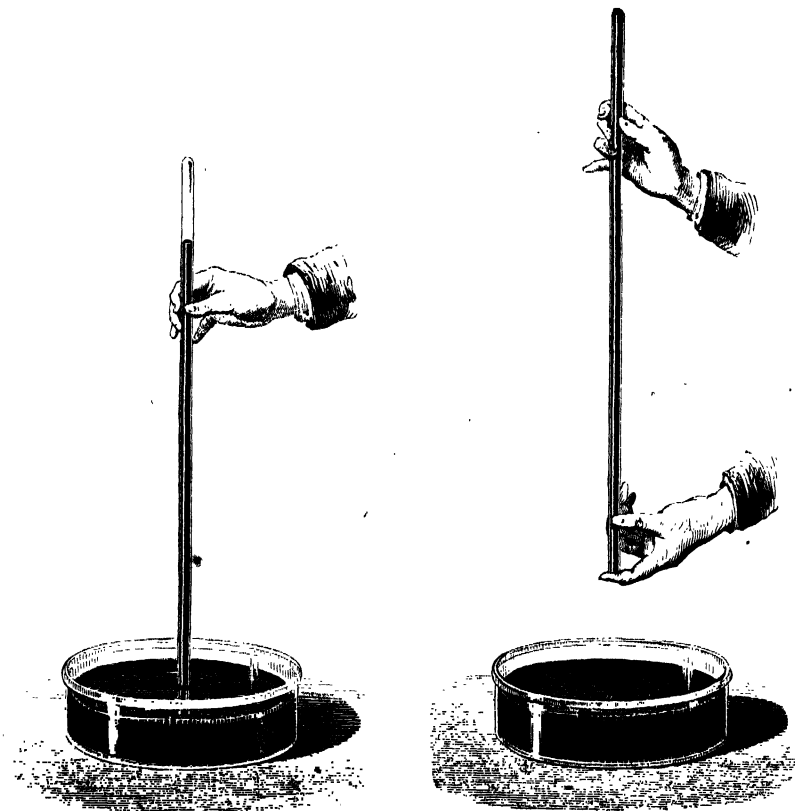


Fig. 108.—Torricellian Experiment.

vessel. In fact, the pressure at the level ABCD must be the same within as without the tube; so that the column of mercury BE exerts a pressure equal to that of the atmosphere.

Accordingly, we conclude from this experiment of Torricelli that *every surface exposed to the atmosphere sustains a normal pressure equal, on an average, to the weight of a column of mercury whose base is this surface, and whose height is 30 inches.*

It is evident that if we performed a similar experiment with water, whose density is to that of mercury as 1 : 13.59, the height of the column sustained would be 13.59 times as much; that is,

30 × 13.59 inches, or about 34 feet. This is the maximum height to which water can be raised in a pump; as was observed by Galileo.

In general the heights of columns of different liquids equal in weight to a column of air on the same base, are inversely proportional to their densities.

103. Pressure of One Atmosphere.—We can easily calculate the amount of this pressure for a given surface, for example, a square inch. It is the weight of a column of mercury whose base is a square inch and height 30 inches, that is, the weight of 30 cubic inches of mercury; and as a cubic inch of mercury weighs about half a pound, the atmospheric pressure on a square inch is about 15 pounds. This pressure of 15 pounds¹ to the square inch is called the pressure of *one atmosphere*; it is exerted in a normal direction at all points on the surface of a body, and in consequence, as in the case of a body wholly immersed in a liquid, the resultant of the different elementary pressures is a vertical upward pressure equal to the weight of the air displaced. The effect of the air, therefore, is not, as was formerly supposed, to press bodies to the surface of the earth; on the contrary, it tends to raise them, as in a liquid, but with comparatively small force, owing to its small density. It is upon this principle that the ascent of balloons depends, as we shall see hereafter.

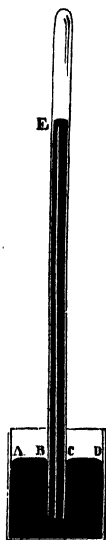


Fig. 109.

104. Pascal's Experiments.—It is supposed, though without any decisive proof, that Torricelli derived from Galileo the definite conception of atmospheric pressure.² However this may be, when the experiment of the Italian philosopher became known in France in 1644, no one was capable of giving the correct explanation of it, and the famous

¹ More accurately, 14.73 pounds, which is equivalent to the weight of 29.922 inches (or 760 millimetres) of pure mercury of the temperature of freezing water.

² In the fountains of the Grand-duke of Tuscany some pumps were required to raise water from a depth of from 40 to 50 feet. When these were worked, it was found that they would not draw. Galileo determined the height to which the water rose in their tubes, and found it to be about 32 feet; and as he had observed and proved that air has weight, he readily conceived that it was the weight of a column of the atmosphere which maintained the water at this height in the pumps. No very useful results, however, were expected from this discovery, until, at a later date, Torricelli adopted and greatly extended it. Desiring to repeat the experiment in a more convenient form, he conceived the idea of substituting for water a liquid that is 14 times as heavy, namely, mercury, rightly imagining that a column of one-fourteenth of the length would balance the force which sustained 32 feet of water (Biot, *Biographie Universelle*, article "Torricelli").—D.

doctrine that "nature abhors a vacuum," by which the rising of water in a pump was accounted for, was generally accepted. Pascal was the first to prove incontestably the falsity of this old doctrine, and to introduce a more rational belief. For this purpose he proposed or executed a series of ingenious experiments, and discussed minutely all the phenomena which were attributed to nature's abhorrence of a vacuum, showing that they were necessary consequences of the pressure of the atmosphere.

We may cite in particular the observation, made at his suggestion, that the height of the mercurial column decreases in proportion as we ascend. This beautiful and decisive experiment, which is repeated as often as heights are measured by the barometer, and which leaves no doubt as to the nature of the force which sustains the mercurial column, was performed for the first time at Clermont, and on the top of the Puy-de-Dôme, on the 19th September, 1648.

105. The Barometer.—By fixing the Torricellian tube in a permanent position, we have a means of measuring the amount of the atmospheric pressure at any moment; and this pressure may be expressed by the height of the column of mercury which it supports. Such an instrument is called a *barometer*. In order that its indications may be accurate, several precautions must be observed. In the first place, the liquid used in different barometers must be identical, for the height of the column supported naturally depends upon the density of the liquid employed, and if this varies, the observations made with different instruments will not be comparable.

The mercury employed is chemically pure, being generally made so by washing with a dilute acid and by subsequent distillation. The barometric tube is filled nearly full, and is then placed upon a sloping furnace, and heated till the mercury boils. The object of this process is to expel the air and moisture which may be contained in the mercurial column, and which, without this precaution, would gradually ascend

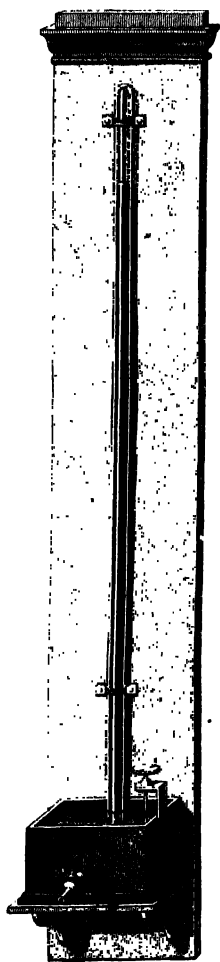


Fig. 110.—Barometer in its simplest form.

into the vacuum above, and cause a downward pressure of unknown amount, which would prevent the mercury from rising to the proper height.

The next step is to fill up the tube with pure mercury, taking care not to introduce any bubble of air. The tube is then inverted in a cistern likewise containing pure mercury recently boiled, and is firmly fixed in a vertical position, as shown in Fig. 110.

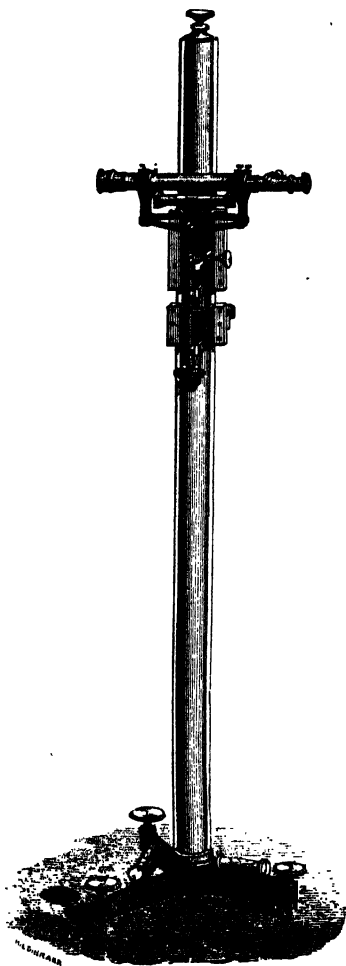


Fig. 111.—Cathetometer.

We have thus a fixed barometer; and in order to ascertain the atmospheric pressure at any moment, it is only necessary to measure the height of the top of the column of mercury above the surface of the mercury in the cistern. For this purpose an iron rod, working in a screw, is fixed vertically above the surface of the mercury in the dish. The extremities of this rod are pointed, and the lower extremity being brought down to touch the surface of the liquid below, the distance of the upper extremity from the top of the column of mercury is measured. Adding to this the length of the rod, which has previously been determined once for all, we have the barometric height. This measurement may be effected with great precision by means of the cathetometer.

105 A. Cathetometer.—This instrument, which is so frequently employed in physics to measure the vertical distance between two points, was invented by Dulong and Petit.

It consists essentially (Fig. 111) of a vertical scale divided usually into half millimetres. This scale forms part of a brass cylinder capable of turning very easily about a strong steel axis. This axis is fixed on a pedestal provided with three levelling screws, and with two spirit-

levels at right angles to each other. Along the scale moves a sliding frame carrying a telescope furnished with crosswires, that is, with two very fine threads, usually spider lines, in the focus of the eye-piece, whose point of intersection serves to determine the line of vision. By means of a clamp and slow-motion screw, the telescope can be fixed with great precision at any required height. The telescope is also provided with a spirit-level and adjusting screw. When the apparatus is in correct adjustment, the line of vision of the telescope is horizontal, and the graduated scale is vertical. If then we wish to measure the difference of level between two points, we have only to sight them successively, and measure the distance passed over on the scale, which is done by means of a vernier attached to the sliding frame.

106. Fortin's Barometer.—The barometer just described is intended to be fixed; when portability is required the barometer invented by Fortin is employed. It is also perfectly adapted to general use. The cistern, which is formed of a tube of boxwood, surmounted by a tube of glass, is closed below by a piece of leather, which can be raised or lowered by means of a screw. This screw works in the bottom of a copper case, which incloses the cistern except at the middle, where it is cut away in front and at the back, so as to leave the surface of the mercury open to view. The barometric tube is encased in a tube of copper, with two slits at opposite sides (Fig. 113); and it is on this tube that the divisions are engraved, the zero point from which they are reckoned being the lower extremity of an ivory point fixed in the covering of the cistern. The temperature of the mercury, which is required for one of the corrections mentioned in next section, is given by a thermometer with its bulb resting against the tube. A sliding

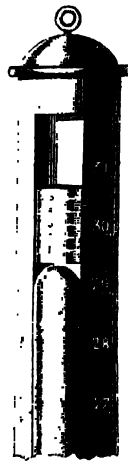


Fig. 113.
Upper portion of
Barometer.

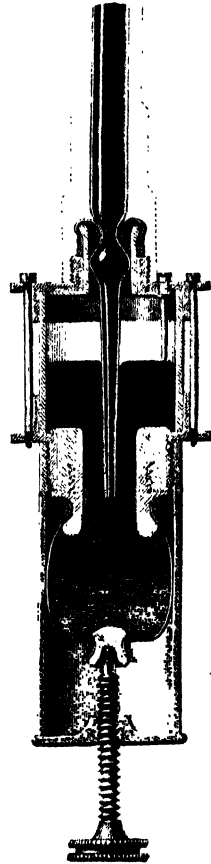


Fig. 112.
Cistern of Fortin's
Barometer.

piece, furnished with a vernier,¹ moves along the tube by means of the screw B, and enables us to determine the height with great precision. Its lower edge is the zero of the vernier. The way in which the barometric tube is fixed upon the cistern is worth notice. In the centre of the upper surface of the copper casing there is an opening, from which rises a short tube of the same metal, lined with a tube of boxwood. The barometric tube is pushed inside, and fitted in with a piece of chamois leather, which prevents the mercury from issuing; but does not exclude the air, which, passing through the pores of the leather, penetrates into the cistern, and so transmits its pressure.

Before taking an observation, the surface of the mercury is adjusted, by means of the lower screw, to touch the ivory point. The observer knows when this condition is fulfilled by seeing the extremity of the point touch its image in the mercury. The vernier is then raised or lowered, until the horizontal plane in which its zero lies is tangential

¹ The vernier is an instrument very largely employed for measuring the fractions of a unit of length on any scale. Suppose we have a scale divided into inches, and another scale containing nine inches divided into ten equal parts. If now we make the end of this latter scale, which is called the vernier, coincide with one of the divisions in the scale of inches, as each division of the vernier is $\frac{1}{10}$ of an inch, it is evident that the first division on the scale will be $\frac{1}{10}$ of an inch beyond the first division on the vernier, the second on the scale $\frac{2}{10}$ beyond the second on the vernier, and so on until the ninth on the scales, which

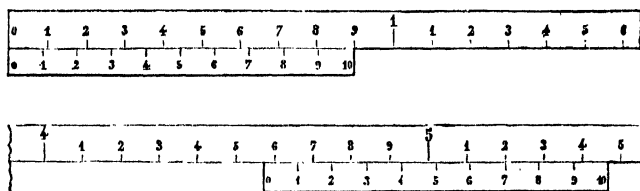


Fig. 114.—Vernier.

will exactly coincide with the tenth on the vernier. Suppose next that in measuring any length we find that its extremity lies between the degrees 5 and 6 on the scale; we bring the zero of the vernier opposite the extremity of the length to be measured, and observe what division on the vernier coincides with one of the divisions on the scale. We see in the figure that it is the seventh, and thus we conclude that the fraction required is $\frac{7}{10}$ of an inch.

If the vernier consisted of 19 inches divided into 20 equal parts, it would read to the $\frac{1}{20}$ of an inch; but there is a limit to the precision that can thus be obtained. An exact coincidence of a division on the vernier with one on the scale seldom or never takes place, and we merely take the division which approaches nearest to this coincidence; so that when the difference between the degrees on the vernier and those on the scale is very small, there may be so much uncertainty in this selection as to nullify the theoretical precision of the instrument. Verniers are also employed to measure angles; when a circle's divided into half degrees, a vernier is used, which gives $\frac{1}{2}$ of a division on the circle, that is, $\frac{1}{4}$ of a half degree, or one minute.—D.

to the upper surface of the mercurial column, as shown in Fig. 113. In making this adjustment, the back of the instrument should be turned towards a good light, in order that the observer may be certain of the position in which the light is just cut off at the summit of the convexity.

When the instrument is to be moved, the screw at the bottom is turned until the tube is filled. The cistern will then be full also, and the barometer should be inverted, as an additional safeguard both against the introduction of air and the escape of mercury. In making observations upon the surface of the ground, the instrument is suspended from a tripod stand by gimbals,¹ so that it always takes a vertical position; or it may be fixed permanently against a wall.

106A. Float Adjustment.—In some barometers the ivory point for indicating the proper level of the mercury in the cistern is replaced by a float. F (Fig. 107)

is a small ivory piston, having the float attached to its foot, and moving freely up and down between the two ivory guides I. A horizontal line (interrupted by the piston) is engraved on the two

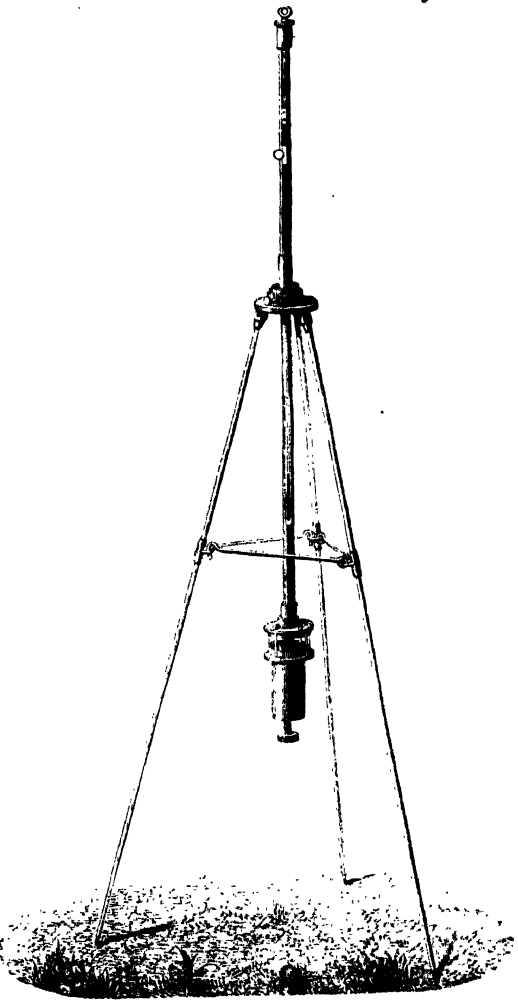


Fig. 115.—Barometer with Tripod Stand.

¹ A kind of universal joint, in common use on board ship for the suspension of compasses, lamps, &c. It is seen in Fig. 115, at the top of the tripod stand.

guides, and another is engraved on the piston, at such a height that the three lines form one straight line when the surface of the mercury in the cistern stands at the zero point of the scale.

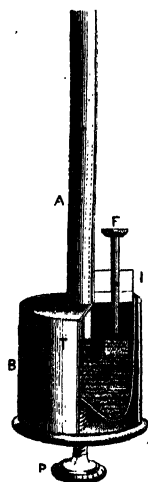


Fig. 107.
Float Adjustment.

107. Barometric Corrections.—In order that barometric heights may be comparable as measures of atmospheric pressure, certain corrections must be applied.

1. Correction for Temperature. As mercury expands with heat, it follows that a column of warm mercury exerts less pressure than a column of the same height at a lower temperature; and it is usual to reduce the actual height of the column to the height of a column at the temperature of freezing water, which would exert the same pressure.

Let h be the observed height at temperature t° Centigrade, and h_0 the height reduced to freezing-point. Then, if m be the coefficient of expansion of mercury per degree Cent., we have

$$h_0 (1 + m t) = h, \text{ whence } h_0 = h - h m t \text{ nearly.}$$

The value of m (Chap. xxii.) is $\frac{1}{5550} = .00018018$. For temperatures Fahrenheit, we have

$$h_0 \{ 1 + m (t - 32) \} = h, \quad h_0 = h - h m (t - 32),$$

where m denotes $\frac{1}{9990} = .0001001$.

But temperature also affects the length of the divisions on the scale by which the height of the mercurial column is measured. If these divisions be true inches at 0° Cent., then at t° the length of n divisions will be $n (1 + l t)$ inches, l denoting the coefficient of linear expansion of the scale, the value of which for brass, the usual material, is .0001878. If then the observed height h amounts to n divisions of the scale, we have

$$h_0 (1 + m t) = h = n (1 + l t);$$

whence

$$h_0 = \frac{n (1 + l t)}{1 + m t} = n - n t (m - l), \text{ nearly;}$$

that is to say, if n be the height read off on the scale, it must be diminished by the correction $n t (m - l)$, t denoting the temperature of the mercury in degrees Centigrade. The value of $m - l$ is .0001614.

For temperatures Fahrenheit, assuming the scale to be of the correct length at 32° Fahr., the formula for the correction (which is still subtractive), is $n(t-32)(m-l)$, where $m-l$ has the value $\cdot 00008967$.¹

2. *Correction for Capillarity.*—In the preceding chapter we have seen that mercury in a glass tube undergoes a capillary depression, whence it follows that the observed barometric height is too small, and that we must add to it the amount of this depression. In all tubes of internal diameter less than about $\frac{1}{4}$ of an inch this correction is sensible; and its amount, for which no simple formula can be given, has been computed, from theoretical considerations, for various sizes of tube, by several eminent mathematicians, and recorded in tables, from which that given below is abridged. These values are applicable on the assumption that the meniscus which forms the summit of the mercurial column is decidedly convex, as it always is when the mercury is rising. When the meniscus is too flat, the mercury must be lowered by the foot-screw, and then screwed up again.

It is found by experiment, that the amount of capillary depression is only half as great when the mercury has been boiled in the tube, as when this precaution has been neglected.

For purposes of special accuracy, tables have been computed,

¹ The correction for temperature is usually made by the help of tables, which give its amount for all ordinary temperatures and heights. These tables, when intended for English barometers, are generally constructed on the assumption that the scale is of the correct length not at 32° Fahr., but at 62° Fahr., which is (by act of Parliament) the temperature at which the British standard yard (preserved in the office of the Exchequer) is correct. On this supposition, the length of n divisions of the scale at temperature t° Fahr., is

$$n\{1+l(t-62)\};$$

and by equating this expression to

$$h_o\{1+m(t-32)\}$$

we find

$$\begin{aligned} h_o &= n\{1-m(t-32)+l(t-62)\} \\ &= n\{1-(m-l)t+(32m-62l)\} \\ &= n\{1-\cdot 00008967t+\cdot 00255654\}; \end{aligned}$$

which, omitting superfluous decimals, may conveniently be put in the form —

$$n - \frac{n}{1000} (\cdot 09t - 2\cdot 56).$$

The correction vanishes when

$$\cdot 09t - 2\cdot 56 = 0;$$

that is, when $t = \frac{256}{9} = 28\cdot 5$.

For all temperatures higher than this the correction is subtractive.

giving the amount of capillary depression for different degrees of convexity, as determined by the sagitta (or height) of the meniscus, taken in conjunction with the diameter of the tube. Such tables, however, are seldom used in this country.¹

TABLE OF CAPILLARY DEPRESSIONS IN UNBOILED TUBES.

(To be halved for Boiled Tubes.)

Diameter of tube in inches.	Depression.	Diameter.	Depression.	Diameter.	Depression.
·10	·140	·20	·058	·40	·015
·11	·126	·22	·050	·42	·013
·12	·114	·24	·044	·44	·011
·13	·104	·26	·038	·46	·009
·14	·094	·28	·033	·48	·008
·15	·086	·30	·029	·50	·007
·16	·079	·32	·026	·55	·005
·17	·073	·34	·023	·60	·004
·18	·068	·36	·020	·65	·003
·19	·063	·38	·017	·70	·002

3. *Correction for Capacity.*—When there is no provision for adjusting the level of the mercury in the cistern to the zero point of the scale, another correction must be applied. It is called the correction for *capacity*. In barometers of this construction, which were formerly much more common than they are at present, there is a certain point in the scale at which the mercurial column stands when the mercury in the cistern is at the correct level. This is called the neutral point. If A be the interior area of the tube, and C the area of the cistern (exclusive of the space occupied by the tube and its contents), when the mercury in the tube rises by the amount x , the mercury in the cistern falls by an amount $y = \frac{A}{C}x$, for the volume of the mercury which has passed from the cistern into the tube is $Cy = Ax$. The change of atmospheric pressure is correctly measured by $x + y = (1 + \frac{A}{C})x$, and if we now take x to denote the distance of the summit of the mercurial column from the neutral point, the corrected distance will be $(1 + \frac{A}{C})x$, and the correction to be applied to the observed reading will be $\frac{A}{C}x$, which is additive if the observed reading be above the neutral point, subtractive if below.

It is worthy of remark that the neutral point depends upon the

¹ The most complete collection of meteorological and physical tables, is that edited by Professor Guyot, and published under the auspices of the Smithsonian Institution, Washington.

volume of mercury. It will be altered if any mercury be lost or added; and as temperature affects the volume, the correction for temperature should be first applied, and the height of the column thus corrected should be used in computing the correction for capacity.

In some modern instruments the correction for capacity is avoided, by making the divisions on the scale less than true inches, in the ratio $\frac{C}{A+C}$, and the effect of capillarity is at the same time compensated by lowering the zero point of the scale. Such instruments, if correctly made, simply require to be corrected for temperature.

4. *Index Errors.*—Under this name are included errors of graduation, and errors in the position of the zero of the graduations. An error of zero makes all readings too high or too low by the same amount. Errors of graduation (which are generally exceedingly small) are different for different parts of the scale.

Barometers intended for accurate observation are now usually examined at Kew Observatory before being sent out; and a table is furnished with each, showing its index error at every half inch of the scale, errors of capillarity and capacity (if any) being included as part of the index error. We may make a remark here once for all respecting the signs attached to errors and corrections. The sign of an error is always opposite to that of its correction. When a reading is too high the index error is one of excess, and is therefore positive; whereas the correction needed to make the reading true is subtractive, and is therefore negative.

5. *Reduction to Sea-level.*—In comparing barometric observations taken over an extensive district for meteorological purposes, it is usual to apply a correction for difference of level. Atmospheric pressure, as we have seen, diminishes as we ascend; and it is usual to add to the observed height the difference of pressure due to the elevation of the place above sea-level. The amount of this correction is proportional to the observed pressure. The law according to which it increases with the height will be discussed in the next chapter.

6. *Correction for Unequal Intensity of Gravity.*—When two barometers indicate the same height, at places where the intensity of gravity is different (for example, at the pole and the equator), the same mass of air is superincumbent over both; but the pressures are unequal, being proportional to the intensity of gravity as measured by the values of g (Chaps. v. vi.) at the two places. When intensity

of pressure is to be expressed in absolute measure, it should be stated in absolute units of force (§ 42) per unit area. If we adopt as our absolute unit of force, that force which, acting on a pound of matter for a second, would generate a velocity of a foot per second, it is necessary that the square foot should be made the unit of area.

Since the force of gravity on a pound contains g absolute units of force, and the weight of 144 cubic inches of mercury at 0° Centigrade is 70.7275 lbs., we have the following rules for reducing pressure per unit area to absolute measure:—

To reduce lbs. per sq. foot to absolute measure, multiply by	g .
„ lbs. per sq. inch	„ „ $\frac{g}{144}$.
„ inches of mercury	„ „ $g \times 70.7275$.

108. Other kinds of Mercurial Barometer.—The *Siphon Barometer*, which is represented in Fig. 116, consists of a bent tube, generally of uniform bore, having two unequal legs. The longer leg, which must be more than 30 inches long, is closed, while the shorter leg is open. A sufficient quantity of mercury having been introduced to fill the longer leg, the instrument is set upright (after boiling to expel air), and the mercury takes such a position, that the difference of levels in the two legs represents the pressure of the atmosphere.

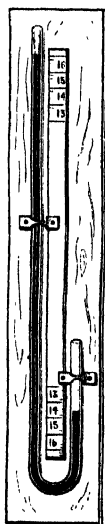


Fig. 116.
Siphon
Barometer.

Supposing the tube to be of uniform section, the mercury will always fall as much in one leg as it rises in the other. Each end of the mercurial column therefore rises or falls through only half the height corresponding to the change of atmospheric pressure.

In the best siphon barometers there are two scales, one for each leg, as indicated in the figure, the divisions on one being reckoned upwards, and on the other downwards, from an intermediate zero point, so that the sum of the two readings is the difference of levels of the mercury in the two branches.

Inasmuch as capillarity tends to depress both extremities of the mercurial column, its effect is generally neglected in siphon barometers; but practically it causes great difficulty in obtaining accurate observations, for according as the mercury is rising or falling its extremity is more or less convex, and a great deal of tapping is usually required to make both ends of the column assume the same

form, which is the condition necessary for annihilating the effect of capillary action.

Wheel Barometer.—The wheel barometer, which is in more general use than its merits deserve, consists of a siphon barometer, the two branches of which have usually the same diameter. On the surface of the mercury of the open branch floats a small piece

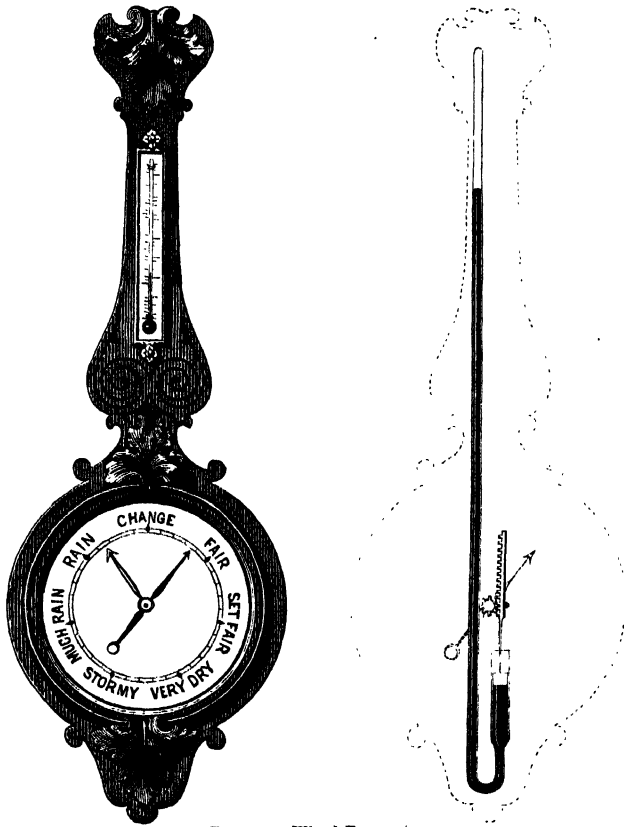


Fig. 118.—Wheel Barometer.

of iron or glass suspended by a thread, the other extremity of which is fixed to a pulley, on which the thread is partly rolled. Another thread, rolled parallel to the first, supports a weight which balances the float. To the axis of the pulley is fixed a needle which moves on a dial. When the level of the mercury varies in either direction, the float follows its movement through the same distance; by the action of the counterpoise the pulley turns, and with it the needle, the extremity of which points to the figures on the dial, marking the baro-

metric heights. The mounting of the dial is usually placed in front of the tube, so as to conceal its presence. The wheel barometer is a very old invention, and was introduced by the celebrated Hooke in 1683. The pulley and strings are sometimes replaced by a rack and pinion, as represented in the figure (Fig. 118).

Besides the faults incidental to the siphon barometer, the wheel barometer is encumbered in its movements by the friction of the additional apparatus. It is quite unsuitable for measuring the exact amount of atmospheric pressure, and is slow in indicating changes.

Marine Barometer.—The ordinary mercurial barometer cannot be used at sea, on account of the violent oscillations which the mercury would experience from the motion of the vessel. In order to meet this difficulty, the tube is contracted in its middle portion nearly to capillary dimensions, so that the motion of the mercury in either direction is hindered. An instrument thus constructed is called a marine barometer. When such an instrument is used on land it is always too slow in its indications.

Adie's Barometer.—A very convenient form of barometer, which is extensively used under the direction of the Board of Trade, is constructed by Adie of London. The error of capillarity is allowed for in fixing the zero point of the scale. The error of capacity is obviated by making the divisions of the scale less than true inches, in such a ratio as exactly to correct for capacity. The observer, therefore, has merely to read the height of the top of the mercurial column, and correct for temperature. The tube is generally contracted in its middle part, to diminish the "pumping" (*i.e.* oscillation), which occurs when it is carried from place to place; but the contraction is much less than in the marine barometer.

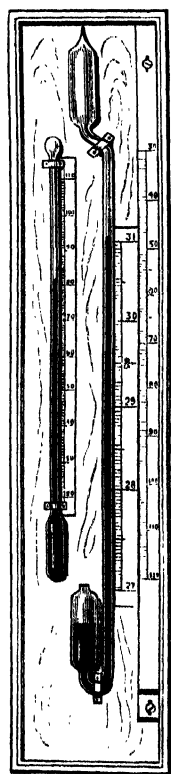


Fig. 117.
Sympiesometer.

108A. *Sympiesometer* ($\sigma\upsilon\upsilon$, $\pi\epsilon\zeta\omega$).—Adie's sympiesometer (Fig. 117) consists of a glass tube 18 inches in length and $\frac{3}{4}$ inch in diameter, with a small chamber at the top, and an open cistern below. In the original construction the upper part of the tube was filled with hydrogen, and the lower part and cistern with oil of almonds. In the construction now employed these materials are replaced by common air and glycerine.

When the pressure of the atmosphere increases, the air in the upper part of the tube is compressed, and the fluid rises; when it diminishes, the fluid falls. The instrument is graduated by comparison with a mercurial barometer. The intervals corresponding to inches of mercurial pressure are much longer than inches, and are of unequal length, becoming shorter as we ascend on the tube. To obviate error from the increased pressure of the inclosed air when its temperature is raised, a thermometer and sliding scale are added to the instrument, so that it may be adjusted for temperature at each observation. The sympiesometer is very quick in its indications, and from its portability is well adapted for being used at sea, but it is not suited for exact observation.

As originally made it was liable to gradual change, from absorption of the hydrogen by the oil of almonds. In the present construction absorption is less liable to occur, at least if the glycerine be of the proper consistency.

109. **Aneroid Barometer** (α , $\nu\eta\rho\omicron\varsigma$).—This barometer depends upon

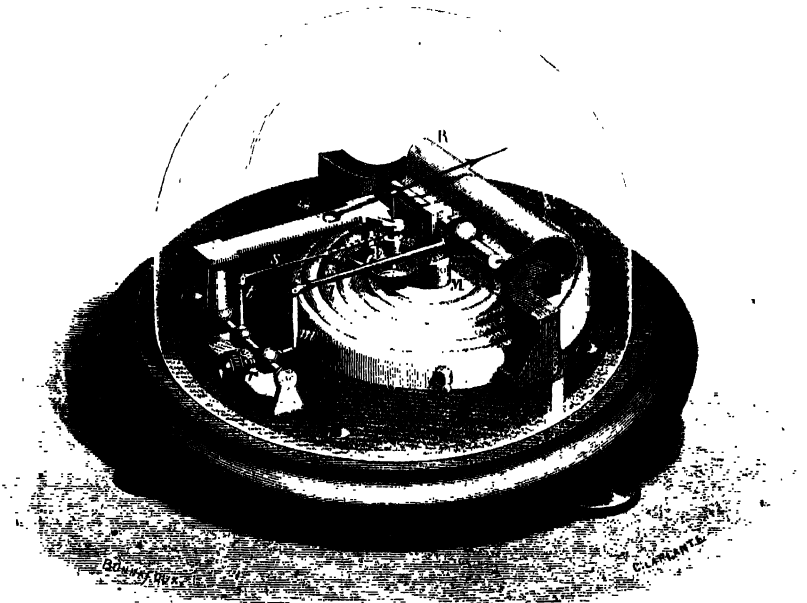


Fig. 119.—Aneroid Barometer.

the changes in the form of a thin metallic vessel, partially exhausted of air, as the atmospheric pressure varies. M. Vidie was the first to overcome the numerous difficulties which were presented in the con-

struction of these instruments. We subjoin a figure of the model which he finally adopted.

The essential part is a cylindrical box partially exhausted of air; the upper surface of which is corrugated in order to make it yield more easily to external pressure. At the centre of the top of the box is a small metallic pillar *M*, which acts upon a powerful steel spring *R*. As the pressure varies, the top of the box rises or falls, transmitting its movement to the spring, and thence, by means of two levers *l* and *m*, to a metallic axis *r*. This latter carries a third lever *t*, the extremity of which is attached to a chain *s* which turns a drum, the axis of which bears the index needle. A spiral spring keeps the chain constantly stretched, and thus makes the needle always take a position corresponding to the shape of the box at the time. The graduation is performed empirically by comparison with a mercurial barometer. The aneroid barometer is very sensitive, and is much more portable than any form of mercurial barometer, being both lighter and less liable to injury. It is sometimes made small enough for the waistcoat pocket. It has the drawback of being affected by temperature to an extent which must be determined for each instrument separately, and of being liable to gradual changes which can only be checked by occasional comparison with a good mercurial barometer.

In the *metallic barometer*, which is a modification of the aneroid, the exhausted box is crescent-shaped, and the horns of the crescent separate or approach according as the external pressure diminishes or increases.

110. Old Forms Revived.—There are two ingenious modifications of the form of the barometer, which, after long neglect, have recently been revived for special purposes.

Counterpoised Barometer.—The invention of this instrument is attributed to Samuel Morland, who constructed it about the year 1680. It depends upon the following principle:—If the barometric tube is suspended from one of the scales of a balance, there will be required to balance it in the other scale a weight equal to the weight of the tube and the mercury contained in it, minus the upward pressure of the liquid against the bottom of the tube and its contents.¹

¹ It may be shown that if *a* be the (annular) area of a section of the tube itself, and *A* the area of the inclosed space (which is filled with mercury), the resultant force to which the tube is subjected from atmospheric and liquid pressure combined is a downward force

$$PA - pa,$$

If the atmospheric pressure increases, the mercury will rise in the tube, and consequently the weight of the floating body will increase, while the upward pressure will be slightly diminished on account of the sinking of the mercury in the cistern. The beam will thus incline to the side of the barometric tube, and the reverse would be the case if the pressure diminished. For the balance may be substituted, as in the figure, a lever carrying a counterpoise; the variations of pressure will be indicated by the movements of this lever.

Such an instrument may very well be used as a *barograph* or recording barometer; for this purpose we have only to attach to the lever an arm with a pencil, which is constantly in contact with a sheet of paper moved uniformly by clock-work. The result will be a continuous trace, whose form corresponds to the variations of pressure. It is very easy to determine, either by calculation or by comparison with a standard barometer, the pressure corresponding to a given position of the pencil on the paper;

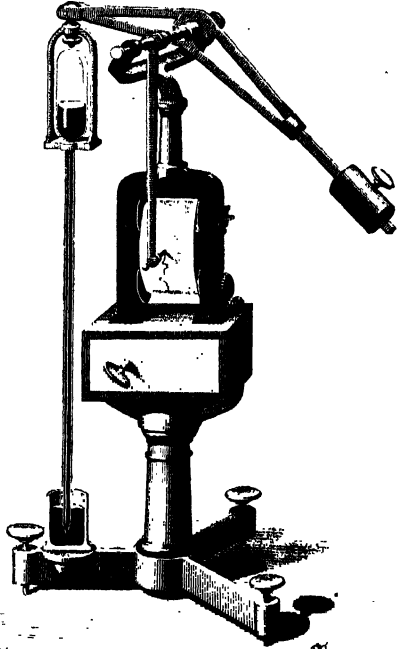


Fig. 120.—Counterpoised Barometer.

P denoting atmospheric pressure, and p the fluid pressure due to the depth of immersion (exclusive of the transmitted atmospheric pressure). This resultant force together with the weight of the tube must be equal to the supporting force at the point of suspension. If the latter be constant, $PA - pa$ must be constant, and the changes in P and p must be inversely as the areas A and a . If these areas are equal P and p will be equal; that is, the tube will descend through the same distance as the mercury in a common barometer would rise; and if A is greater than a , the movement will be proportionately magnified. For great sensitiveness, therefore, the tube should be large and thin.

We have here neglected the changes of level in the mercury in which the tube is immersed. These changes tend to increase the distance moved by the tube, and must be added to the movements as above calculated.

thus, if the paper is ruled with twenty-four equidistant lines, corresponding to the twenty-four hours of the day, we can see at a glance what was the pressure at any given time. An arrangement of this kind has been adopted by the Abbé Secchi for the meteorograph of the observatory at Rome. It has also been successfully employed at the Liverpool Exchange.

Fahrenheit's Barometer.—Fahrenheit's barometer consists of a tube

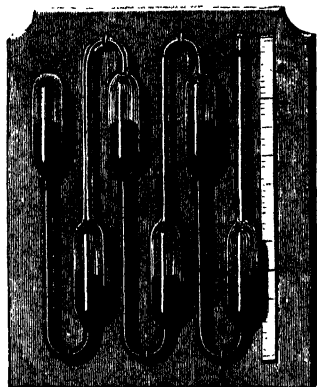


Fig. 121.—Fahrenheit's Barometer.

bent several times, the lower portions of which contain mercury; the upper portions are filled with water, or any other liquid, usually coloured. It is evident that the atmospheric pressure is balanced by the sum of the differences of level of the columns of mercury, diminished by the sum of the corresponding differences for the columns of water; whence it follows that, by employing a considerable number of tubes, we may greatly reduce the height of the barometric column. This circumstance renders the instrument interesting as a scientific curiosity,

but at the same time diminishes its sensitiveness, and renders it unfit for purposes of precision. It is therefore never used for the measurement of atmospheric pressure; but an instrument upon the same principle has recently been employed for the measurement of very high pressures, as will be explained in Chap. xiv.

110A. Photographic Registration.—Since the year 1847 various meteorological instruments at the Royal Observatory, Greenwich, have been made to yield continuous traces of their indications by the aid of photography, and the method is now generally employed at meteorological observatories in this country. The Greenwich system is fully described in the *Greenwich Magnetical and Meteorological Observations* for 1847, pp. lxiii.–xc. (published in 1849).

The general principle adopted for all the instruments is the same. The photographic paper is wrapped round a glass cylinder, and the axis of the cylinder is made parallel to the direction of the movement which is to be registered. The cylinder is turned by clockwork, with uniform velocity. The spot of light (for the magnets and barometer), or the boundary of the line of light (for the thermometers), moves, with the movements which are to be registered, backwards

and forwards in the direction of the axis of the cylinder, while the cylinder itself is turned round. Consequently (as in Morin's machine, Chap. v.), when the paper is unwrapped from its cylindrical form, there is traced upon it a curve of which the abscissa is proportional to the time, while the ordinate is proportional to the movement which is the subject of measure.

The barometer employed in connection with this system is a large siphon barometer, the bore of the upper and lower extremities of its arms being about 1·1 inch. A glass float in the quicksilver of the lower extremity is partially supported by a counterpoise acting on a light lever (which turns on delicate pivots), so that the wire supporting the float is constantly stretched, leaving a definite part of the weight of the float to be supported by the quicksilver. This lever is lengthened to carry a vertical plate of opaque mica with a small aperture, whose distance from the fulcrum is eight times the distance of the point of attachment of the float-wire, and whose movement, therefore (§ 108), is four times the movement of the column of a cistern barometer. Through this hole the light of a lamp, collected by a cylindrical lens, shines upon the photographic paper.

Every part of the cylinder, except that on which the spot of light falls, is covered with a case of blackened zinc, having a slit parallel to the axis of the cylinder; and by means of a second lamp shining through a small fixed aperture, and a second cylindrical lens, a base line is traced upon the paper, which serves for reference in subsequent measurements.

The whole apparatus, or any other apparatus which serves to give a continuous trace of barometric indications, is called a *barograph*; and the names *thermograph*, *magnetograph*, *anemograph*, &c., are similarly applied to other instruments for automatic registration.

CHAPTER XIII.

VARIATIONS OF THE BAROMETER.

111. Measurement of Heights by the Barometer.—As the height of the barometric column diminishes when we ascend in the atmosphere, it is natural to seek in this phenomenon a means of measuring heights. The problem would be extremely simple, if the air had everywhere the same density as at the surface of the earth. In fact, the density of the air at sea-level being about 10,500 times less than that of mercury, it follows that, on the hypothesis of uniform density, the mercurial column would fall an inch for every 10,500 inches, or 875 feet, that we ascend. This result, however, is far from being in exact accordance with fact, inasmuch as the density of the air diminishes very rapidly as we ascend, on account of its great compressibility.

111A. Height of Homogeneous Atmosphere.—If the atmosphere were of uniform and constant density, its height would be approximately obtained by multiplying 30 inches by 10,500, which gives 26,250 feet, or about 5 miles.

More accurately, if we denote by H the height of the atmosphere at a given time and place, on the assumption that the density throughout is the same as the observed density D at the base, and if we denote by P the observed pressure at the base, expressed in absolute units of force per unit area (§ 107, 6), then since the pressure P must be equal to the weight of a column of volume H and of mass HD , we have .

$$P = g HD \quad H = \frac{P}{g D} \quad (1)$$

The height H , computed on this imaginary assumption, is called the *height of the homogeneous atmosphere*, corresponding to the pressure P , density D , and intensity of gravity g , and is frequently introduced in physical formulæ.

The expression for H shows that its value is not affected if P and D vary in the same ratio, as is the case in barometric fluctuations when the temperature is constant; but that increase of temperature and increase of moisture increase H , since warm air and moist air are less dense than cold and dry air at the same pressure.

It is not necessary that the height H should be reckoned from the surface of the earth. It may be reckoned upwards from any point in the atmosphere, and denotes the height which the air above this point would have, if reduced to the density D which exists at the point.

Neglecting differences of temperature and moisture, and the trifling diminution of gravity as we ascend, the value of H is the same for all points in the same vertical column, because, as we ascend, P and D diminish in the same ratio.

112. Principles of Hypsometry.—Supposing the temperature, moisture, and intensity of gravity to be uniform in a vertical column of air, it is easy to state the law according to which the pressure would diminish as we ascend. Consider, for example, three layers of equal thickness, which is so small that we may regard the density as constant within the limits of each layer, though varying from each layer to the next. Let D, D', D'' be their densities, and P, P', P'' the pressures at their lower faces, the weights of the two lower layers are $P - P'$ and $P' - P''$, and these must be proportional to their densities; hence we have

$$\frac{P - P'}{P' - P''} = \frac{D}{D'}$$

but by Boyle's law $\frac{D}{D'} = \frac{P}{P'}$; consequently we have $\frac{P - P'}{P' - P''} = \frac{P}{P'}$, whence it easily follows that $\frac{P}{P'} = \frac{P'}{P''}$; that is to say, the ratio of the density of the first layer to that of the second, is the same as of the second to the third. Applying this principle to any number of consecutive layers of equal thickness, we see that the ratio of the density of each to that of the next will be the same for the whole series. It follows that, *as the heights increase in arithmetical progression, the pressures diminish in geometrical progression.*

This proposition may be put into the algebraical form:

$$x_2 - x_1 = H \log \frac{P_1}{P_2}, \quad \left\{ \right.$$

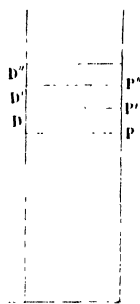


Fig. 122.

where x_1, x_2 are the heights of two stations above a fixed level, P_1, P_2 the pressures at the two stations, and H some constant. The proof given in the note¹ shows, that if the logarithms are Napierian, H is equal to the height of the homogeneous atmosphere. If the logarithms be of the common kind, H is equal to the height of the homogeneous atmosphere multiplied by 2.3026, the value of which product for the latitude of Great Britain, and for the temperature of freezing water, is about 60,360 feet.

This formula has been deduced on the supposition that the temperature and the intensity of gravity are uniform through the whole extent of the air between the two stations. If these two elements vary, they cause the value of H to vary; and it would be necessary for accuracy to employ in the formula the mean value of H for the stratum of air which intervenes between the two stations. The variation in the intensity of gravity is usually insignificant, and it is customary to assume as the mean temperature, the arithmetical mean of the temperatures t_1 and t_2 of the two stations. On these assumptions the value of H , if the temperatures be expressed in degrees Fahrenheit, will (by the law of expansion of air, Chap. xxiii.) be $60,360 \left(1 + \frac{t_1 + t_2 - 64}{986}\right)$.

It is proved in treatises on logarithms that if $\frac{P_1}{P_2}$ be but little greater than unity,

$$\text{Nap. log } \frac{P_1}{P_2} = 2 \frac{P_1 - P_2}{P_1 + P_2} \text{ nearly;}$$

and since the height of the homogeneous atmosphere at freezing temperature in these latitudes is about 26,214 feet, we obtain the formula—

$$\text{Difference of level in feet} = 52428 \frac{P_1 - P_2}{P_1 + P_2} \left(1 + \frac{t_1 + t_2 - 64}{986}\right),$$

which may be used for differences of level not exceeding about 3000 feet.

¹ Let x denote distance measured upwards from a fixed level, then, using the notation of § 111A, the pressure due to the weight of a layer of thickness dx is $g D dx$; but this is the amount by which the pressure diminishes as x is increased by the amount dx ; we have therefore

$$-dP = g D dx = \frac{P}{H} dx,$$

since by § 111A, $g D = \frac{P}{H}$. We have therefore

$$-\frac{dP}{P} = \frac{dx}{H}, \text{ whence } \log P_1 - \log P_2 = \frac{x_2 - x_1}{H}.$$

The determination of heights by means of atmospheric pressure, whether the pressure be observed directly by the barometer or indirectly by the boiling-point thermometer (see Chap. xxvi.), is called *hypsimetry* ($\psi\phi\sigma\varsigma$, height).

As a rough rule, it may be stated that, in ordinary circumstances, the barometer falls an inch in ascending 900 feet.

113. Diurnal Oscillation of the Barometer.—In these latitudes, the mercurial column is in a continual state of irregular oscillation; but in the tropics it rises and falls with great regularity according to the hour of the day, attaining two maxima in the twenty-four hours.

It generally rises from 4 A.M. to 10 A.M., when it attains its first maximum; it then falls till 4 P.M., when it attains its first minimum; a second maximum is observed at 10 P.M., and a second minimum at 4 A.M. The hours of maxima and minima are called the tropical hours ($\tau\rho\epsilon\pi\omega$, to turn), and vary a little with the season of the year. The difference between the highest maximum and lowest minimum is called the diurnal¹ *range*, and the half of this is called the *amplitude* of the diurnal oscillation. The amount of the former does not exceed about a tenth of an inch.

The character of this diurnal oscillation is represented in Fig. 123. The vertical lines correspond to the hours of the day; lengths have been measured upwards upon them proportional to the barometric heights at the respective hours, diminished by a constant quantity; and the points thus determined have been connected by a continuous curve. It will be observed that the two lower curves, one of which relates to Cumana, a town of Venezuela, situated in about 10° north latitude, show strongly marked oscillations corresponding to the maxima and minima. In our own country the regular diurnal oscillation is marked by irregular fluctuations, so that a single day's observations give no clue to its existence. Nevertheless, on taking observations at regular hours for a number of consecutive days, and comparing the mean heights for the different hours, some indications of the law will be found. A month's observations will be sufficient

¹ The epithets *annual* and *diurnal*, when prefixed to the words *variation*, *range*, *amplitude*, denote the *period* of the variation in question; that is, the time of a complete oscillation. Diurnal variation does not denote variation from one day to another, but the variation which goes through its cycle of values in one day of twenty-four hours. Annual range denotes the range that occurs within a year. This rule is universally observed by writers of high scientific authority.

A table, exhibiting the values of an element for each month in the year, is a table of annual (not monthly) variation; or it may be more particularly described as a table of variations from month to month.

for an approximate indication of the law; but observations, extending over some years, will be required to establish with anything like precision the hours of maxima and the amplitude of the oscillation.

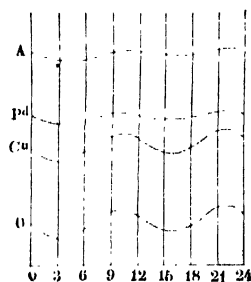


Fig 123.
Curves of Diurnal Variation.

The two upper curves represent the diurnal variation of the barometer at Padua (lat. $45^{\circ} 24'$) and Abo (lat. $60^{\circ} 56'$), the data having been extracted from Kaemtz's *Meteorology*. We see, by inspection of the figure, that the oscillation in question becomes less strongly marked as the latitude increases. The range at Abo is less than half a millimetre. At about the 70th degree of north latitude it becomes insensible;

and in approaching still nearer to the pole, it appears from observations, which however need further confirmation, that the oscillation is reversed; that is to say, that the maxima here are contemporaneous with the minima in lower latitudes.

There can be little doubt that the diurnal oscillation of the barometer is in some way attributable to the heat received from the sun, which produces expansion of the air, both directly, as a mere consequence of heating, and indirectly, by promoting evaporation, and thus increasing the volume of the air (as well as diminishing its sp. gravity) by the addition of aqueous vapour. The precise nature of the connection between this cause and the diurnal barometric oscillation has not, however, as yet been satisfactorily established.

114. Irregular Variations of the Barometer.—The height of the barometer, at least in the temperate zones, depends on the state of the atmosphere, and its variations often serve to predict the changes of weather with more or less certainty. In this country the barometer generally falls for rain or S.W. wind, and rises for fine weather or N.E. wind.

Barometers for popular use have generally the words—

Set fair.	Fair.	Change.	Rain.	Much rain.	Stormy.
30.5	30	29.5	29	28.5	28 inches.

marked at the respective heights. These words must not, however, be understood as absolute predictions. A low barometer rising is generally a sign of fine, and a high barometer falling of wet weather. Moreover, it is to be borne in mind that the barometer stands about a tenth of an inch lower for every hundred feet that we ascend above sea-level.

The connection between a low or falling barometer and wet weather is to be found in the fact that moist air is specifically lighter than dry, even at the same temperature, and still more when, as usually happens, moist air is warmer than dry.

115. Inverse March of Barometer and Thermometer.—It is impossible to lay down universal rules for the connection between the indications of the barometer and the state of the weather, since rules which would usually hold true in one place might be quite inapplicable at another. We may, however, state a principle which is of very extensive application, namely, that warm winds, especially when they have passed over considerable masses of water, are likely to be accompanied by rain; for they are charged with vapour which is liable to be condensed as its temperature falls. Cold winds, on the contrary, contain vapour which was taken up at a lower temperature than it now has, and is therefore far from a state of saturation. They are therefore unlikely to produce rain unless it be when they first begin to blow, when they may condense vapour previously existing in the air.

These characteristics are very marked in our own country, where the warm winds from the south-west have passed over the Atlantic, while the cold winds from the north-east have for the most part traversed dry land.

Again, the march of the barometer is in general opposite to that of the thermometer; that is to say, *the barometer usually falls when the thermometer rises, and vice versa*. This law is one of the most general in meteorology, and is easily explained; in fact, when the temperature rises at any place, it produces a dilatation of the air, and consequently an overflow into neighbouring regions; the weight of air over the place is thus diminished. On the contrary, a fall of temperature produces an inflow of air and an increase of pressure.

It is therefore to be expected that the mean barometric height should be lower during warm and rainy winds than during cold and dry winds; and that this is the case is rendered extremely evident by the annexed figure, which represents the mean barometric rose of the winds at Paris. Upon each of the eight lines which represent the principal directions of the wind, have been laid

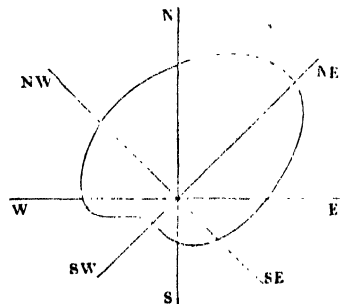


Fig. 124.
Barometric Rose of Winds at Paris.

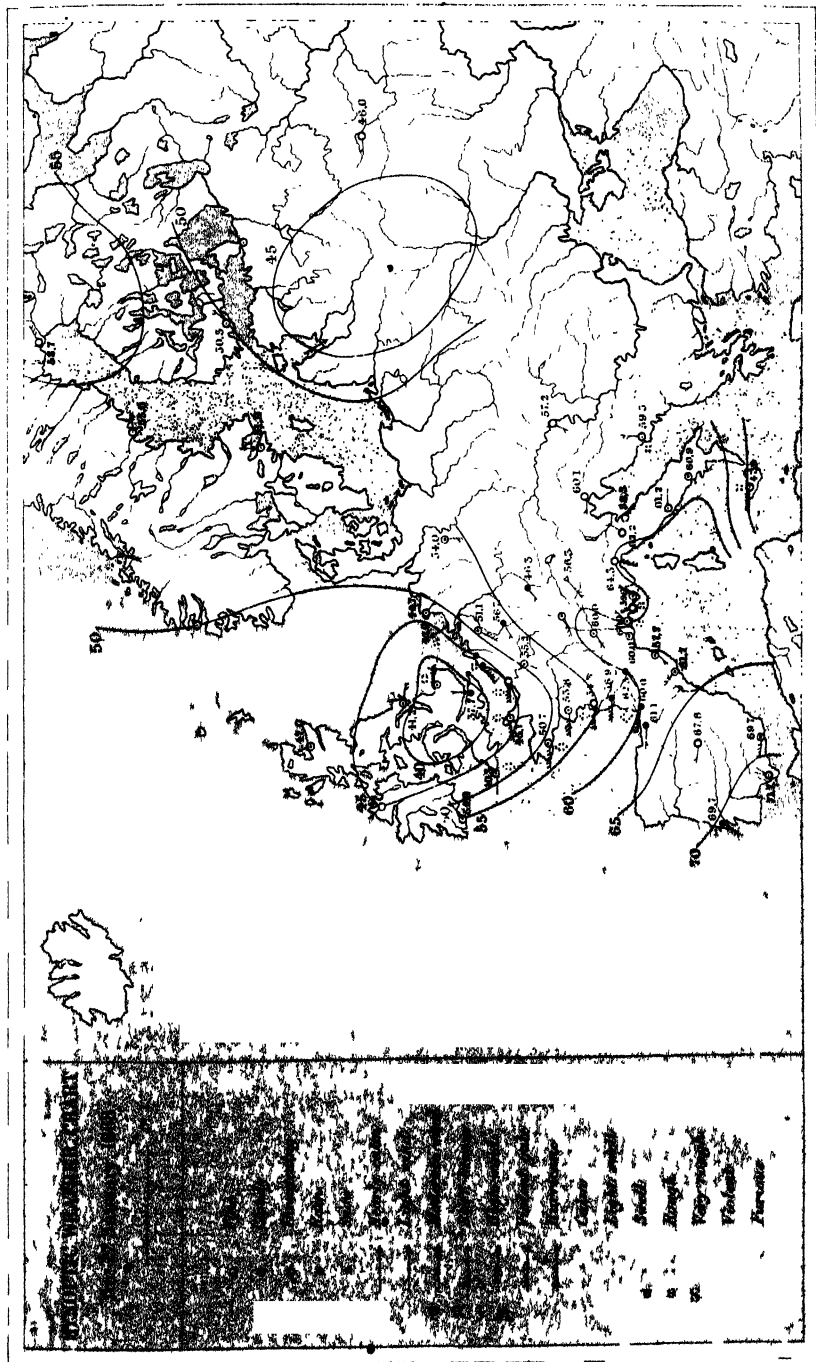
off lengths proportional to the corresponding barometric pressures diminished by a constant. The figure shows a very sudden increase in passing from S.W. to W. It is, in fact, during the change of the wind from one of these quarters to the other, that the greatest atmospheric perturbations occur.

116. Synoptic Weather Charts. Isobaric Lines.—The extension of telegraphic communication over Europe has led to the establishment of a system of correspondence by which the barometric pressures, at a given moment, at a number of stations which have been selected for meteorological observation, are known at one or more stations appointed for receiving the reports. From the information thus furnished, curves (called isobaric lines) are drawn upon a chart through those places at which the pressure is the same. The barometric condition of an extensive region is thus rendered intelligible at a glance. Plate I. is a specimen of these synoptic charts,¹ which are prepared every day at the observatory of Paris; it refers to the 22d of January, 1868. Besides the isobaric lines, these charts indicate, by the system of notation explained at the left of the figure, the general state of the weather, the strength of wind, and state of the sea. The isobaric curves correspond to differences of five millimetres (about 0·2 inch) of pressure, and according as they are near together, or far apart, the variation of pressure in passing from one to another is more or less sudden (or to use a very expressive modern phrase, the barometric gradient is more or less steep), just as the contour lines on a map of hilly ground approach each other most nearly where the ground is steepest. Generally speaking, the wind blows from regions of high to regions of low barometer, and with greater force as the barometric gradient is steeper.

The isobaric lines frequently, as in the example here selected, form closed curves encircling a region of barometric depression. Two such centres are here exhibited—one in the south of England and the other in the west of Russia. Such centres of depression always accompany great atmospheric disturbances. The air, in fact, rushes

¹ The curves drawn upon this chart are isobaric lines, each corresponding to a particular barometric pressure, which is indicated by the numerals marked against it. These denote the pressure in millimetres diminished by 700. For example, the line which passes through the south of Spain corresponds to the pressure 770 millimetres; that through the north of Spain to 785 millimetres. The curves are drawn for every fifth millimetre. The smaller numerals, which are given to one place of decimals, indicate the pressures actually observed at the different stations, from which the isobaric lines are drawn by estimation.

The other symbols refer to cloud, wind, and sea, and are explained at the left of the chart.



in from all sides, usually with a spiral motion, towards these centres of depression, the direction of rotation in the spiral being, for the northern hemisphere, opposite to the motion of the hands of a watch with its face upwards. The centrifugal force due to this rotation tends to increase the central depression, and thus protracts the duration of the phenomenon.

These revolving storms are called cyclones. They attain their greatest violence in tropical regions, the West Indies being especially noted for their destructive effect. They frequently proceed from the Gulf of Mexico in a north-easterly direction, increasing in diameter as they proceed, but diminishing in violence. Their velocity of translation is usually from ten to twenty miles an hour.

The storm-warnings inaugurated by the late Admiral Fitzroy are based partly upon information received by telegraph of storms that have actually commenced at some distant locality, and partly upon a comparison of barometric pressures at different localities.¹

¹ For fuller information respecting the laws of storms, which is a purely modern subject, and is continually receiving fresh developments, we would refer to Mr. Buchan's *Handy Book of Meteorology*.

CHAPTER XIV.

BOYLE'S (OR MARIOTTE'S) LAW.¹

117. Boyle's Law.—As gases are composed of molecules in a state of permanent repulsion, they may be compared to springs constantly bent, and making constant efforts to free themselves. The amount of pressure which they exert against the sides of the vessels which contain them, depends upon the volume which they occupy, increasing as this volume diminishes. By a number of careful experiments upon this point, Boyle and Mariotte independently established the law that this volume varies inversely as the pressure, provided that the temperature remain constant. As the density evidently varies inversely as the volume, we may express the law in other words by saying that at the same temperature the density varies directly as the pressure.

If V and V' be the volumes of the same quantity of gas, P and P' , D and D' , the corresponding pressures and densities, Boyle's law will be expressed by the equations

$$\frac{V}{V'} = \frac{P'}{P} = \frac{D}{D'}.$$

118. Mariotte's Tube.—The correctness of this law may be verified by means of the following apparatus, which was employed by both the experimenters above named. It consists (Fig. 125) of a bent tube with branches of unequal length; the long branch is open, and the

¹ Boyle, in his *Defence of the Doctrine touching the Spring and Weight of the Air against the Objections of Franciscus Linus*, appended to *New Experiments, Physico-mechanical, &c.* (second edition, 4to, Oxford, 1662), describes the two kinds of apparatus represented in Figs. 125, 126 as having been employed by him, and gives in tabular form the lengths of tube occupied by a body of air at various pressures. These observed lengths he compares with the theoretical lengths computed on the assumption that volume varies reciprocally as pressure, and points out that they agree within the limits of experimental error.

Mariotte's treatise, *De la Nature de l'Air*, is stated in the *Biographie Universelle* to have been published in 1679. (See Preface to Tait's *Thermodynamics*, p. iv.)

short branch closed. The tube is fastened to a board provided with two scales; one by the side of the long branch, divided into parts of equal length; the other by the side of the short branch, having divisions which correspond to parts of equal volume. The graduation of both scales begins from the same horizontal line through 0, 0. Mercury is first poured in at the extremity of the long branch, and by inclining the apparatus to either side, and cautiously adding more of the liquid if required, the mercury can be made to stand at the same level in both branches, and at the zero of both scales. Thus we have, in the short branch, a quantity of air separated from the external air, and at the same pressure. Mercury is then poured into the long branch, so as to reduce the volume of this inclosed air by one-half; it will then be found that the difference of level of the mercury in the two branches is equal to the height of the barometer at the time of the experiment; the compressed air therefore exerts a pressure equal to that of two atmospheres. If more mercury be poured in so as to reduce the volume of the air to one-third or one-fourth of the original volume, it will be found that the difference of level is respectively two or three times the height of the barometer; that is, that the compressed air exerts a pressure equal respectively to that of three or four atmospheres. This experiment therefore shows that if the volume of the gas becomes two, three, four times as small, the pressure becomes two, three, four times as great. This is the principle expressed in Boyle's law.



Fig. 125.
Mariotte's Tube.

The law may also be verified in the case where the gas expands, and where its pressure consequently diminishes. For this purpose a barometric tube (Fig. 126), partially filled with mercury, is inverted in a tall vessel, containing mercury also, and is held in such a position that the level of the liquid is the same in the tube and in the vessel. The volume occupied by the gas is marked, and the tube is raised; the gas expands, its pressure diminishes, and, in virtue of the excess of the atmospheric pressure, a column of mercury *ab* rises in the tube, so that its height, added to the pressure of the expanded air, is

equal to the atmospheric pressure. It will then be seen that if the volume of air becomes double what it was before, the height of the column raised is one-half that of the barometer; that is, the expanded air exerts a pressure equal to half that of the atmosphere. If the volume is trebled, the height of the column is two-thirds that of the barometer; that is, the pressure of the expanded air is one-third that of the atmosphere, a result which is in accordance with Boyle's law.

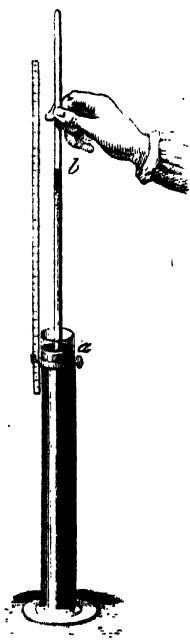


Fig. 126.—Proof of Boyle's Law for Expanding Air.

119. Despretz's Experiments.—The simplicity of Boyle's law, taken in conjunction with its apparent agreement with facts, led to its general acceptance as a rigorous truth of nature, until in 1825 Despretz published an account of experiments, showing that different gases are unequally compressible. He inverted in a cistern of mercury several cylindrical tubes of equal height, and filled them with different gases. The whole apparatus was then inclosed in a strong glass vessel filled with water, and having a screw-piston as in CErsted's piesometer (§ 22). On pressure being applied, the mercury rose to unequal heights in the different tubes, carbonic acid for example being more reduced in volume than air. These

experiments proved that though Boyle's law might possibly be true for one of the gases employed, it could not be rigorously true for more than one.

In 1829 Dulong and Arago undertook a laborious series of experiments with the view of testing the accuracy of the law as applied to air; and the results which they obtained, even when the pressure was increased to twenty-seven atmospheres, agreed so nearly with it as to confirm them in the conviction that, for air at least, it was rigorously true. When re-examined, in the light of later researches, the results obtained by Dulong and Arago seem to point to a different conclusion.

120. Unequal Compressibility of Different Gases.—The unequal compressibility of different gases, which was first established by Despretz's experiments above described, is now usually exhibited by the aid of the following apparatus designed by Pouillet. A is a cast-iron reservoir, containing mercury surmounted by oil. In this latter liquid dips a

bronze plunger P, the upper part of which has a thread cut upon it, and works in a nut, so that the plunger can be screwed up or down by means of the lever L. The reservoir A communicates by an iron tube with another cast-iron vessel, into which are firmly fastened two tubes T T about six feet in length and $\frac{1}{16}$ th of an inch in internal diameter, very carefully calibrated (§ 180). Equal volumes of two gases, perfectly dry, are introduced into these tubes through their upper ends, which are then hermetically sealed. The plunger is then made to descend, and a gradually increasing pressure is exerted, the volumes occupied by the gases are measured, and it is ascertained that no two gases follow precisely the same law of compression. The difference, however, is almost insensible when the gases employed are non-liquefiable, as air, oxygen, hydrogen, nitrogen, nitric oxide, and marsh-gas. But when we compare any one of these with a liquefiable gas, such as carbonic acid, cyanogen, or ammonia, the difference is rapidly and distinctly manifested. Thus, under a pressure of twenty-five atmospheres, carbonic acid occupies a volume which is only $\frac{1}{4}$ ths of that occupied by air.

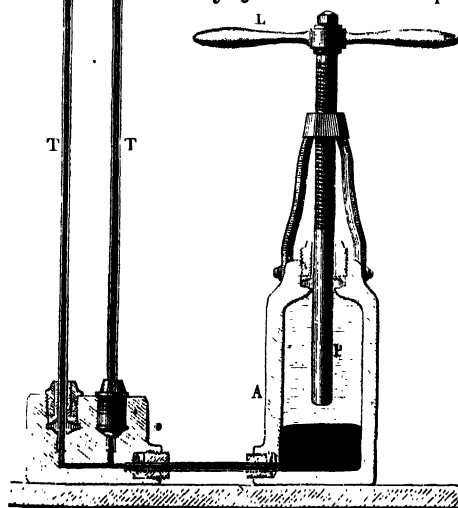


Fig. 127.—Pouillet's Apparatus for showing Unequal Compressibility of Different Gases.

121. Regnault's Experiments.—Boyle's law, therefore, is not to be considered as rigorously exact; but it is certainly a very close approximation to the truth, except for gases near their point of liquefaction. In order to demonstrate the inaccuracy of the law for air, or any gas that is not liquefiable, and more especially if it is required to determine the law of deviation for each particular gas, it is necessary to employ very precise methods of measurement. In

ordinary experiments on compression, and even in the elaborate investigations of Dulong and Arago, a definite portion of gas is taken

and successively diminished in volume by the application of continually increasing pressure. Now it is evident that, in experiments of this kind, in proportion as the pressure increases, the variations in volume become smaller, and the precision with which they can be determined consequently diminishes. Regnault adopted the plan of operating in all cases upon the same volume of gas, which, being initially at different pressures, was always reduced to one-half. The pressure was observed before and after this operation, and, if Boyle's law were true, its value should be found to be doubled. In this way the same precision of measurement is obtained at high as at low pressures:

A general view of Regnault's apparatus is given in Fig. 128. It consists of an iron reservoir containing mercury, furnished at the top with a force-pump for water. The lower part of this reservoir communicates with a cylinder which is also of iron, and in which are two openings to admit tubes. Communication between the reservoir and the cylinder can be established or interrupted by means of a stop-cock R, of very exact workmanship. Into one

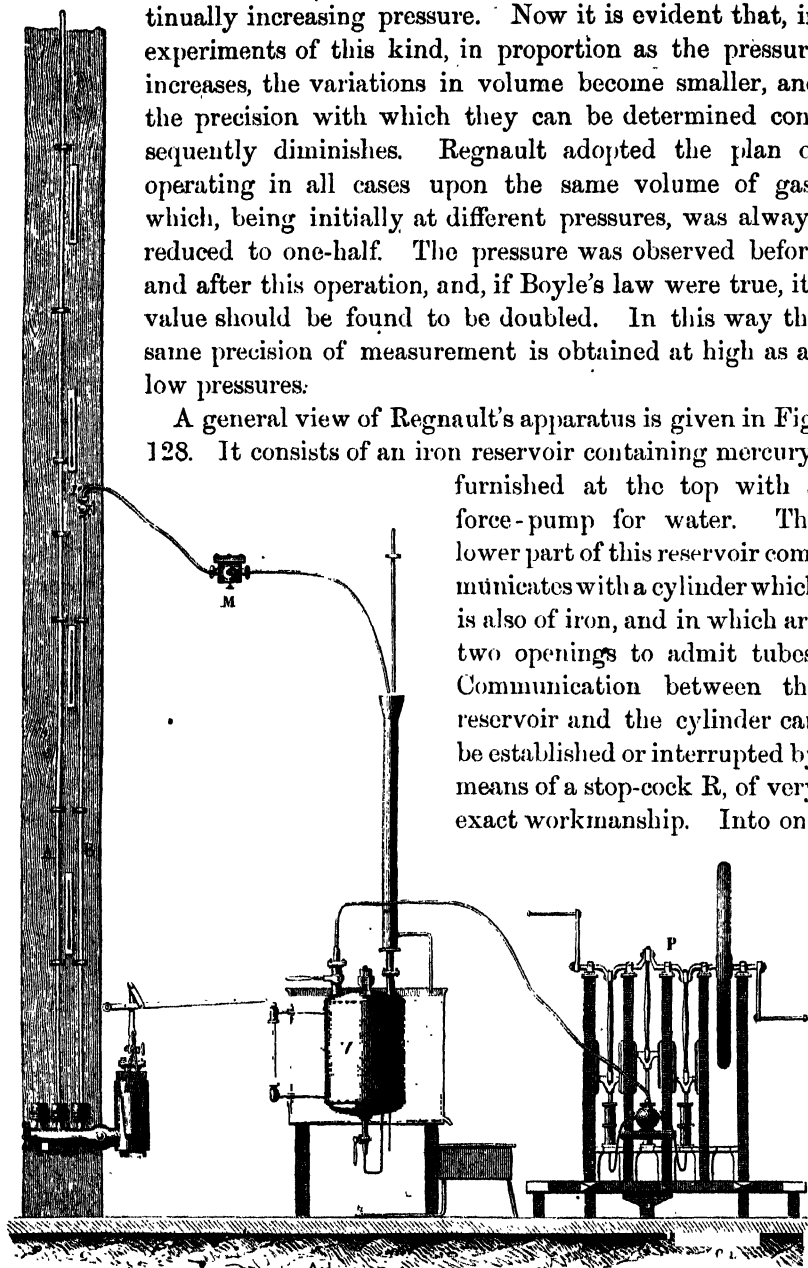


Fig. 128.—Regnault's Apparatus for Testing Boyle's Law.

of the openings is fitted the lowest of a series of glass tubes A, which are placed end to end, and firmly joined to each other by metal fittings, so as to form a vertical column of about twenty-five metres in height.

The height of the mercurial column in this long manometric tube could be exactly determined by means of reference marks placed at distances of about $\cdot 95$ of a metre, and by the graduation on the tubes forming the upper part of the column. The mean temperature of the mercurial column was given by thermometers placed at different heights. Into the second opening in the cylinder fits the lower extremity of the tube B, which is divided into millimetres, and also gauged with great accuracy. This tube has at its upper end a stop-cock *r* which can open communication with the reservoir V, into which the gas to be operated on is forced and compressed by means of the pump P.

An outer tube, which is not shown in the figure, envelops the tube B, and, being kept full of water, which is continually renewed, enables the operator to maintain the tube at a temperature sensibly constant, which is indicated by a very delicate thermometer. Before fixing the tube in its place, the point corresponding to the middle of its volume is carefully ascertained, and after the tube has been permanently fixed, the distance of this point from the nearest of the reference marks is observed.¹

After these explanatory remarks we may describe the mode of conducting the experiments. The gas to be operated on, after being first thoroughly dried, is introduced at the upper part of the tube B, the stop-cock of the pump being kept open, so as to enable the gas to expel the mercury and occupy the entire length of the tube. The force-pump is then brought into play, and the gas is reduced to about half of its former volume; the pressure in both cases being ascertained by observing the height of the mercury in the long tube above the nearest mark. It is important to remark that it is not at all necessary to operate always upon exactly the same initial volume, and reduce it exactly to one-half, which would be a very tedious opera-

¹ Regnault's apparatus was fixed in a small square tower of about fifteen metres in height, forming part of the buildings of the Collège de France, and which had formerly been built by Savart for experiments in hydraulics. The tower could therefore contain only the lower part of the manometric column; the upper part rose above the platform at the top of the tower, resting against a sort of mast which could be ascended by the observer. The readings inside the tower could be made by means of a cathetometer, but this was impossible in the upper portion of the column, and for this reason the tubes forming this portion were graduated.—D.

tion; these two conditions are approximately fulfilled, and the graduation of the tube enables the observer always to ascertain the actual volumes.

122. Results.—The general result of the investigations of Regnault is, that Boyle's law does not exactly represent the compressibility of even non-liquefiable gases, such as air, hydrogen, nitrogen, which, with carbonic acid, were the gases operated on by him. But the resulting differences are so small that they would not be detected by a mere inspection of the numbers which represent volumes and pressures. They may, however, be clearly exhibited by submitting the results to the following test:—Suppose we take a certain quantity of gas which, under the pressure P , occupies the volume V , and that we reduce it to a volume V' , when the pressure becomes P' , and, if Boyle's law were accurate, we should have the equation

$$\frac{V}{V'} = \frac{P'}{P}, \text{ or } VP = V'P'$$

Or, in another form, $\frac{VP}{V'P'} - 1 = 0$.

Now it is found that for all gases except hydrogen this difference, instead of being always zero, is constantly positive, and has not only a sensible value, but, which is of especial importance, it increases regularly with the pressure, which shows that it cannot be attributed to the inevitable errors of observation.

If we measure off upon any line lengths proportional to the different pressures, and raise perpendiculars proportional to the differences $\frac{VP}{V'P'} - 1$, by joining the extremities of these perpendiculars by a continuous line an uninterrupted curve is obtained, which evidently is a graphic representation of the departure of the gas in question from Boyle's law. These curves have been very carefully traced by Regnault; and their algebraic equations can be found by the ordinary methods of interpolation. These equations are employed when it is required to calculate rigorously the change of volume corresponding to a very high pressure.

Since the difference $\frac{VP}{V'P'} - 1$ is positive, $V'P'$ must be less than VP , and consequently the volume V' corresponding to the pressure P' is less than that given by Boyle's law. We thus see that, in general, gases are more compressible than Boyle's law would indicate; and in the case of gases that are liquefiable, this difference of compressibility is, as we have said, considerable.

In this respect hydrogen is a remarkable exception, as was originally shown by Despretz in the experiments which we have mentioned; it is less compressible than it should be by Boyle's law, the difference $\frac{VP}{V'P'} - 1$ being negative.

This singular peculiarity of hydrogen is quite in harmony with the views which are entertained as to the nature of this gas. It has been observed, from several comparative experiments performed upon carbonic acid, that at the temperature of 100° Cent. the law of compressibility of this gas differs from Boyle's law much less than at ordinary temperatures. We may thus fairly suppose that if we were to operate at a still higher temperature, we should approach still more nearly to the law, which would doubtless be verified at a particular temperature, beyond which the error would be in the opposite direction. It would thus appear that for each gas there is a sort of normal temperature, at which the compressibility is exactly represented by Boyle's law.

The compressibility of a gas should also increase as the temperature decreases, as is proved by the experiment on carbonic acid. With the exception of hydrogen, all gases under ordinary conditions are below this normal temperature. But if, as chemical phenomena tend to prove, hydrogen is a kind of metal, we must suppose it to be relatively in a high state of rarefaction, which accounts for the peculiarity presented by its compressibility.

123. Manometers or Pressure-gauges.—Manometers are instruments for measuring the elastic force of a gas or vapour contained in the interior of a closed space. This elastic force is generally expressed in units called atmospheres (§103), and is often measured by means of a column of mercury.

When the column of mercury moves freely in an open tube, the manometer is said to be open [*à air libre*]; it was a manometer of this kind that Regnault employed to measure the successive pressures to which the volume of gas was subjected.

The open mercurial pressure-gauge is often used in the arts to measure pressures that are not very considerable. The figure represents one of the simplest forms. The apparatus consists of a box, generally of iron, at the top of which is an opening closed by a screw stopper, which is traversed by the tube *b*, open at both ends, and dipping into the mercury in the box. The air or vapour whose elastic force is to be measured enters by the tube *a*, and presses upon

the mercury. It is evident that if the level of the liquid in the box is the same as in the tube, the pressure in the box must be exactly equal to that of the atmosphere. If the mercury in the tube rises above that in the box, the pressure of the air in the box must exceed that of the atmosphere by a pressure corresponding to the height of the column raised. The pressures are generally marked in atmospheres upon a scale beside the tube.

124. Multiple Branch Manometer.—When the pressures to be measured are considerable, as in the boiler of a high-pressure steam-engine, the

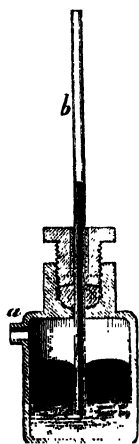


Fig. 129.—Open Mercurial Manometer.

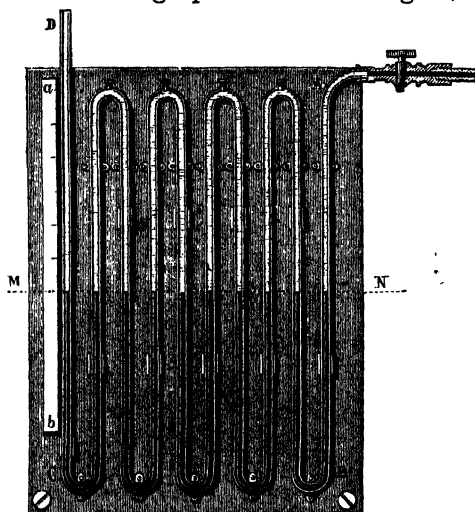


Fig. 130.—Multiple Branch Manometer.

above instrument, if employed at all, must be of a length corresponding to the pressure. If, for instance, the pressure in question is eight atmospheres, the length of the tube must be at least 8×30 inches = 20 feet. Such an arrangement is inconvenient even for stationary machines, and is entirely inapplicable to movable machines.

Without departing from the principle of the open mercurial pressure-gauge, namely, the balancing of the pressure to be observed against the weight of a liquid increased by one atmosphere, we may reduce the length of the instrument by an artifice already employed by Fahrenheit in his barometer (§110).

The apparatus for this purpose consists of an iron tube ABCD bent back upon itself a certain number of times. The extremity A communicates with the boiler by a stop-cock, and the last branch CD is of glass, and has a scale by its side.

The first step is to fill the tube with mercury as far as the level MN. At this height are holes by which the mercury escapes when it reaches them, and which are afterwards hermetically sealed. The upper portions are filled with water through openings which are also stopped after the tube has been filled. If the mercury in the first tube, which is in communication with the reservoir of gas, falls through a certain distance h , it will alternately fall and rise through the same distance in each of the tubes, and will consequently rise through the same distance in the last tube; now this distance corresponds to an effective pressure represented by a column of mercury of height $10h$, diminished by ten times the same height of water; that is, to a height of mercury equal to $10h (1 - \frac{1}{13.59})$. It will thus be seen that a very considerable pressure will be indicated by a comparatively small variation of the mercurial column. If, for instance, beginning with the atmospheric pressure, an additional pressure of five atmospheres is exerted, that is, an effective pressure of six atmospheres, the quantity h will be given in metres by the equation

$$5 \times .76 = 10h (1 - \frac{1}{13.59}),$$

whence

$$5 \times .76 \times 13.59 = 10h \times 12.59, \\ h = .40m.$$

125. Compressed-air Manometer.—This instrument, which may assume different forms, sometimes consists, as in Fig. 131, of a bent tube AB closed at one end a , and containing within the space Aa a quantity of air, which is cut off from external communication by a column of mercury. The apparatus has been so constructed, that when the pressure on B is equal to that of the atmosphere, the mercury stands at the same height in both branches; so that, under these circumstances, the inclosed air is exactly at atmospheric pressure. But if the pressure increases, the mercury is forced into the left branch, so that the air in that branch is compressed, and its tension gradually increases until equilibrium is established. The pressure of the gas exerted at B is then equal to the pressure of the compressed air, together with that of a column of mercury equal to the difference of level of the liquid in the two branches. This pressure is expressed in atmospheres on the scale ab .

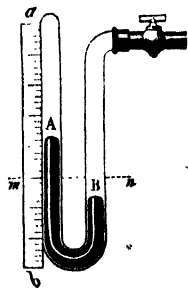


Fig. 131.—Compressed-air Manometer.

The graduation of this scale is effected directly in practice; by placing the manometer in communication with a reservoir of compressed air whose pressure is given by an open mercurial gauge, or by a standard manometer of any kind.

If the tube AB be supposed cylindrical, the graduation can be previously effected by an application of Boyle's law.

Let l be the length of the tube occupied by the inclosed air when its pressure is equal to that of one atmosphere; at the point to which the level of the mercury rises is marked the number 1. It is required to find to what point the end of the liquid column should reach when a pressure of n atmospheres is exerted at B. Let x be the height of this point above 1; then the volume of the air, which was originally l , has become $l-x$, and its pressure is therefore equal to $H \frac{l}{l-x}$, H being the mean height of the barometer. This pressure, together with that due to the difference of level $2x$, is equivalent to n atmospheres. We have thus the equation—

$$H \frac{l}{l-x} + 2x = nH,$$

whence

$$2x^2 - (nH + 2l)x + (n-1)Hl = 0.$$

$$x = \frac{nH + 2l \pm \sqrt{(nH + 2l)^2 - 8(n-1)Hl}}{4}.$$

We thus find two values of x ; but that given by taking the positive sign of the radical is inadmissible; for if we put $n=1$, we ought to have $x=0$, which cannot be the case unless the sign of the radical is negative.

By giving n the successive values of $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, &c., in this expression for x , we have the points on the scale corresponding to pressures of one atmosphere and a half, two atmospheres, &c.

As we have before remarked, the sensibility of the instrument decreases as the pressure increases, and the distance traversed by the mercury for an increment of pressure equal to one atmosphere becomes less and less. This inconvenience is partly avoided by the arrangement shown in Fig. 132. The branch

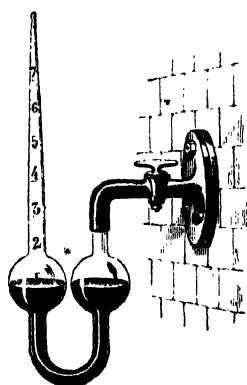


Fig. 132.
Compressed-air Manometer.

containing the air is of a conical form; in this way, as the mercury rises, equal changes of volume correspond to increasing lengths. The

effect of this arrangement is seen by an inspection of the scale, on which the numbers corresponding to successive atmospheres of pressure are nearly equidistant, whereas when the tube is cylindrical they rapidly approach each other.

126. Metallic Manometers.—The fragility of glass tubes, and the fact that they are liable to become opaque by the mercury clinging to their sides, are serious drawbacks to their use, especially in machines in motion. Accordingly, metallic manometers are often employed, depending upon the changes of form effected by the pressure of gas on its containing vessel, when suitably constructed. We shall here mention only Bourdon's gauge (Fig. 133). It consists essentially of a copper tube of elliptic section, which is bent through about 540° as represented in Fig. 133. One of the extremities communicates by a stop-cock with the reservoir of steam or compressed gas; to the other extremity is attached a steel needle which traverses a scale. When the stop-cock permits communication with the atmosphere, the end of the needle stands at the mark 1; but if the pressure increases the curvature diminishes, the free extremity of the tube moves away from the fixed extremity, and the needle traverses the scale.

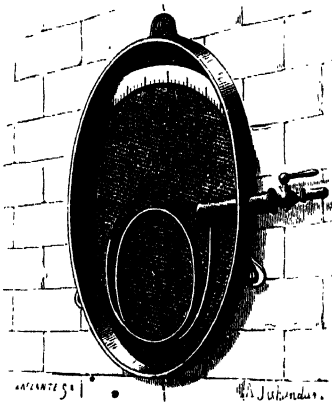


Fig. 133.—Bourdon's Pressure-gauge.

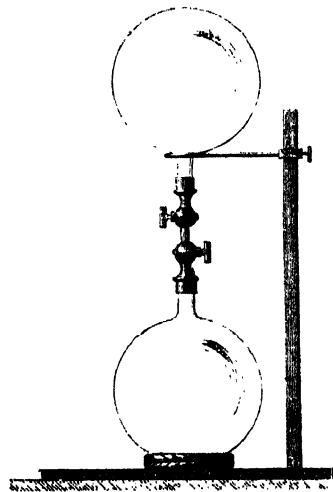


Fig. 134.—Mixture of Gases.

127. Mixture of Gases.—When gases of different densities are enclosed in the same space, experiment shows that, even under the most unfavourable circumstances, an intimate mixture takes place, so that each gas becomes uniformly diffused through the entire space. This

fact has been shown by a decisive experiment due to Berthollet. He took two globes (Fig. 134) which could be screwed together, and placed them in a cellar. The lower globe was filled with carbonic acid, the upper globe with hydrogen. Communication was established between them, and at the end of a certain time it was ascertained that the gases had become intimately mixed; in fact, the proportion of carbonic acid and of hydrogen was exactly the same in both globes. The fact that the composition of the air is the same at all heights is another striking proof.

If several gases are inclosed in the same space, each of them exerts the same pressure as if the others were absent, and consequently the pressure exerted by the mixture is equal to the sum of the pressures due to each gas separately. These separate pressures can easily be calculated by Boyle's law, when the original pressure and volume of each gas are known.

For example, let V and P , V' and P' , V'' and P'' be the volumes and pressures of the gases which are made to pass into a vessel of volume U . The first gas exerts, when in this vessel, a pressure equal to $\frac{VP}{U}$, the second a pressure equal to $\frac{V'P'}{U}$, the third a pressure equal to $\frac{V''P''}{U}$, and so on, so that the total pressure M is equal to $\frac{VP}{U} + \frac{V'P'}{U} + \frac{V''P''}{U}$, whence $MU = VP + V'P' + V''P''$.

This formula expresses the law of pressure for a mixture of gases; it may easily be verified by passing different volumes of gas into a graduated glass jar inverted over mercury, after having first measured their volumes and pressures.

128. Absorption of Gases by Liquids and Solids.—All gases are to a greater or less extent soluble in water. This property is of considerable importance in the economy of nature; thus the life of aquatic animals and plants is sustained by the oxygen of the air which the water holds in solution. The *volume* of a given gas that can be dissolved in water at a given temperature is found to be in general the same at all pressures,¹ and the ratio of this volume to that of the water which dissolves it is called the *coefficient of solubility or of absorption*. At the temperature 0° Cent. the coefficient of solubility for carbonic acid is 1, for oxygen .04, and for ammonia 600.

If a mixture of two or more gases be placed in contact with water,

¹ Hence the *weight* of gas absorbed is directly as the pressure.

each gas will be dissolved to the same extent as if it were the only gas present.

Other liquids as well as water possess the power of absorbing gases, according to the same laws, but with coefficients of solubility which are different for each liquid.

Increase of temperature diminishes the coefficient of solubility, which is reduced to zero when the liquid boils.

Some solids, especially charcoal, possess the power of absorbing gases. Boxwood charcoal absorbs about nine times its volume of oxygen, and about ninety times its volume of ammonia. When saturated with one gas, if put into a different gas, it gives up a portion of that which it first absorbed, and takes up in its place a quantity of the second. Finely-divided platinum condenses on the surface of its particles a large quantity of many gases, amounting in the case of oxygen to many times its own volume. If a jet of hydrogen gas be allowed to fall, in air, upon a ball of spongy platinum, the gas combines rapidly, in the pores of the metal, with the oxygen of the air, giving out an amount of heat which renders the platinum incandescent and usually sets fire to the jet of hydrogen.

Most solids have in ordinary circumstances a film of air adhering to their surfaces. Hence iron filings, if carefully sprinkled on water, will not be wetted, but will float on the surface, and hence also the power which many insects have of running on the surface of water without wetting their feet. The film of air in these cases prevents wetting, and hence, by the principles of capillarity, produces increased buoyancy.

CHAPTER XV.

AIR-PUMP.

129. Air-pump.—The air-pump was invented by Otto Guericke about 1650, and has since undergone some improvements in detail which have not altered the essential parts of its construction.

It consists of a glass or metal cylinder called the barrel, in which a piston works. This piston has an opening through it which is closed at the lower end by a valve *S* opening upwards. The barrel communicates with a passage leading to the centre of a brass surface carefully polished, which is called the plate of the air-pump. The entrance to the passage is closed by a conical stopper *S'*, at the extremity of a metal rod which passes through the piston-head, and works in it tightly, so as to be carried up and down with the motion

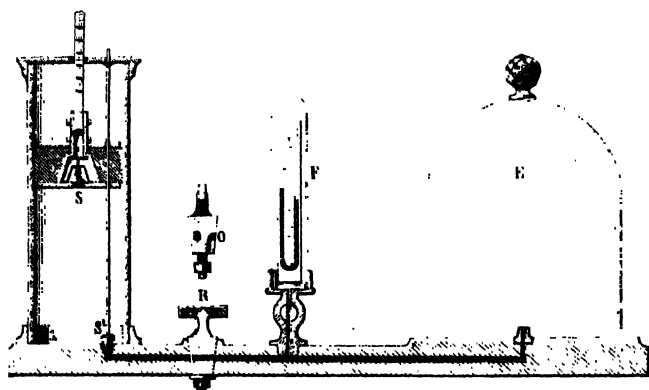


Fig. 135.—Air-pump.

of the piston. A catch at the upper part of the rod confines its motion within very narrow limits, and only permits the stopper to rise a small distance above the opening.

Suppose now that the piston is at the bottom of the cylinder, and

is raised. The valve S' is opened, and the air of the receiver E rushes into the cylinder. On lowering the piston, the valve S' closes its opening, the air which has entered the cylinder cannot return into the receiver, and, on being compressed, raises the valve in the piston, and escapes into the air outside. On raising the piston again, a portion of the air remaining in the receiver will pass into the cylinder, whence it will escape on pushing down the piston, and so on.

We see, then, that if this motion be continued, a fresh portion of the air in the receiver will be removed at each successive stroke. But as the quantity of air removed at each stroke is only a fraction of the quantity remaining, we can never produce a perfect vacuum, though we might approach as near to it as we pleased if this were the only obstacle.

130. Calculation of the Degree of Exhaustion.—It is easy to calculate the quantity of air left in the receiver after a given number of strokes of the piston. Let V be the volume of the cylinder, V' that of the receiver, and M the mass of air in the receiver at first. On raising the piston, the air which occupied the volume V' occupies a volume $V' + V$; of the air thus expanded the volume V is removed, and the volume V' left, being $\frac{V'}{V' + V}$ of the whole quantity or mass M . The quantity remaining after the second stroke is $\frac{V'}{V' + V}$ of that after the first, or is $\left(\frac{V'}{V' + V}\right)^2 M$; and after n strokes $\left(\frac{V'}{V' + V}\right)^n M$. Hence the density and (by Boyle's law) the pressure are each reduced by n strokes to $\left(\frac{V'}{V' + V}\right)^n$ of their original values.

We see, then, that the pressure goes on decreasing indefinitely, and that, consequently, the elasticity of the air may, theoretically at least, be rendered less than any assigned quantity.

131. Mercurial Gauge.—In order to follow the steps of the operation, and to observe at each instant the elastic force of the air in the receiver, the instrument is provided with a siphon-barometer, called the mercurial gauge, inclosed in a bell-shaped vessel of glass F , and communicating by a stop-cock with the receiver. This barometer consists of a bent tube, the branches of which are about a foot in length; one of these is closed and filled with mercury, the other is open. When the pressure of the air in the receiver becomes less than that represented by a column of mercury equal in length to the closed branch of the gauge, the mercury falls, and the elastic force of the air at any moment is given by the difference of level of the

mercury in the two branches; this difference can be measured on a graduated scale. The mercurial gauge serves to show whether the instrument is working properly; for instance, in the case of air getting in anywhere, this would be shown by the fluctuations of the mercurial column. It also shows when the greatest possible effect has been attained, by the level of the mercury remaining stationary notwithstanding the motion of the piston. In theory, as we have said above, there is no limit to the action of the machine, and at each stroke of the piston the elastic force of the air should decrease; but in reality this is not the case, on account of the inevitable imperfections of the apparatus; there is always a limit, extending further in proportion to the excellence of the machine, and the barometer shows the moment when this limit is reached. Instead of a siphon-barometer, we might have an ordinary barometer in connection with the receiver, and thus observe the progress of the vacuum from the first strokes of the piston.

132. Admission Stop-cock.—After the receiver has been exhausted of air, if it was required to raise it from the plate, a very considerable force would be necessary, amounting to as many times fifteen pounds as the area of the plate contained square inches. It would, therefore, be in general impossible to raise the receiver. This is, however, rendered possible by means of the stop-cock R, which is shown in section above. It is perforated by a straight channel, which, when the machine is being worked, forms part of the communicating passage. At 90° from the extremities of this channel is another opening O, forming the mouth of a bent passage, leading to the external air. When we wish to admit the air into the receiver, we have only to turn the stop-cock so as to bring the opening O to the side next the receiver; if, on the contrary, we turn it towards the pump-barrel, all communication between the pump and the receiver is stopped, the risk of air entering is diminished, and the vacuum remains good for a greater length of time. This precaution is taken when we wish to leave bodies in a vacuum for a considerable time. Another method is to employ a separate plate, which can be detached so as to leave the machine available for other purposes.

133. Double-barrelled Air-pump.—The machine just described has only a single pump-barrel; air-pumps of this kind are sometimes employed, and are usually worked by a lever like a pump-handle. With this arrangement, it is evidently necessary that the piston, after having ascended, should descend again to expel the air from the

pump-barrel, and it is only after this double stroke that the operation can begin anew.

• Double-barrelled pumps are more frequently used. An idea of their general arrangement may be formed from Figs. 136, 137, and 138. Fig. 138 gives the machine in perspective, Fig. 136 is a section through the axes of the pump-barrels, and Fig. 137 shows the manner in which communication is established between the

receiver and the two barrels. It will be observed that the two passages from the barrels unite in a single passage to the centre of the plate *p*.

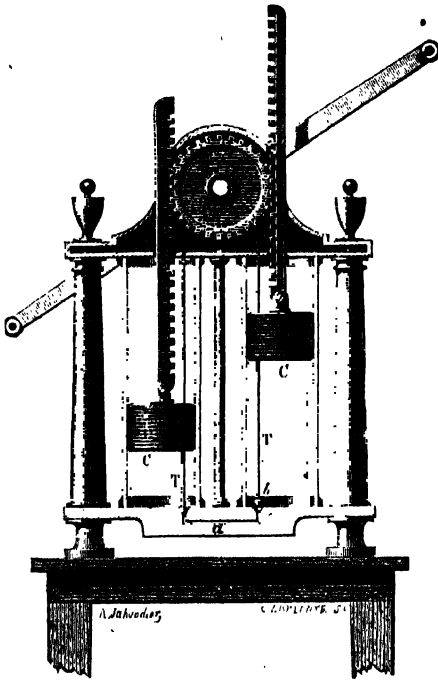


Fig. 136.

Double-barrelled Air-pump.

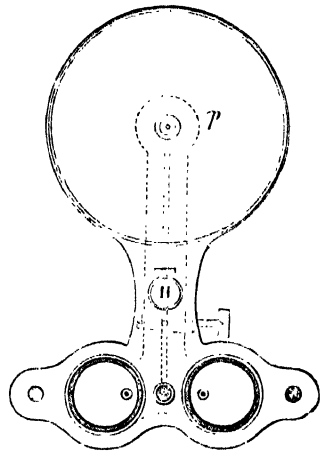


Fig. 137.

The piston-rods *C* are two racks working with the pinion *P*. This pinion is turned by a double-handed lever, which is worked alternately in opposite directions. In this arrangement, when one piston ascends the other descends, and consequently in each single stroke the air of the receiver passes into one or other pump-barrel. A vacuum is thus produced by half the number of strokes which would be required with a single-barrelled pump. It has besides another advantage. In the single-barrelled pump the force required to raise the piston increases as the exhaustion proceeds, and when it is nearly completed there is the resistance of almost an atmosphere to be overcome; that is, nearly 15 pounds to the square inch. In the double-

barrelled pump, at the moment when one piston is at the top, and the other at the bottom, the force opposing the ascent of the one is precisely equal to that assisting the descent of the other. We must

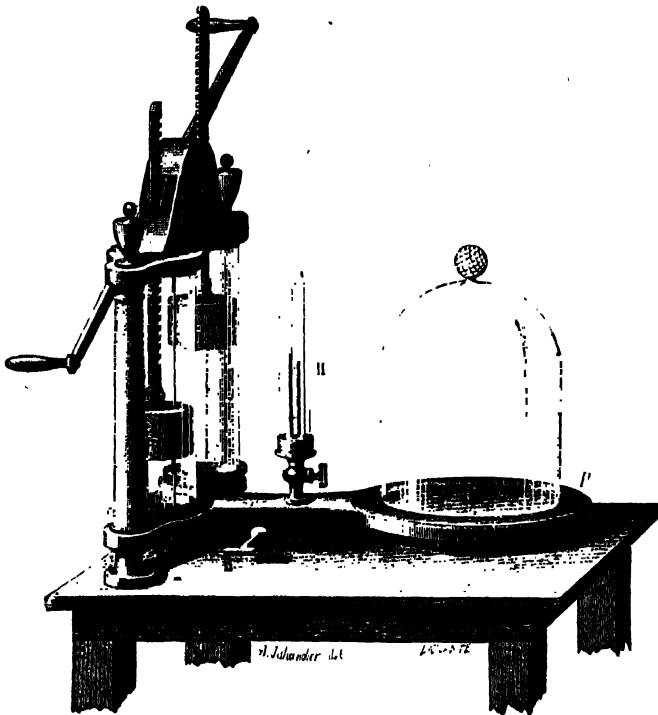


Fig 138 —Air pump.

observe, however, that this equality exists only at the beginning of the stroke; for when one of the pistons descends, the air below it is compressed, its tension becoming greater and greater, until it reaches that of the atmosphere and raises the piston-valve. At this moment the resistance to the ascent of the other piston is entirely uncompensated, and up to this point the compensation has been gradually diminishing. But the more nearly we approach to a perfect vacuum, the more slowly does the tension of the air compressed beneath the piston increase, so that, unlike the single-barrelled pump, it becomes easier to work as the exhaustion proceeds.

134. Single-barrelled Pumps with Double Action.— We do not, however, require two pump-barrels in order to obtain double action, as the same effect may be obtained with a single barrel. An arrangement for this purpose was long ago suggested by Delahire for water-

pumps; but the principle has only lately been applied to the construction of air-pumps.

Fig. 139 represents the single barrel of the double-acting pump of Bianchi. It will be seen that the piston-valve opens into the hollow piston-rod; a second valve, also opening upward, is placed at the top of the pump-barrel. Two other openings, one above, the other below, serve to establish communication by means of a bent vertical tube between the pump-barrel and the passage to the plate. These openings are closed alternately by two conical stoppers at the two extremities of a metal rod passing through the piston, and carried with it in its vertical movement by means of friction. When the piston ascends, as in the figure, the upper opening is closed and the lower one is open; when the piston begins to descend, the opposite effect is immediately produced. Accordingly we see that, whichever be the direction in which the piston is moving, the receiver is being exhausted of air. In fact, when the piston ascends, air from the receiver will enter by the lower opening, and the air above the piston will be gradually compressed, and will finally escape by the valve above. In the descending movement, air will enter by the upper opening, and the compressed air beneath the piston will escape by the piston-valve. The movement of the piston is produced by a peculiar arrangement shown in Fig. 140, which gives a general view of the apparatus.

The pump-barrel, which is composed entirely of cast-iron, oscillates about an axis passing through its base. On the top are guides in which the end of a crank travels. The pump is worked by turning a heavy fly-wheel of cast-iron, on the axis of which is a pinion which drives a toothed wheel on the axis of the crank. The end of the crank is attached to the extremity of the piston-rod. It is evident that on turning the fly-wheel the pump-barrel will oscillate from side to side, following the motions of the crank, and the piston will alternately ascend and descend in the barrel, the length of which should be equal to the diameter of the circle described by the end of the crank.

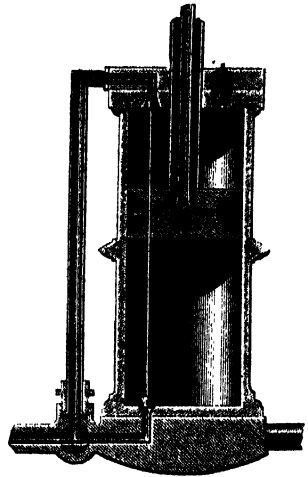
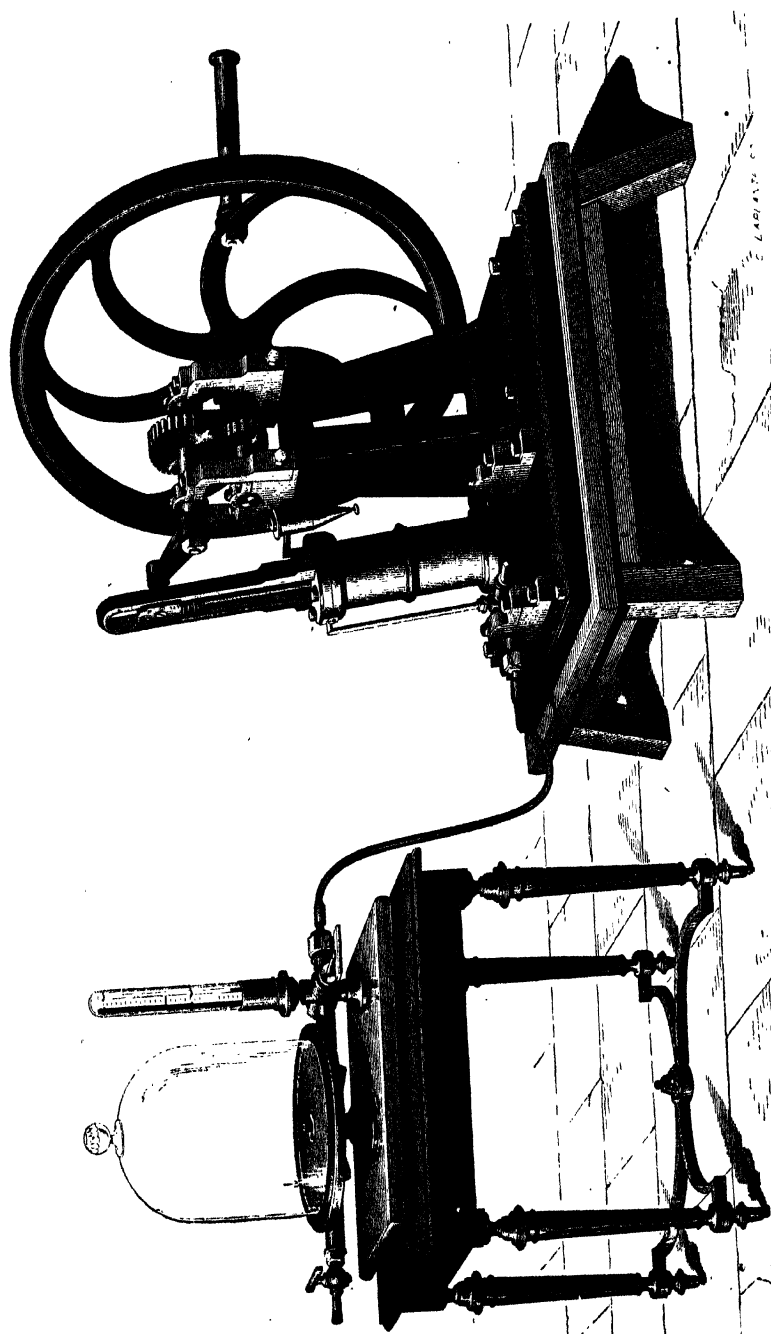


Fig. 139.
Barrel of Bianchi's Air-pump.



135. Various Experiments with the Air-pump.—At the time when the air-pump was invented, several experiments were devised to show the effects of a vacuum, some of which have become classical, and are usually repeated in courses of experimental physics.

Experiment of the Burst Bladder.—On the plate of an air-pump is placed a glass cylinder open at the bottom, and having a piece of bladder tightly stretched over the top. As the exhaustion proceeds, the bladder bends inwards in consequence of the atmospheric pressure above it, and finally bursts with a loud report.

It often happens that, notwithstanding the strong exterior pressure, the bladder does not give way, its molecules preserving their equilibrium of cohesion. But this equilibrium is, so to speak, unstable, and a few taps are sufficient to destroy it and cause the bladder to burst.

Magdeburg Hemispheres.

—We take two hemispheres (Fig 142), which can be exactly fitted on each other; their exact adjustment is further assisted by a projecting internal rim, which is smeared with lard. The apparatus is exhausted of air

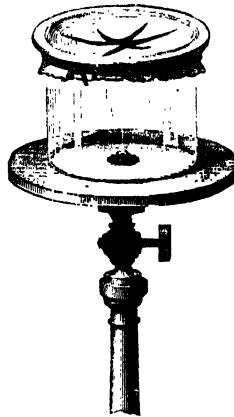


Fig. 141.
Burst Bladder.

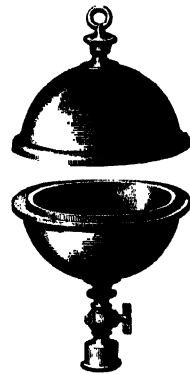


Fig. 142.
Magdeburg Hemispheres.

through the medium of the stop-cock attached to one of the hemispheres, and when a vacuum has been produced, it will be found that a considerable force is required to separate the two parts, and this force increases with the size of the hemispheres.

This resistance to a force of separation is due to the normal exterior pressure of the air on every point of the surface, a pressure which is counterbalanced by only a very feeble pressure from the interior. In order to estimate the resultant effect of these different pressures, let us suppose that the external surface, instead of being spherical, is formed of a series of steps; that is to say, of alternate vertical and horizontal elements. It is evident that the pressures exerted upon these latter will have no influence upon the adhesion of the hemispheres; the first alone produce this effect, and the sum of these is

evidently equal to the pressure of the atmosphere upon the circular area forming the common base of the hemispheres. For example, if this area is ten square inches, the hemispheres will be pressed against each other with a force of 150 pounds.

Fountain in Vacuo.—The apparatus for this experiment consists of a bell-shaped vessel of glass (Fig. 143), the base of which is pierced

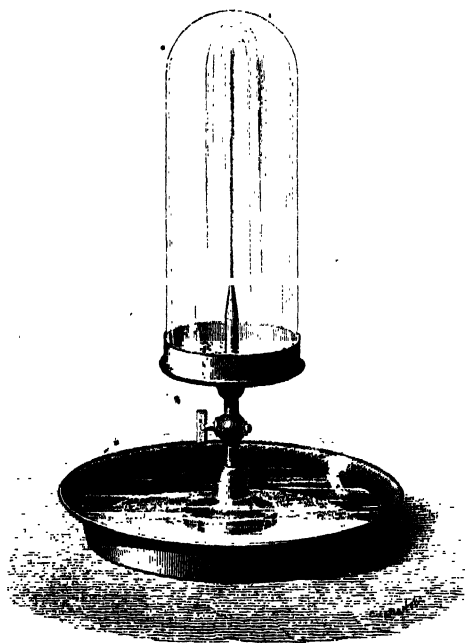


Fig. 143.—Fountain in Vacuo.

by a tube fitted with a stop-cock which enables us to exhaust the vessel of air. If, after a vacuum has been produced, we place the lower end of the tube in a vessel of water, and open the stop-cock, the liquid, being pressed externally by the atmosphere, mounts up the tube and ascends in a jet into the interior of the vessel. This experiment is often made in the opposite manner. Under the receiver of the air-pump is placed a vial

partly filled with water, and having its cork pierced by a tube open at both ends, the lower end being beneath the surface of the water. As the exhaustion proceeds, the air in the vial, by its excess of pressure, acts upon the liquid and makes it issue in a jet.

136. Limit to the Action of the Air-pump.—We have said above (§ 131) that the air-pump does not continue the process of rarefaction indefinitely, but that at a certain stage its effect ceases, and the tension of the air in the receiver undergoes no further diminution. If the pump is very badly made, this tension is considerable; but even with the most perfect machines it is always sensible. A pump such as we have described may be considered as very good if it reduces the tension of the air in the receiver to one-fiftieth of an inch of mercury: it is very rarely that a lower limit is reached.

LEAKAGE.—This limit to the action of the machine is due to various causes. One of these is evidently the leakage at different parts of the apparatus. It is impossible to prevent the air from getting in at several points; and although at the beginning of the operation the quantity of air which thus enters is small in comparison with that which is pumped out, still, as the exhaustion proceeds, the air enters faster, on account of the diminished internal pressure, and at the same time the quantity expelled at each stroke becomes less, so that at length a point is reached at which the inflow and outflow are equal.

SPACE UNTRAVERSED BY PISTON.—Another reason of imperfect exhaustion is that, after all possible precautions, a space is still left between the bottom of the pump-barrel and the lower surface of the piston when the latter is at the end of its downward stroke. It is evident that at this moment the air contained in this *untraversed space* is of the same tension as the atmosphere. On raising the piston, this air is indeed rarefied; but it still preserves a certain tension, and it is evident that when the air in the receiver has been brought to this stage of rarefaction, the machine will cease to produce any effect.

If v is the volume of this space, V the volume of the pump-barrel, the air, which at volume v has a tension H equal to that of the atmosphere, will have, at volume V , a tension equal to $H\frac{v}{V}$. This gives the limit to the action of the machine as deduced from the consideration of the untraversed space.

AIR GIVEN OUT BY OIL.—Finally, perhaps the most important cause, and the most difficult to remedy, is the absorption of air by the oil used for lubricating the pistons. This oil is poured on the top of the piston, but the pressure of the external air forces it between the piston and the barrel, whence it falls in greater or less quantity to the bottom of the barrel, where it absorbs air, and partially yields it up at the moment when the piston begins to rise, thus evidently tending to derange the working of the machine. It has been attempted to get rid of untraversed space by employing a kind of piston of mercury. This has also the advantage of fitting the barrel more accurately, and thus preventing the entrance of air. The use of oil is at the same time avoided, and we thus escape the injurious effects mentioned above. We proceed to describe two machines founded upon this principle.

137. **Kravogl's Air-pump.**—This contains a hollow glass cylinder AB tapering at the upper end, and surmounted by a kind of funnel. The piston is of the same shape as the cylinder, and is covered with a layer of mercury, whose depth over the point of the piston is about

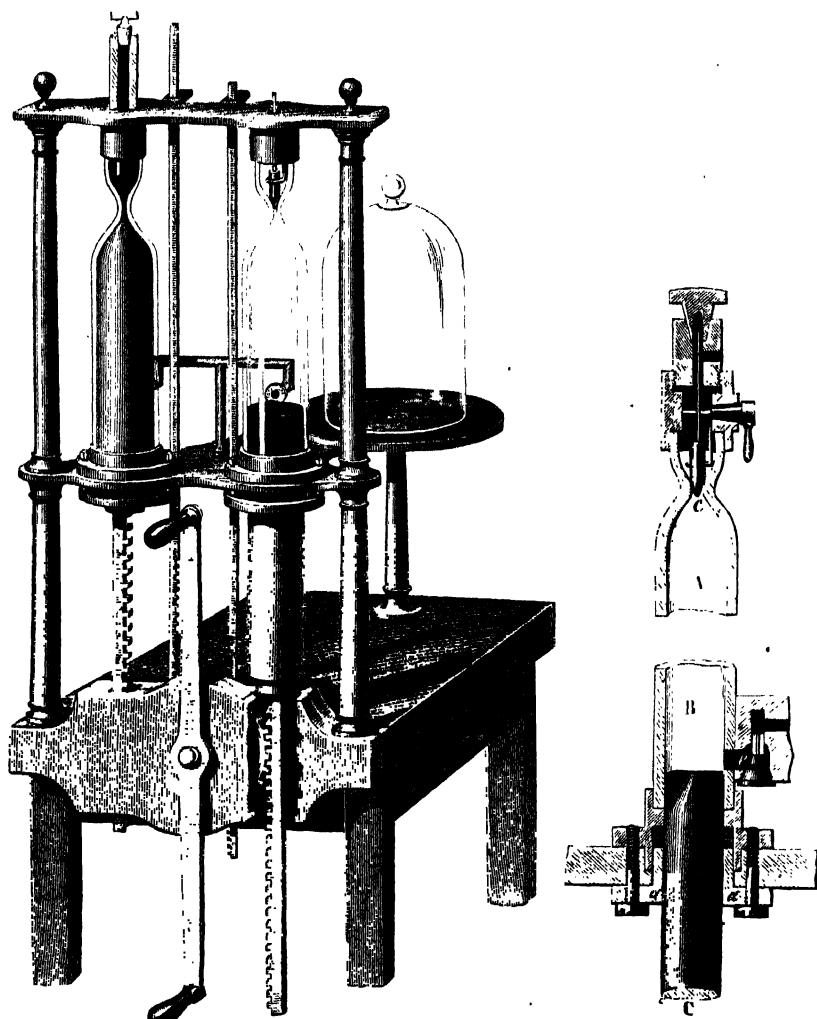


Fig. 145.—Kravogl's Air-pump.

$\frac{1}{16}$ th of an inch when the piston is at the bottom of its stroke, but is nearly an inch when the piston rises and fills the funnel-shaped cavity in which the pump-barrel terminates. A small interval, filled by the liquid, is left between the barrel and the piston; but at the

bottom of the barrel the piston passes through a leather box carefully made, so as to be perfectly air tight.

The air from the receiver passes through the lateral opening *e*; it is driven before the mercury into the funnel above. With the air passes a certain quantity of mercury, which is detained by a steel valve *c* at the narrowest part of the funnel. This valve rises automatically when the surface of the mercury is at a distance of about half an inch from the funnel, and falls back into its former position when the piston is at the end of its upward stroke. In the downward stroke, when the mercury is again half an inch from the funnel, the valve opens again and allows a portion of the mercury to pass.

The effect of this arrangement is easily understood; there is no "untraversed space," the presence of the mercury above and around the piston causes a very complete fit, and excludes the external air; and hence the machine, when well made, is very effective.

When this is the case, and when the mercury used in the apparatus is perfectly dry, a vacuum of about $\frac{1}{25}$ th of an inch can be obtained. The dryness of the mercury is a very important condition, for at ordinary temperatures the elastic force of the vapour of water has a very sensible value. If we wish to employ the full powers of the machine, we must have, between the vessel to be exhausted of air and the pump-barrel, a desiccating apparatus.

The arrangement of the valve *c* is peculiar. It is of a conical form, so as, in its lowest position, to permit the passage of air coming from the receiver. Its ascent is produced by the pressure of the mercury, which forces it against the conical extremity of the passage, and the liquid is thus prevented from escaping.

The figure represents a double-barrelled machine analogous to the ordinary air-pump. Besides the pinion working with the racks of the pistons, there is a second smaller pinion, not shown in the figure, which governs the movements of the valves *c*. All the parts of this machine, as the stop-cocks, valves, pipes, &c., must be of steel, to avoid the action which the mercury would have upon any other metal.

138. Geissler's Machine.—Geissler, of Bonn, has invented a mercurial air-pump, in which the vacuum is produced by communication of the receiver with the Torricellian vacuum. Fig. 146 represents this machine as constructed by Alvergnyat. It consists of a vertical tube, which serves as a barometric tube, and communicates at the bottom, by means of a caoutchouc tube, with a globe which serves as the cistern.

At the top of the tube is a three-way stop-cock, by which communication can be established either with the receiver to the left, or with a funnel to the right, which latter has an ordinary stop-cock at the bottom.

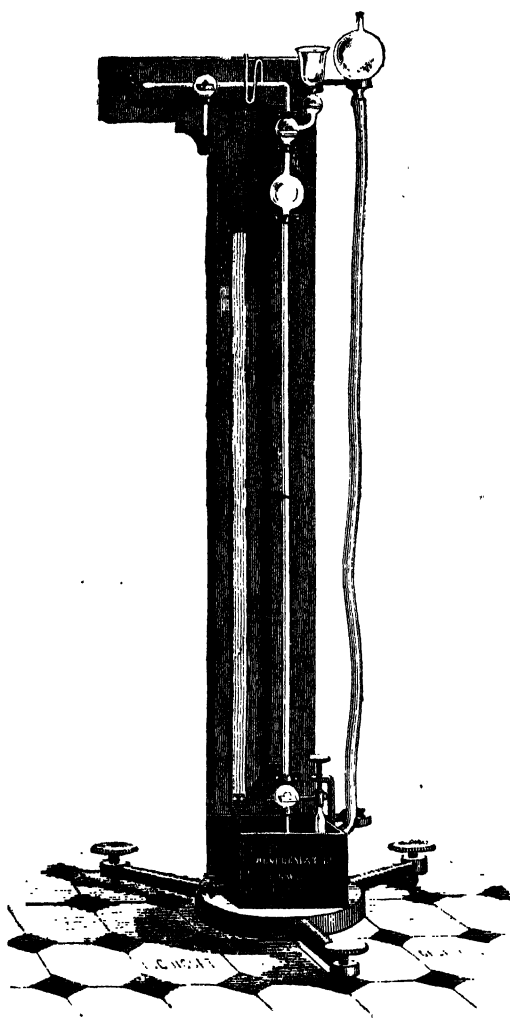


Fig. 146.—Geissler's Machine.

By means of another stop-cock on the left, communication with the receiver can be opened or closed. These stop-cocks are made entirely of glass. The machine works in the following manner: communication being established with the funnel, the globe which serves as cistern is raised, and placed, as shown in the figure, at a higher level than the stop-cock of the funnel. By the law of equilibrium in communicating vessels, the mercury fills the barometric tube, the neck of the funnel, and part of the funnel itself. If the communication between the funnel and tube be now stopped, and the globe lowered, a Torricellian vacuum is produced in the upper part of the vertical tube.

Communication is now opened with the receiver; the air rushes into the vacuum, and the column of mercury falls a little. Communication is now stopped between the tube and receiver, and opened between the tube and the funnel, the simple stop-cock of the funnel being, however, left shut. If at this moment the globe is replaced in the posi-

tion shown in the figure, the air endeavours to escape by the funnel, and it is easy to allow it to do so. Thus, a part of the air of the receiver has been removed, and the apparatus is in the same position as at the beginning. The operation described is equivalent to a stroke of the piston in the ordinary machine, and this process must be repeated till the receiver is exhausted.

As the only mechanical parts of this machine are glass stop-cocks, which are now executed with great perfection, it is capable of giving very good results. With dry mercury a vacuum of $\frac{1}{25}$ th of an inch may very easily be obtained. The working of the machine, however, is inconvenient, and becomes exceedingly laborious when the receiver is large. It is therefore employed directly only for producing a vacuum in very small vessels; when the spaces to be exhausted of air are at all large, the operation is begun with the ordinary machine, and the mercurial air-pump is only employed to render the vacuum thus obtained more perfect.

138 A. Sprengel's Air-pump.—

This instrument, which may be regarded as an improvement upon Geissler's, is represented in its simplest form in Fig. 146 A. *cd* is a glass tube longer than a barometer tube, down which mercury is allowed to fall from the funnel A. Its lower end dips into the glass vessel B, into which it is fixed by means of a cork. This vessel has a spout at its side, a few millimetres higher than the lower end of the tube. The first portions of mercury which

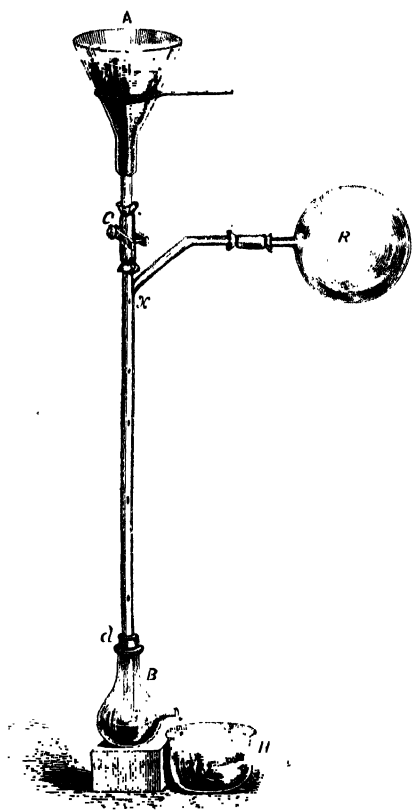


Fig. 146 A.—Sprengel's Air-pump.

run down will consequently close the tube, and prevent the possibility of air entering it from below. The upper part of *cd* branches off at *x* into a lateral tube communicating with the re-

ceiver R, which it is required to exhaust. A convenient height for the whole instrument is six feet. The funnel A is supported by a ring as shown in the figure, or by a board with a hole cut in it. The tube *cd* consists of two parts, connected by a piece of india-rubber tubing, which can be compressed by a clamp so as to keep the tube closed when desired. As soon as the mercury is allowed to run down the exhaustion begins, and the whole length of the tube, from *x* to *d*, is seen to be filled with cylinders of mercury separated by cylinders of air, all moving downwards. Air and mercury escape through the spout of the bulb B, which is above the basin H, where the mercury is collected. This has to be poured back from time to time into the funnel A, to pass through the tube again and again until the exhaustion is completed.

As the exhaustion is progressing, it will be noticed that the inclosed air between the mercury cylinders becomes less and less, until the lower part of *cd* presents the aspect of a continuous column of mercury about 30 inches high. Towards this stage of the operation a considerable noise begins to be heard, similar to that of a shaken water-hammer, and common to all liquids shaken in a vacuum. The operation may be considered completed when the column of mercury does not inclose any air, and when a drop of mercury falls upon the top of this column without inclosing the slightest air-bubble. The height of this column now corresponds exactly with the height of the column of mercury in a barometer; or, what is the same, it represents a barometer whose vacuum is the receiver R and connecting tube.

Dr. Sprengel recommends the employment of an auxiliary air-pump of the ordinary kind, to commence the exhaustion when time is an object, as without this from 20 to 30 minutes are required to exhaust a receiver of the capacity of half a litre. As, however, the employment of the auxiliary pump involves additional connections and increased leakage, it should be avoided when the best possible exhaustion is desired. The fall tube must not exceed about a tenth of an inch in diameter, and special precautions must be employed to make the india-rubber connections air-tight. (See *Chemical Journal* for 1865, p. 9.)

By this instrument air has been reduced to $\frac{1}{1000000}$ th of atmospheric density, and the average exhaustion attainable by its use is about one-millionth, which is equivalent to $\cdot 00003$ of an inch of mercury.

139. Double Exhaustion.—In the mercurial machines just described there is no “untraversed space,” as the liquid completely expels all

the air from the pump-barrel. These machines are of very recent invention. Babinet long before introduced an arrangement for the purpose, not of getting rid of this space, but of exhausting it of air.

For this purpose, when the machine ceases to work with the ordinary arrangement, the communication of the receiver with one of the pump-barrels is shut off, and this barrel is employed to exhaust the air from the other. This change is effected by means of a stop-cock at the point of junction of the passages leading from the two barrels (Fig. 147). The stop-cock has a T-shaped aperture, the point of intersection of the two branches being in constant communication with the receiver.

In a different plane from that of the T-shaped aperture is another aperture *mn*, which, by means of the tube *l*, establishes communication between the pump-barrel B and the communicating passage of the pump-barrel A. From this explanation it will be seen that if the stop-cock be turned as shown in the first figure, the two pump-barrels both communicate with the receiver, and the operation proceeds in the ordinary manner.

But if the stop-cock be turned through a quarter of a revolution, as shown in the second figure, the pump-barrel B alone communicates with the receiver, while it is itself exhausted of air by the barrel A.

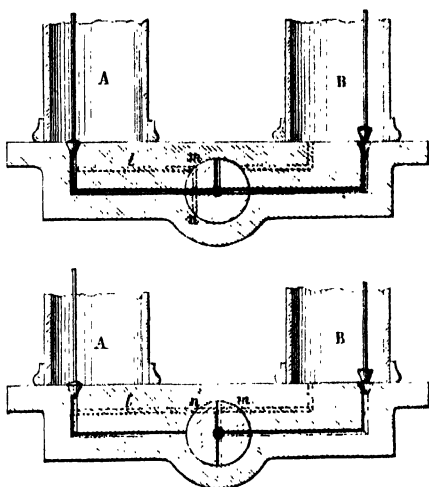


Fig. 147.—Babinet's Doubly-exhausting Stop-cock.

It is easy to express by a formula the effect of this double exhaustion. Suppose the pump to have ceased, under the ordinary method of working, to produce any farther exhaustion, the air in the receiver has therefore reached a tension nearly equal to H_V^r (§ 136). At this moment the stop-cock is turned into its second position. When the piston B descends, the piston A rises, and the air of the "untraversed space" in B is drawn into A and rarefied. During the inverse operation the air in A is prevented from returning to B, and thus the rarefied air from B, becoming still further rarefied,

will draw a fresh quantity of air from the receiver. This air will then be driven into A, where it will be compressed by the descending movement of the piston, and will find its way into the air outside.¹

This double exhaustion will itself cease to work when air ceases to pass from the pump-barrel B into the pump-barrel A. Now when the piston in this latter is raised, the elastic force of the air which was contained in its "untraversed space" is equal to H_V^v , for on the last opening of the valve, the air in this space escaped into the atmosphere. On the other hand, when the piston in B is at the end of its upward stroke, the tension of the air is the same as in the receiver. Let this be denoted by x . When the piston in B descends the air is compressed into the "untraversed space" and the passage leading to A. Let the volume of this passage be l . Then the tension will increase, and become $x \frac{V + l}{v + l}$. When the machine ceases to produce any farther effect, this tension cannot be greater than that in the pump-barrel A, which is H_V^v ; we have thus, to determine the limit to the action of the pump, the equation

$$x \frac{V + l}{v + l} = H_V^v, \text{ whence}$$

$$x = H_V^v \cdot \frac{v + l}{V + l}.$$

140. Air-pump with Free Piston.—We shall describe one more air-pump, constructed by Deleuil, and founded upon an interesting principle. We know that gases possess a remarkable power of adhesion for solids, so that a body placed in the atmosphere may be considered as covered with a very thin coat of air, forming, so to speak, a permanent envelope. On account of this circumstance, gases find very great difficulty in moving in very narrow spaces. On these facts depends the principle of what is called the air-pump with free piston.

The piston P (Fig. 149), which is composed entirely of metal, is of a considerable length, and on its outer surface is a series of parallel circular grooves very close together. It does not touch the pump-barrel at any point; but the distance between the two is very small, about .001 of an inch. This free piston is surrounded by a cushion of gas, which forms its only stuffing, and is sufficient to enable the

¹ It will be observed that during the process of double exhaustion the piston of B behaves like a solid piston; its valve never opens, because the pressure below it is always less than atmospheric.

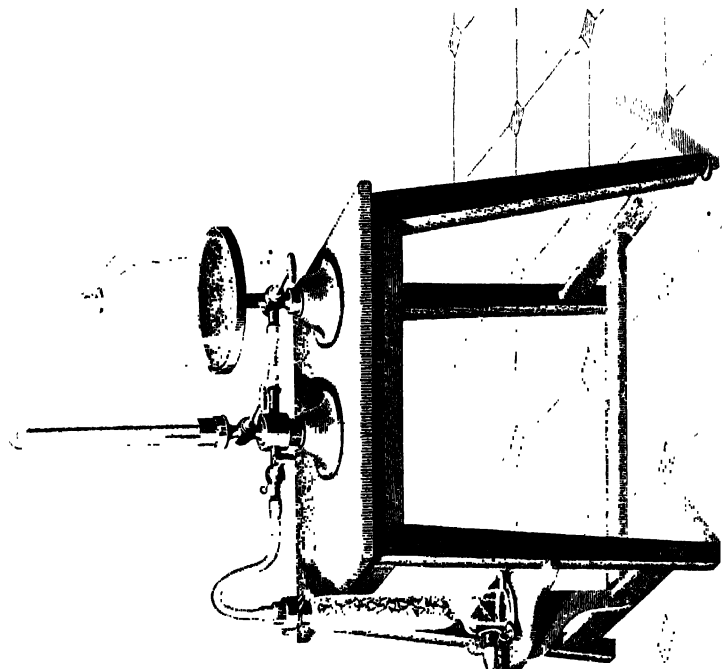
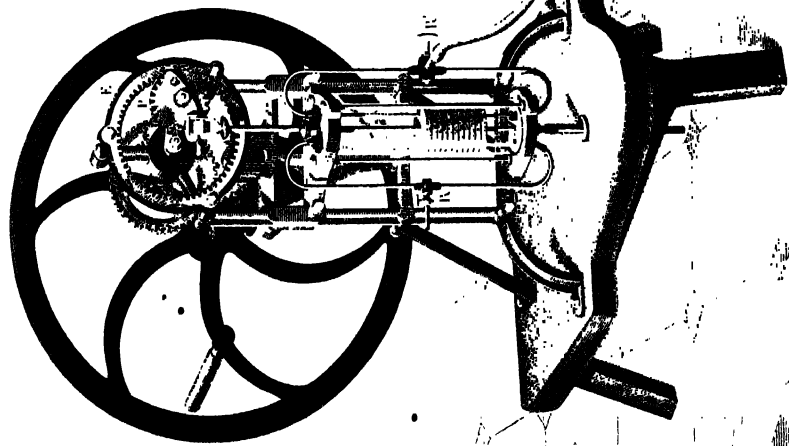


Fig. 148. — DeLaval's Air-pump

machine to work in the ordinary manner, notwithstanding the permanent communication between the upper and lower surfaces of the piston. This machine gives a vacuum about as good as is obtainable

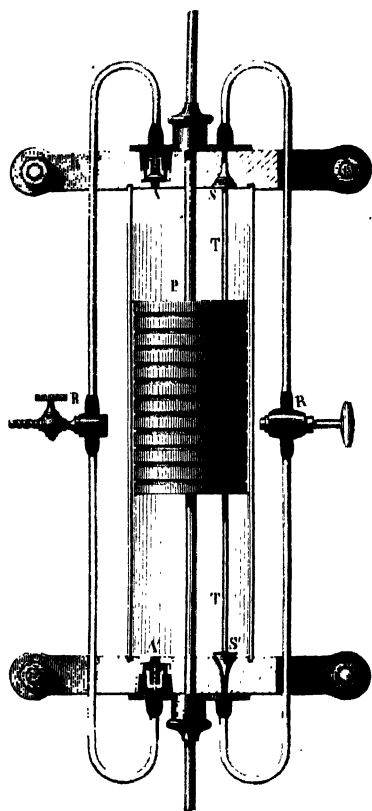


Fig. 149.

Piston and Barrel of Delaunay's Air-pump.

by ordinary pumps, and it has the important advantages of not requiring oil, and of having less friction. It consequently wears better, and is less liable to the development of heat, which is a frequent source of annoyance in air-pumps. It is single-barrelled with double action, like Bianchi's. The two openings S and S' are to admit air from the receiver; they are closed and opened alternately by conical stoppers at the ends of the rod T, which passes through the piston, and is carried with it by friction in its movement. They communicate with tubes which unite at R' with a tube leading from the receiver. A and A' are valves for the expulsion of the air, which escapes by tubes uniting at R. The alternate movement of the piston is produced by what is called Delahire's gearing. This depends on the principle, that *when a circle rolls without sliding in the interior of another circle of double the diameter, any point on the circum-*

ference of the rolling circle describes a diameter of the fixed circle. In order to utilize this property, the end of the piston-rod is jointed to the extremity of a piece of metal which is rigidly attached to the pinion P, the joint being exactly opposite the circumference of the pinion. This latter is driven by a fly-wheel with suitable gearing, and works with the fixed wheel E, which is toothed on the inside. Thus the piston will freely, and without any lateral effort, describe a vertical line, the length of the stroke being equal to the diameter of the fixed wheel.

141. Condensing Pump.—It can easily be seen from the description

of the air-pump, that if the expulsion-valves were connected with a tube communicating with a reservoir, the air removed by the pump would be forced into this reservoir. This communication is established in the instrument just described. If, therefore, R' be made to communicate with the external air, this air will be continually drawn in at that point and forced back into the reservoir connected with R , so that the instrument will act as a condensing pump. The condensing pump is thus seen to be the same instrument as the air-pump, the only difference being that the receiver is connected with the expulsion-valves, instead of with the exhaustion-valves; it is thus, so to speak, the air-pump reversed.

This fact can be very well seen in the structure of a small pump frequently employed in the laboratory, and represented in Fig. 150.

At the bottom of the pump-barrel are two valves, communicating with two separate reservoirs, that on the left being an admission-valve, and that on the right an expulsion-valve.

When the piston is now raised, rarefaction is produced in the reservoir to the left; and when it is pushed down, the air in the reservoir to the right is compressed.

142. In Fig. 151 is represented a condensing pump often employed. At the bottom of the pump-barrel is a valve b opening downward; in a lateral tube is an admission-valve a opening inward. The position of these valves is shown in the figure. They are conical metal stoppers, fitted with a rod passing through a hole in a small plate behind, an arrangement which prevents the valve from overturning. The rod is surrounded by a small spiral spring, which keeps the valve pressed against the opening. If now the lower part of the pump-barrel be screwed upon a reservoir, at each upward stroke of

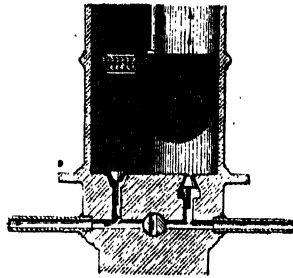


Fig. 150.—Barrel of Condensing Pump.

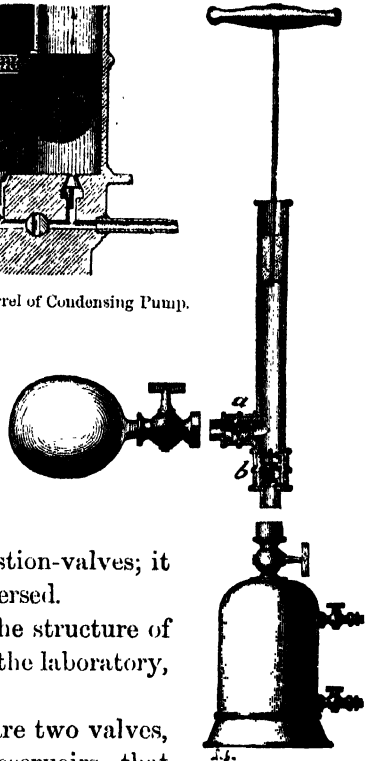


Fig. 151.
Condensing Pump.

the piston the barrel will be filled with air through the valve a , and at every downward stroke this air will be forced into the reservoir.

If the lateral tube be made to communicate with a bladder or gas-holder filled with any gas, this gas will be forced into the reservoir, and compressed.

143. Calculation of the Effect of the Instrument.—The density of the compressed air after a given number of strokes of the piston may easily be calculated. If v be the volume of the pump-barrel, and V that of the reservoir; at each stroke of the piston there is forced into the reservoir a volume of air equal to that of the pump-barrel, which gives a volume nv at the end of n strokes. The air in the reservoir, accordingly, which when at atmospheric pressure had density D , and occupied a volume $V + nv$, will, when the volume is reduced to V , have the density $D \frac{V + nv}{V}$, and the pressure will, by Boyle's law, be $\frac{V + nv}{V}$ atmospheres.

If this formula were rigorously applicable in all cases, there would be no limits to the pressure attainable, except those depending on the strength of the reservoir and the motive power available.

But, in fact, the untraversed space left below the piston when at the end of its downward stroke, sets a limit to the action of the instrument, just as in the common air-pump. For when the air in the barrel is reduced from the volume of the barrel v to that of the untraversed space v' , its tension becomes $H \frac{v}{v'}$, and this air cannot pass into the reservoir unless the tension of the air in the reservoir is less than this quantity. This is accordingly the utmost limit of compression that can be attained.

We must, however, carefully distinguish between the effects of untraversed space in the air-pump and in the compression-pump. In the first of these instruments the object aimed at is to rarefy the air to as great a degree as possible, and untraversed space must consequently be regarded as a defect of the most serious importance.

The object of the condensing pump, on the contrary, is to compress the air, not indefinitely, but up to a certain point. Thus, for instance, one pump is intended to give a compression of five atmospheres, another of ten, &c. In each of these cases the maker provides that this limit shall be reached, and accordingly the untraversed space can have no injurious effect beyond increasing the number of strokes required to produce the desired amount of condensation.

144. Various Contrivances for producing Compression.—In order to expedite the process of compression, several pumps such as we have described are combined, which may be done in various ways. Fig. 152 represents the system employed by Regnault in his investigations connected with Boyle's law and the elastic force of vapour. It consists of three pumps, the piston-rods of which are jointed to

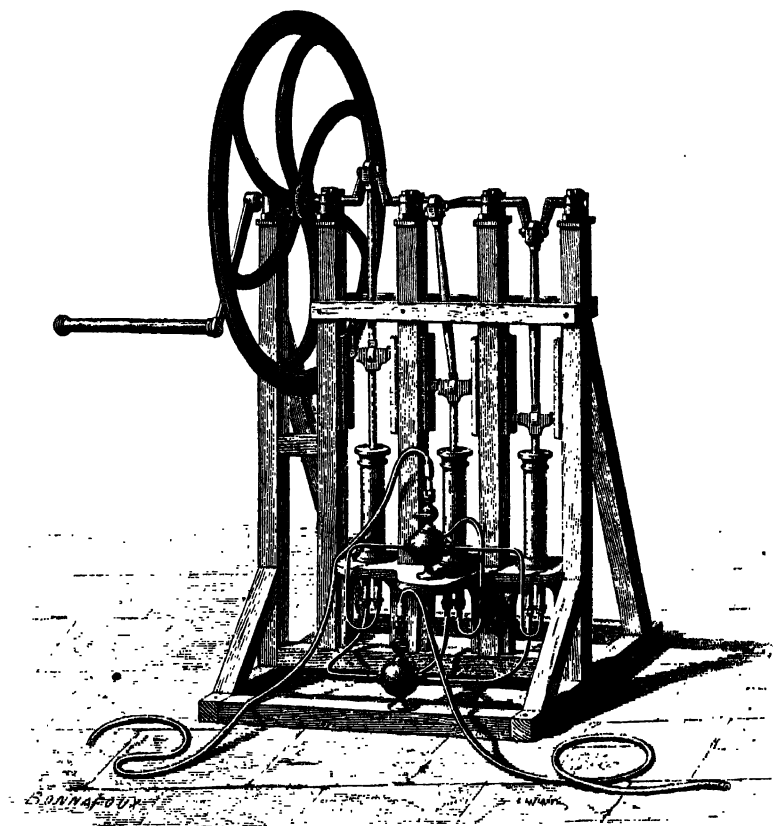


Fig. 152.—Connected Pumps.

three cranks on a horizontal axle, by means of three connecting-rods. This axle, which carries a fly-wheel, is turned by means of one or two handles. The different admission-valves are in communication with a single reservoir in connection with the external air, and the compressed gas is forced into another reservoir which is in communication with the experimental apparatus.

A serious obstacle to the working of these instruments is the heat

generated by the compression of the air, which expands the different parts of the instrument unequally, and often renders the piston so tight that it can scarcely be driven. In some of these instruments which are employed in the arts, this inconvenience is lessened by keeping the lower valves covered with water, which has the additional advantage of getting rid of "untraversed space." In this way a pressure of forty atmospheres may easily be obtained with air. Air may also be compressed directly, without the intervention of pumps, when a sufficient height of water can be obtained. It is only necessary to lead the liquid in a tube to the bottom of a reservoir containing air. This air will be compressed until its tension exceeds that of the atmosphere by the amount due to the height of the summit of the tube. It is by a contrivance of this kind that the compressed air is obtained which drives the boring-machines employed in the tunnel through Mont Cenis.

145. Practical Applications of the Air-pump and of Compressed Air.

—Besides the use made of the air-pump and the compression-pump in the laboratory, these instruments are variously employed in the arts.

The air-pump is employed by sugar-refiners to lower the boiling point of the syrup. Compression-pumps are used by soda-water manufacturers to force the carbonic acid into the reservoirs containing the water which is to be aerated. The small apparatus described above (Fig. 151) is sufficient for this purpose; it is only necessary to fill the side-vessel with carbonic acid, and to pour a certain quantity of water into the reservoir below. Compressed air has for several years been employed to assist in laying the foundations of bridges in rivers where the sandy nature of the soil requires very deep excavations. Large tubes called *caissons*, in connection with a condensing pump, are gradually let down into the river; the air by its pressure keeps out the water, and the workmen, who are admitted into the apparatus by a sort of lock, are thus enabled to walk on dry ground.

In pneumatic despatch tubes, which have recently been established in many places, a kind of train is employed, consisting of a piston preceded by boxes containing the despatches. By exhausting the air at the forward end of the tube, or forcing in compressed air at the other end, the train is blown through the tube with great velocity.

The atmospheric railway, which was for a few years in existence, was worked upon the same principle: an air-tight piston travelled

through a fixed tube, and was connected by an ingenious arrangement with the train above.

Excavating machines driven by compressed air are coming into extensive use in mining operations. They have the advantage of assisting ventilation, inasmuch as the compressed air, which at each stroke of the machine escapes into the air of the mine, cools as it expands.

In the air-gun the bullet is projected by a portion of compressed air which, on pulling the trigger, escapes into the barrel from a reservoir in which it has been artificially compressed.

We may add that the large machines employed in iron-works for supplying air to the furnaces, are really compression-pumps.

CHAPTER XVI.

UPWARD PRESSURE OF THE AIR.

146. The Baroscope.—Atmospheric air exerts, as we have already mentioned (§ 101), an upward pressure on bodies surrounded by it. This pressure, according to the principle of Archimedes, which applies to gases as well as to liquids, is equal to the weight of the air displaced. Hence it follows that the weight of a body in the air is not its actual weight, but differs from it by a quantity equal to the upward pressure on the body. This principle is illustrated by the baroscope.

This is a kind of balance, the beam of which supports two balls of very unequal sizes, which balance each other in the air. If the

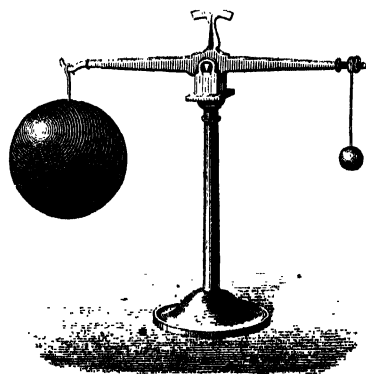


Fig. 153.—Baroscope.

apparatus is placed under the receiver of an air-pump, after a few strokes of the piston, the beam will be seen to incline towards the larger ball, and the inclination will increase as the exhaustion proceeds. The reason is that the air, before it was pumped out, produced an upward pressure, which is now removed. The weight of each ball is thus increased by that of an equal volume of air, whose density is the difference between the densities at

the beginning and end of the experiment. This addition is greater for the larger ball, which therefore preponderates.

If after exhausting the air, carbonic acid, which is heavier than air, were allowed to enter the receiver, the large ball would be subjected to a greater increase of upward pressure than the small one, and the beam would incline to the side of the latter.

147. Balloons.—Suppose a body to be lighter than an equal volume of air, then this body will rise in the atmosphere. For example, if we fill soap-bubbles with hydrogen, and shake them off from the end of the tube at which they are formed, they will be seen, if sufficiently large, to ascend in the air. This curious experiment is due to the philosopher Cavallo, who announced it in 1782.¹

The same principle applies to balloons, which may essentially be reduced to an envelope inclosing a gas lighter than air. In consequence of this difference of density, we can always, by taking a

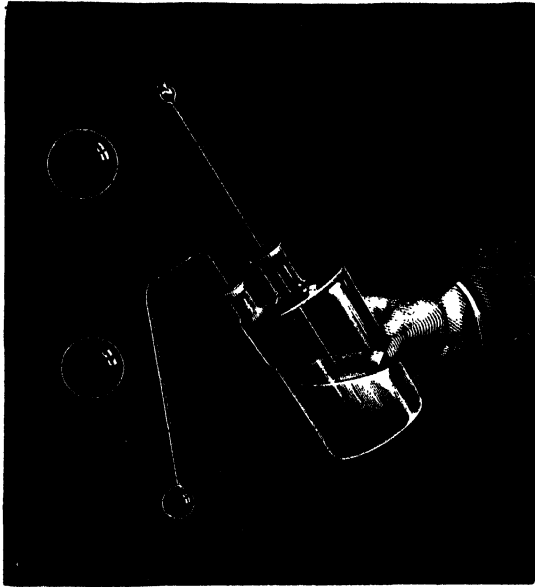


Fig. 154.—Ascent of Soap-bubbles filled with Hydrogen.

sufficiently large volume, make the weight of the gas and containing envelope less than that of the air displaced. In this case the balloon will ascend.

The invention of balloons is due to the brothers Joseph and Stephen Montgolfier. The balloons made by them were globe-shaped, and

¹ The first idea of a balloon must be attributed to Francisco de Lana, who, about 1670, proposed to exhaust the air in globes of copper of sufficient size and thinness to weigh less, under these conditions, than the air displaced. The experiment was not tried, and would certainly not have succeeded, for the pressure of the atmosphere would have caused the globes to collapse. The theory, however, was thoroughly understood by the author, who made an exact calculation of the amount of force tending to make the globes ascend.

—D.

constructed of paper, or of paper covered with cloth, the air inside being rarefied by the action of heat. It is curious to remark that in their first attempts they employed hydrogen gas, and showed that balloons filled with this gas could ascend. But as the hydrogen readily escaped through the paper, the flight of the balloons was short, and thus the use of hydrogen was abandoned, and hot air was alone employed.

The name *montgolfières* is still applied in France to fire-balloons. They generally consist of an envelope with a wide opening below,

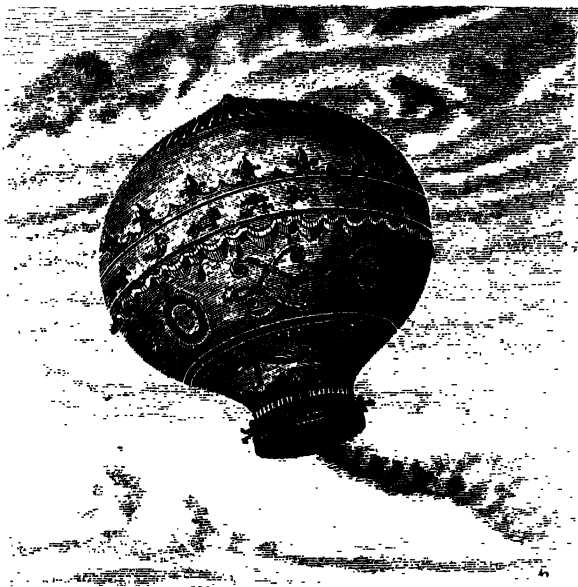


Fig. 156.—Fire-balloon of Pilatre de Rozier.

under which is hung a brazier,¹ in which, at the moment of ascent, combustibles are placed, and the ascending power of the balloon is thus kept up for some time.

The first public experiment of the ascent of a balloon was performed at Annonay on the 5th June, 1783. On October 21st of the same year, Pilatre de Rozier and the Marquis d'Arlandes achieved the first aerial voyage in a fire-balloon, represented in our figure.

Charles proposed to reintroduce the use of hydrogen by employing

¹ A sponge, dipped in spirits of wine, and ignited, is frequently employed as the source of heat, and is fixed in its place by a light wire-frame.

an envelope impermeable to the gas. This is usually made of silk varnished on both sides, or of two sheets of silk with a sheet of india-rubber between. Instead of hydrogen, coal-gas is now generally employed, on account of its cheapness and of the facility with which it can be procured.

148. Buoyancy.—The buoyancy or lifting power of a balloon is the difference between its weight and that of the air displaced. It is easy to compare the three modes of inflation with respect to the buoyancy which they respectively afford.

A cubic metre of air weighs	1·300 grammes.
A cubic metre of hydrogen	·089 ..
A cubic metre of coal-gas	about ·750 ..
A cubic metre of air heated to 200° Cent.	·800 ..

We thus see that the buoyancy per cubic metre with hydrogen is 1·211, with coal-gas ·550, and with hot air about ·500 grammes. If, for instance, the total weight to be raised is estimated at 1500 grammes, the volume of a balloon filled with hydrogen capable of raising the weight will be $\frac{1500}{1·210} = 1239$ cubic metres. If coal-gas were employed the required volume would be $\frac{1500}{·550} = 2727$ cubic metres.

The car in which the aeronauts sit is usually made of wicker-work or whalebone. It is sustained by cords attached to a net-work covering the entire upper half of the balloon, so as to distribute the weight as evenly as possible. The balloon terminates below in a kind of neck opening freely into the air. At the top there is another opening in the inside, which is closed by a valve held to by a spring. Attached to the valve is a cord which passes through the interior of the balloon, and hangs above the car within reach of the hand of the aeronaut.

When the aeronaut wishes to descend, he opens the valve for a few moments and allows some of the gas to escape. An important part of the equipment consists of sand-bags for ballast, which are gradually emptied to check too rapid descent. In the figure is represented a contrivance called a parachute, by means of which the descent is sometimes effected. This is a kind of large umbrella with a hole at the top, from the circumference of which hang cords supporting a small car. When the parachute is left to itself, it opens out, and the resistance of the air, acting upon a large surface, moderates the rate of descent. The hole at the top is essential to safety, as it affords a regular passage for air which would otherwise

escape from time to time from under the edge of the parachute, thus producing oscillations which might prove fatal to the aeronaut.

One very important precaution to be observed is not to inflate the

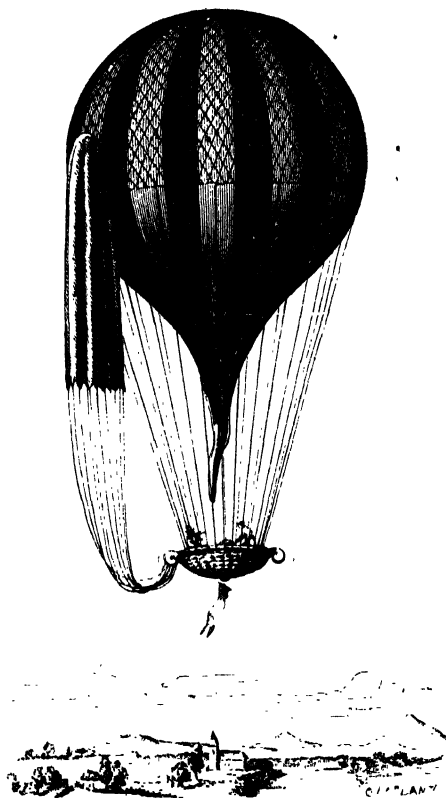


Fig. 156.— Balloon with Car and Parachute.

balloon completely at the commencement of the ascent. The reason is, that the atmospheric pressure diminishing as the balloon ascends, the expansive power of the gas contained produces an increasing effect, as in the experiment described in § 18, and the result would probably be the bursting of the balloon. As the balloon ascends, it increases in volume; but until it is completely inflated, the buoyancy remains constant. Suppose, for instance, that the atmospheric pressure is reduced by one-half, the volume of the balloon will be doubled; it will thus displace a volume of air twice as great as before, but the density of this air will only be half as much, so that the buoyancy remains the same. This conclusion,

however, is not quite exact, because part of the balloon, as the cords, the car, &c., are of invariable volume; the density of the air displaced by them is constantly diminishing, and consequently the buoyancy diminishes also. If the balloon continues to rise after it is completely inflated, its buoyancy diminishes rapidly, becoming zero when a stratum of air is reached in which the weight of the volume displaced is equal to that of the balloon itself. It is carried past this stratum in the first instance in virtue of the velocity which it has acquired, and finally comes to rest in it after a number of oscillations.

149. Theory of the Balloon.—The tension of the air in this stratum, the radius of the balloon, and the weight of the different parts, are

connected by a relation which can be very easily established, if we neglect variations of temperature, and which may serve as a guide in the construction of the balloon. If V be the volume of the balloon in litres, the weight of air displaced in grammes is¹

$$1.293 V \frac{h}{H},$$

h being the pressure in the stratum of equilibrium, and H that at the surface of the earth. If w and v be the weight and volume of the solid parts, including the aeronauts themselves, δ the density of the gas in the balloon, the equation of equilibrium will be $(V+v) 1.293 \frac{h}{H} = V \delta \frac{h}{H} + w$.

In this equation w and v include the weight and volume of the substance composing the balloon and net-work, and therefore are not altogether independent of V , the volume of the balloon. The equation is thus in reality rather complicated, but it may be solved approximately by trial, or by known algebraic methods, and we may find from it the size of balloon necessary for reaching a stratum of a given pressure; and by § 112 we can find the height corresponding to this pressure.

150. Effect of the Air upon the Weight of Bodies.—The upward pressure of the air impairs the exactness of weights obtained even with a perfectly true balance, tending, by the principle of the baroscope, to make the denser of two equal masses preponderate. The stamped weights used in weighing are, strictly speaking, standards of mass, and will equilibrate any equal masses in vacuo; but in air the equilibrium will be destroyed by the greater upward pressure of the air upon the larger and less dense body. When the specific gravities of the weights and of the body weighed are known, it is easy from the apparent weight to deduce the true weight (that is to say, the mass) of the body.

Let x be the real weight of a body which balances a weight marked P pounds.

The apparent weight of the body is $x - \frac{x}{D}$, $a = x \left(1 - \frac{1}{D}\right)$, D being the density of the body, and a that of air.² The body marked as

¹ The weight of a litre (or cubic decimetre = 61.027 cubic inches) of dry air at temperature 0° Cent., and pressure of 760 millimetres of mercury, is 1.293 grammes. Reduced to British measure, this gives 32.7 grains as the weight of 100 cubic inches.

² The value of a varies according to the temperature and pressure (see Chap. xxiii.) Its ordinary value is about $\frac{1}{770}$. (See § 100.)

weighing P pounds has in air the apparent weight $P - \frac{P}{M} a = P \left(1 - \frac{a}{M}\right)$, M denoting the density of the substance of which it is composed. These two apparent weights balance one another; hence we have the equation

$$x \left(1 - \frac{a}{D}\right) = P \left(1 - \frac{a}{M}\right),$$

whence

$$x = P \frac{1 - \frac{a}{M}}{1 - \frac{a}{D}} = P \left\{ 1 + a \left(\frac{1}{D} - \frac{1}{M} \right) \right\} \text{ nearly.}$$

Let us take, for instance, a piece of sulphur whose weight has been found to be 100 grains, the weights being of copper, the density of which is 8.8. The density of sulphur is 2.

We have, by applying the formula,

$$x = 100 \left\{ 1 + \frac{1}{770} \left(\frac{1}{2} - \frac{1}{8.8} \right) \right\} = 100.05 \text{ grains.}$$

We see then that the difference is not altogether insensible. It varies in sign, as the formula shows, according as M or D is the greater. When the density of the body to be weighed is less than that of the weights used, the real weight is greater than the apparent weight; if the contrary, the case is reversed. If the body to be weighed were of the same density as the weights used, the real and apparent weights would be equal. We may remark, that in determining the ratio of the weights of two bodies of the same density, we need not concern ourselves with the effect of the upward pressure of the air, as the correcting factor, which has the same value for both cases, will disappear in the quotient.

CHAPTER XVII.

PUMPS FOR LIQUIDS.

151. Machines for raising water have been known from very early ages, and the invention of the common pump is pretty generally ascribed to Ctesibius, teacher of the celebrated Hero of Alexandria; but the true theory of its action was not understood till the time of Galileo and Torricelli.

152. Reason of the Rising of Water in Pumps.—Suppose we take a tube with a piston at the bottom, and immerse the lower end of it in water. The raising of the piston tends to produce a vacuum below it, and the atmospheric pressure, acting upon the external surface of the liquid, compels it to rise in the tube and follow the upward motion of the piston. This upward movement of the water would take place even if some air were interposed between the piston and the water; for on raising the piston this air would be rarefied, and its pressure no longer balancing that of the atmosphere, this latter pressure would cause the liquid to ascend in a column, whose weight, added to the pressure of the air below the piston, would be equal to the atmospheric pressure. This is the principle on which water rises in pumps. These instruments have a considerable variety of forms, of which we shall describe the most important types.

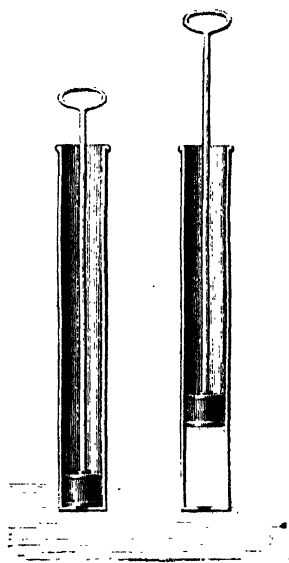


Fig. 157.—Principle of Suction pump.

153. Suction-pump.—The suction-pump consists of a cylindrical

pump-barrel traversed by a piston, and communicating by means of a smaller tube, called the suction-tube, with the water in the pump-well. At the junction of the pump-barrel and the tube is a valve opening upward, called the suction-valve, and in the piston is an opening closed by another valve, also opening upward.

Suppose now the suction-tube to be filled with air at the atmospheric pressure, and the water consequently to be at the same level

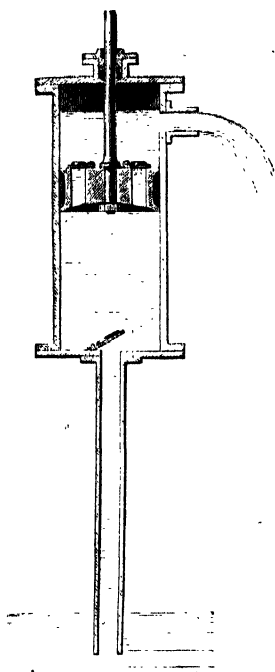


Fig. 158.—Suction-pump.

inside the tube and in the well. Suppose the piston to be at the end of its downward stroke, and to be now raised. This motion tends to produce a vacuum below the piston, hence the air contained in the suction-tube will open the suction-valve, and rush into the pump-barrel. Its elastic force being thus diminished, the atmospheric pressure will cause the water to rise in the tube to a height such that the pressure due to this height, increased by the pressure of the air inside, will exactly counter-balance the pressure of the atmosphere. If the piston now descends, the suction-valve closes, the water remains at the level to which it has been raised, and the air, being compressed in the barrel, opens the piston-valve and escapes. At the next stroke of the piston the water will rise still further, and a fresh portion of air will escape.

If, then, the length of the suction-tube is less than about 30 feet, the water will, after a certain number of strokes of the piston, be able to reach the suction-valve and rise into the pump-barrel. When this point has been reached the action changes. The piston in its downward stroke compresses the air, which escapes through it, but the water also passes through, so that the piston when at the bottom of the pump-barrel will have above it all the water which has previously risen into the barrel. If the piston be now raised, supposing the total height to which it is raised to be not more than 34 feet above the level of the water in the well, as should always be the case, the water will follow it in its upward movement, and will fill the pump-barrel. In the downward stroke this water will be forced through the piston-valve,

and in the following upward stroke will be discharged at the spout. A fresh quantity of water will by this time have risen into the pump-barrel, and the same operations will be repeated.

We thus see that from the time when the water has entered the pump-barrel, at each upward stroke of the piston a volume of water is ejected equal to the contents of the pump-barrel.

In order that the water may be able to rise into the pump-barrel, the suction-valve must not be more than 34 feet above the level of the water in the well, otherwise the water would stop at a certain point of the tube, and could not be raised higher by any farther motion of the piston.

Moreover, in order that the working of the pump may be such as we have described, that is, that at each upward stroke of the piston a quantity of water may be removed equal to the volume of the pump-barrel, it is necessary that the piston when at the top of its stroke should not be more than 34 feet above the water in the well.

154. Condition that the Water may reach the Pump-barrel.—If the piston does not descend to the bottom of the barrel, the water may fall short of rising to the suction-valve, even though the total height reached by the piston be less than 34 feet. For, if the piston when at the end of its downward stroke leaves below it a space containing air, the tension which this air possesses when the piston is raised diminishes by a corresponding quantity the height to which the water can attain. If, for instance, the length of the suction-tube is 33·5 feet, and the tension of the air remaining above it is, when at its least value, equal to the pressure of 1 foot of water, it is evident that the total height to which the water can rise will be less than 33 feet, and it will, in consequence, be unable to reach the pump-barrel.

Example. The suction-valve of a pump is at a height of 27 feet above the surface of the water, and the piston, the entire length of whose stroke is 7·8 inches, when at the lowest point is 3·1 inches from the fixed valve; find whether the water will be able to rise into the pump-barrel.

When the piston is at the end of its downward stroke, the air which it leaves below it is at the atmospheric pressure; when the piston is raised this air becomes rarefied, and its pressure, by Boyle's law, becomes $\frac{3\cdot1}{10\cdot9}$ that of the atmosphere; this pressure can therefore balance a column of water whose height is $34 \times \frac{3\cdot1}{10\cdot9}$ feet, or 9·67 feet. Hence, the maximum height to which the water can attain is 34 —

9.67 feet = 24.33 feet; and consequently, as the suction-tube is 27 feet long, the water will not rise into the pump-barrel, even supposing the pump to be perfectly free from leakage.

Practically, the pump-barrel should not be more than about 25 feet above the surface of the water in the well; but the spout may be more than 34 feet above the barrel, as the water after rising above the piston is simply pushed up by the latter, an operation which is independent of atmospheric pressure. Pumps in which the spout is at a great height above the barrel are commonly called *lift-pumps*, but they are not essentially different from the suction-pump.

155. Force necessary to raise the Piston.—The force which must be expended in order to raise the piston, is equal to the weight of a column of water, whose base is the section of the piston, and whose height is that to which the water is raised. Let S be the section of the piston, P the atmospheric pressure upon this area, h the height of the column of water which is above the piston in its present position, and h' the height of the column of water below it; then the upper surface of the piston is subjected to a pressure equal to $P + Sh$; the lower face is subjected to a pressure in the opposite direction equal to $P - Sh'$, and the entire downward pressure is represented by the difference between these two, that is, by $S(h + h')$.

The same conclusion would be arrived at even if the water had not yet reached the piston. In this case, let l be the height of the column of water raised; then the pressure below the piston is $P - Sl$; the pressure above is simply the atmospheric pressure P , and, consequently, the difference of these pressures acts downward, and its value is Sl .

156. Efficiency of Pumps.—From the results of last section it would appear that the force required to raise the piston, multiplied by the height through which it is raised, is equal to the weight of water discharged multiplied by the height of the spout above the water in the well. This is an illustration of the principle of work (§17A). As this result has been obtained from merely statical considerations, and on the hypothesis of no friction, it presents too favourable a view of the actual efficiency of the pump.

Besides the friction of the solid parts of the mechanism, there is work wasted in generating the velocity with which the fluid, as a whole, is discharged at the spout, and also in producing eddies and other internal motions of the fluid. These eddies are especially produced at the sudden enlargements and contractions of the passages

through which the fluid flows. To these drawbacks must be added loss from leakage of water, and at the commencement of the operation from leakage of air, through the valves and at the circumference of the piston. In common household pumps, which are generally roughly made, the *efficiency* may be as small as $\cdot 25$ or $\cdot 3$; that is to say, the product of the weight of water raised, and the height through which it is raised, may be less than the work done in driving the pump in the ratio of one of these numbers to unity.

In Figs. 159 and 160 are shown the means usually employed for working the piston. The first figure needs no explanation; it will be seen that the upward and downward movement of the piston is effected by means of a lever. The second figure represents an arrangement often employed, in which the alternate motion of

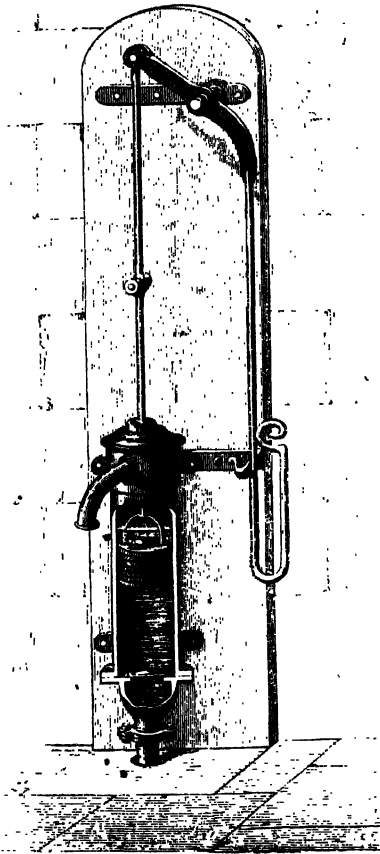


Fig. 159.

Suction-pump.

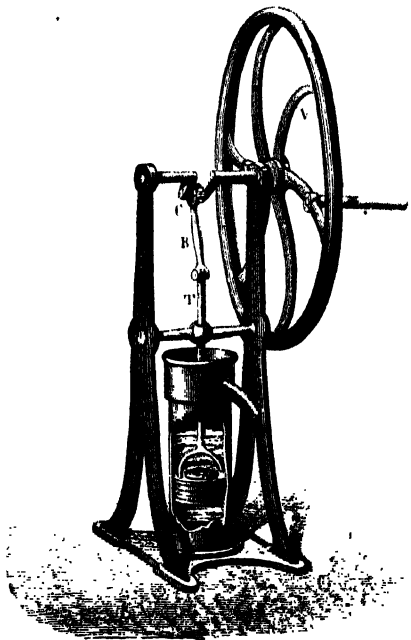


Fig. 160.

the piston is effected by means of a rotatory motion. For this purpose the piston-rod T is joined by means of the connecting-rod

B to the crank C of an axle turned by a handle attached to the fly-wheel V.

157. Forcing-pump.—The forcing-pump consists of a pump-barrel dipping into water, and having at the bottom a valve opening upward.

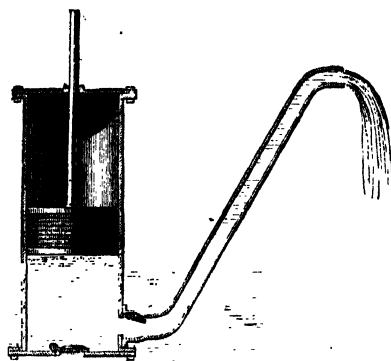


Fig. 161.-- Forcing-pump.

In communication with the pump-barrel is a side-tube with a valve at the point of junction opening from the barrel into the tube. A solid piston moves up and down the pump-barrel, and it is evident that when this piston is raised, water enters the barrel by the lower valve, and that when the piston descends, this water is forced into the side-tube. The greater the height of this tube, the greater will be the

force required to push the piston down, for the resistance to be overcome is the pressure due to the column of water raised.

The forcing-pump most frequently has a short suction-pipe leading from the reservoir, as represented in Fig. 163. In this case the water is raised from the reservoir into the barrel by atmospheric pressure during the up-stroke, and is forced from the barrel into the ascending pipe in the down-stroke.

158. Plunger.—When the height to which the water is to be forced is very considerable, the different parts of the pump must be very strongly made and fitted together, in order to resist the enormous pressure produced by the column of water, and to prevent leakage. In this case the ordinary piston stuffed with tow or leather washers cannot be used, but is replaced by a solid cylinder of metal called a *plunger*. Fig. 164 represents a section of a pump thus constructed. The plunger is of smaller section than the barrel, and passes through a stuffing-box in which it fits air-tight. The volume of water which enters the barrel at each up-stroke, and is expelled at the down-stroke, is the same as the volume of a length of the piston equal to the length of stroke; and the hydrostatic pressure to be overcome is proportional to the section of the plunger, not to that of the barrel. As the operation proceeds, air is set free from the water, and would eventually impede the working of the pump were it not permitted to escape.

For this purpose the plunger is pierced with a narrow passage, which is opened from time to time to blow out the air.

The drainage of deep mines is usually effected by a series of pumps. The water is first raised by one pump to a reservoir, into which dips the suction-tube of a second pump, which sends the water up to a second reservoir, and so on. The piston-rods of the different pumps are all joined to a single rod called the *spear*, which receives its motion from a steam-engine.

159. Fire-engine.—The ordinary fire-engine is formed by the union of two forcing-pumps which play into a common reservoir, containing in its upper portion (called the air-chamber) air compressed by the working of the engine.

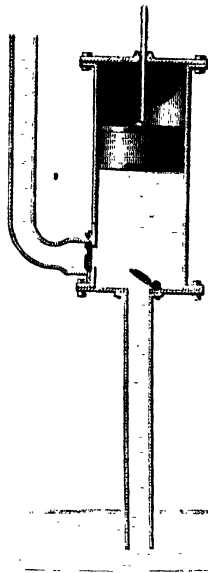


Fig. 163.

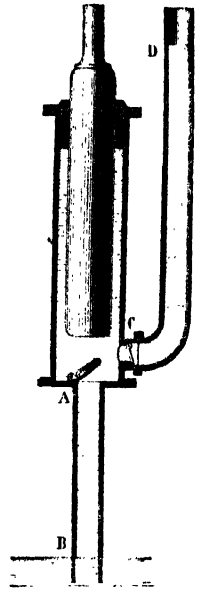


Fig. 164.

Suction and Force Pumps.

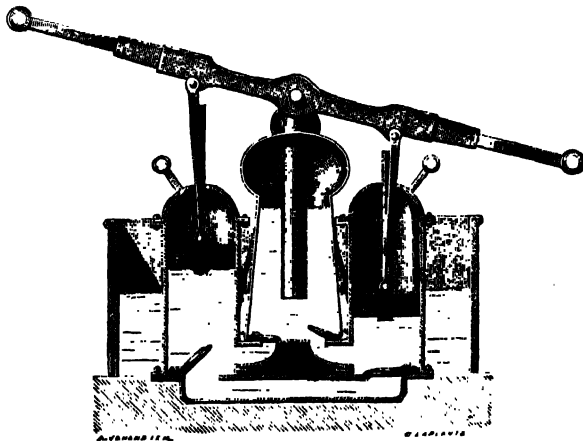


Fig. 162.—Fire engine.

A tube dips into the water in this reservoir, and to the upper end of this tube is screwed the leather hose through which the water is

discharged. The piston-rods are jointed to a lever, the ends of which are raised and depressed alternately, so that one piston is ascending while the other is descending. Water is thus continually being forced into the common reservoir except at the instant of reversing stroke, and as the compressed air in the air-chamber performs the part of a reservoir of work (nearly analogous to the fly-wheel), the discharge of water from the nozzle of the hose is very steady.

The engine is sometimes supplied with water by means of an attached cistern (as in Fig. 162) into which water is poured; but it is more usually furnished with a suction-pipe which renders it self-feeding.

160. Double-acting Pumps.—These pumps, the invention of which is due to Delahire, are often employed for household purposes. They consist of a pump-barrel VV (Fig. 165), with four openings in it, A, A', B, B'. The openings A and B' are in communication with the suction-tube C; A' and B are in communication with the ejection-tube C'. The four openings are fitted with four valves opening all in the same direction, that is, from right to left, whence it follows that A and B' act as suction-valves, and A' and B as ejection-valves, and, consequently, in whichever direction the piston may be moving, the suction and ejection of water are taking place at the same time.

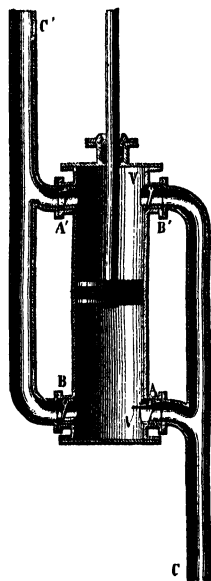


Fig. 165
Double-action Pump.

161. Rotatory Pump.—Double-acting pumps produce a continuous suction of water. The same end may be attained by means of rotatory pumps, which are largely used in some countries. The figure represents one of these pumps as constructed by Dietz.

The pump-barrel consists of a cylindrical drum B (Fig. 166), containing within it a second cylinder A, of less diameter, and of nearly the same length, open at the ends, which can be turned about its axis by means of a handle; around this axis is a cam or guide $mn\ m'n'$, rigidly fixed to one of the ends of the drum B. In the box A are four slits, into which fit four blades p ; these latter are constantly guided by the movement, of one of their ends upon the cam, while the other ends move along the interior surface of the drum B, and the broad iron plate $b'a\ ab$, whose

distance from the cam is everywhere equal to the length of one of the blades. By means of two holes in this plate, communication is established between the pump-barrel and the suction-tube C, and between the pump-barrel and the ejection-tube C'. From this arrangement it follows, that on causing the box B to rotate in the direction shown by the arrow, a partial vacuum will be produced behind the tongues, and the water will be drawn in at this side, and ejected at the opposite side.

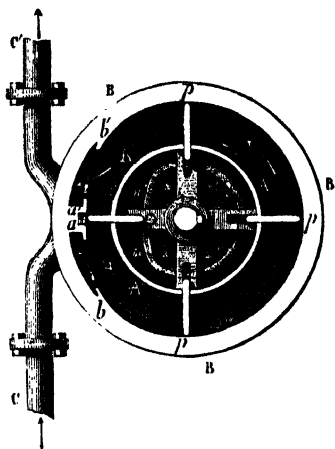


Fig. 166.—Rotatory Pump.

162. Hydraulic Press.—The hydraulic press (Fig. 167) consists of a suction and force pump *aa* worked by means of a lever turning about an axis *O*. The water drawn from the reservoir *BB'* is forced along the tube *CC* into the cistern *V*. In the top of the cistern is an opening through which moves a heavy metal plunger *AA*. This carries on its

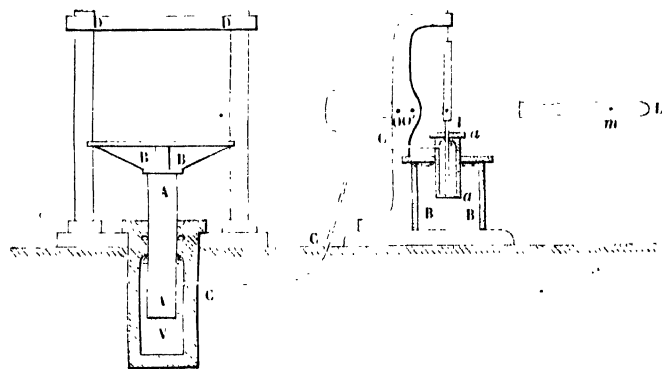


Fig. 167.—Bramah Press.

upper end a large plate *BB*, upon which are placed the objects to be pressed. Suppose the cistern *V* to be at first empty, and the piston *A* to be carried by its own weight to the bottom of the cistern; under these circumstances, suppose the pump to begin to work. The cistern first begins to fill with water; then the pressure exerted by the piston of the pump is transmitted, according to the principles laid down in § 63, to the bottom of the piston *A*; this piston

accordingly rises, and the objects to be pressed, being intercepted between the plate and the top of a fixed frame, are subjected to the transmitted pressure. The amount of this pressure depends both on the ratio of the sections of the pistons and on the length of the lever used to work the force-pump. Suppose, for instance, that the distance of the point *m*, where the hand is applied, from the point *O*, is equal to twelve times the distance *IO*, and suppose the force exerted to be equal to fifty pounds. By the principle of the lever this is equivalent to a force of 50×12 at the point *I*; and if the section of the piston *A* be at the same time 100 times that of the piston of the pump, the pressure transmitted to *A* will be $50 \times 12 \times 100 = 60,000$ pounds. These are the ordinary conditions of the press usually employed in workshops. By drawing out the pin which serves as an axis at *O*, and introducing it at *O'*, we can increase the mechanical advantage of the lever.

Two parts essential to the working of the hydraulic press are not represented in the figure. These are a safety-valve, which opens when the pressure attains the limit which is not to be exceeded; and, secondly, a tap in the tube *C*, which is opened when we wish to put an end to the action of the press. The water then runs off, and the piston *A* descends again to the bottom of the cistern.

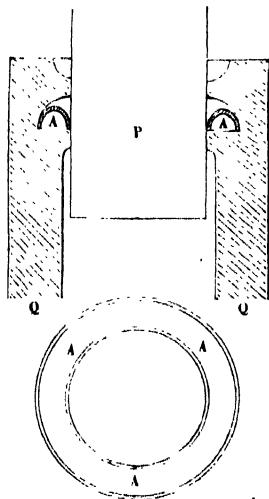


Fig. 168.—Cup—Leather.

The hydraulic press was clearly described by Pascal, and at a still earlier date by Stevinus, but for a long time remained practically useless; because as soon as the pressure began to be at all strong, the water escaped at the surface of the piston *A*. Bramah invented the *cupped leather collar*, which prevents the liquid from escaping, and thus enables us to utilize all the power of the machine. It consists of a leather ring *AA* (Fig. 168), bent so as to have a semi-circular section. This is fitted into a hollow in the interior of the sides of the cistern, so that water passing between the piston and cylinder will fill the concavity of the cupped leather collar, and by pressing on it will produce a packing that fits more tightly as

the pressure on the piston increases.

The hydraulic press is very extensively employed in the arts. 'It

is of great power, and may be constructed to give pressures of two or three hundred tons. It is the instrument generally employed in cases where very great force is required, as in testing anchors or raising very heavy weights. It was used for raising the sections of the Britannia tubular bridge, and for launching the *Great Eastern*.

CHAPTER XVIII.

EFFLUX OF LIQUIDS.—TORRICELLI'S THEOREM.

163. If an opening is made in the side of a vessel containing water, the liquid escapes with a velocity which is greater as the surface of the liquid in the vessel is higher above the orifice, or to employ the usual phrase, as the *head* of liquid is greater. This point in the dynamics of liquids was made the subject of experiments by Torricelli, and the result arrived at by him was that the velocity of efflux is equal to that which would be acquired by a body falling freely from the upper surface of the liquid to the centre of the orifice. If h be this height, the velocity of efflux is given by the formula

$$V = \sqrt{2gh}.$$

This is called Torricelli's theorem; it supposes the sides of the vessel to be thin, and the diameter of the orifice to be very small compared with that of the vessel. It is further assumed that the orifice and the upper surface are under the same conditions as regards atmospheric pressure.

Torricelli's theorem has been regarded as an immediate consequence of the theory of gravitation; according to which, whatever be the path of a heavy body, its velocity depends only on the height of the point of starting above the point finally reached. If this height be h , the velocity is always $\sqrt{2gh}$.

But it is not evident that the molecules of a liquid which is escaping are subjected to no force but that of gravity. Besides, the first portions which escape from the vessel do not come from the free surface, and their velocity is due solely to the pressure exerted by the liquid column. It will thus be seen that the velocity of efflux, owing to the complex nature of the phenomenon, can be rigorously established only by the experimental method. It is very easy to

perform a simple experiment upon this point. In fact, the molecules issuing from the orifice are ejected with a certain velocity, and should therefore, by the theory of projectiles, describe parabolic paths. The jet issuing from the vessel should accordingly be parabolic, and by measuring its range, we can calculate the velocity of efflux.

The experiment may easily be made by means of the apparatus represented in Fig. 169. It consists of a cylinder in which are a

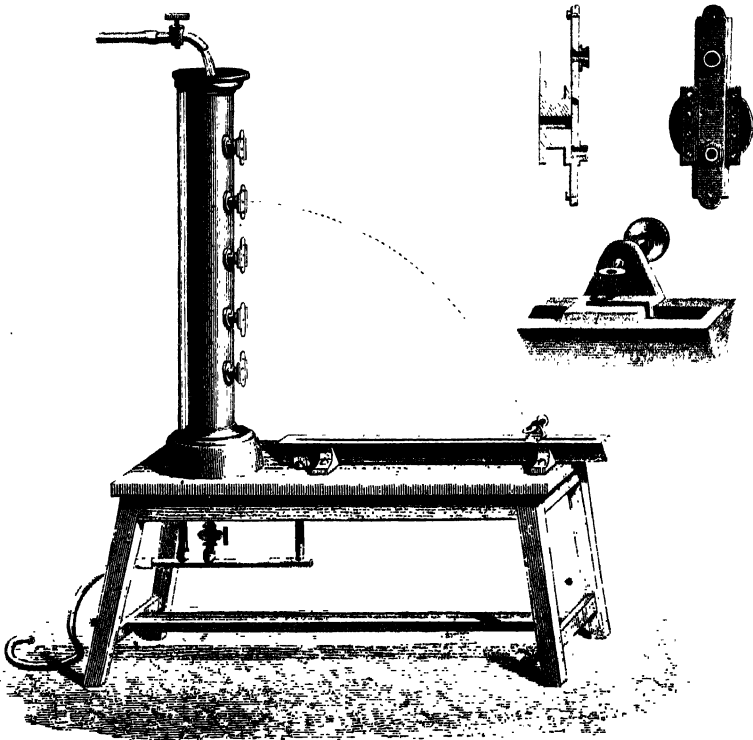


Fig. 169.—Apparatus for verifying Torricelli's Theorem.

number of equidistant orifices in the same vertical line. A tap placed above the cylinder supplies the vessel with water, and with the help of an overflow-pipe, maintains the liquid at a constant level, which is as much above the highest orifice as each orifice is above that next below it.

The liquid, which escapes is received in a trough, the edge of which is graduated. A travelling piece with an index line engraved on it slides along the trough; it carries, as shown in one of the

separate figures, a disc pierced with a circular hole, and capable of being turned in any direction about a horizontal axis passing through its centre. In this way the disc can always be placed in such a position that its plane shall be at right angles to the liquid jet, and that the jet shall pass freely and exactly through its centre. The index line then indicates the range of the parabolic jet with considerable precision. This range is reckoned from the vertical plane containing the orifices, and is measured on the horizontal plane passing through the centre of the disc. The distance of this latter plane below the lowest orifice is equal to that between any two consecutive orifices.

The following is the way in which the result of an experiment is estimated. Let b be the height of the orifice above the horizontal plane through the centre of the ring, and let a be the range of the jet. If the liquid molecules were simply falling from a height, b , they would traverse this space in a time given by the formula

$$b = gt^2$$

On the other hand, if they were simply obeying the force of ejection at the orifice, they would, by virtue of their initial velocity x , traverse the distance a in the same time t , whence we have

$$a = xt.$$

Eliminating t between these two equations, we have

$$b = \frac{ga^2}{2x^2};$$

whence

$$x = a \sqrt{\frac{g}{2b}}.$$

On comparing this velocity with that given by Torricelli's theorem, there is generally found to be a small difference between them, as is shown in the subjoined table:—

Head.	Jet.		Velocity.		Ratio of actual to theoretical velocity.
	a .	b .	Actual.	Theoretical.	
metres.	metres.	metres.			
2.29	6.28	7.53	6.65	6.70	.993
3.93	4.66	8.45	8.67	8.70	.988
7.17	1.41	6.25	11.67	11.88	.983

164. Intersection of Jets.—If Torricelli's theorem is correct, the

value of α just found ought to be equal to $\sqrt{2gh}$; thus, we have the equation

$$\alpha \sqrt{\frac{a}{2b}} = \sqrt{2gh}, \text{ whence } a^2 = 4bh. \quad (3)$$

From this a curious result may be deduced: it will be seen that if h and b vary in such a manner that their produce remains constant, the value of a will remain unchanged. This law may easily be verified experimentally. It is only necessary to open at the same time the top and bottom orifices, or the second and fourth; the result will be two jets which will intersect one another at the centre of the ring.

We may remark, however, that the verification of this law does not prove Torricelli's theorem, for the result would be the same, if in (3) the constant 4 were replaced by any other constant; it proves, however, that the velocity is proportional to the square root of the head of water, and not to the head itself, as was formerly believed.

165. Quantity of Liquid Discharged.—It would appear at first sight that Torricelli's theorem could be tested by a very simple and decisive experiment. Suppose the level of the liquid in a vessel to be maintained constant, and the volume of the liquid which escapes through an orifice during a certain time to be measured. This can be compared with the volume calculated *a priori*, by assuming that the quantity discharged in a unit of time is equal to a cylinder whose base is the section of the orifice and height the velocity; so that the quantity which escapes in time t will be given by the formula

$$Q = ts \sqrt{2gh};$$

s being the section of the orifice. Now, in all the experiments which have been performed, when the orifice is a simple perforation in a thin plate, the actual discharge has been found to be less than this, being generally about $\frac{6}{10}$ of it.

This discrepancy arises from neglecting the fact, that the particles of liquid at the margin of the jet have a converging motion, in consequence of which the jet contracts rapidly for a small distance after issuing from the orifice. Beyond this small distance the contraction is very gradual, depending only on the continued action of gravity. The portion which forms the termination of the rapid contraction, is called the contracted vein, or *vena contracta*, and its section appears to be about $\frac{6}{10}$ of that of the orifice. If, then, in the above formula we make s denote the section of the contracted vein, we shall obtain results agreeing with experiment.

166. Efflux-tubes.—This explanation is confirmed by the effect of efflux-tubes. These are pipes not exceeding a few inches in length, which are fitted to the holes in the side of the vessel. If, for instance, a tube of cylindrical form is employed, the contraction will be prevented by the adherence of the liquid to the sides of the tube; the section of the jet is then the same as that of the tube. In this case, if the velocity be measured, it will be found to be less than when the orifice is a hole in a thin plate; but, on multiplying the velocity thus obtained by the section of the tube, we shall obtain a result agreeing with the actual discharge.

The apparatus above described enables us to estimate the differences in velocity caused by efflux-tubes. For this purpose a sliding plate is used, with one orifice and two efflux-tubes, one cylindrical, the other conical; by sliding the plate the liquid can be made to flow out of either of these tubes.

167. Efflux through Pipes.—When the liquid, instead of escaping through a short spout, flows through a long tube, the velocity is considerably reduced by the friction of the molecules against each other, and against the sides of the tube. This velocity is also not the same at all points in the same section; it is least where the liquid is in contact with the tube, and is greatest in the centre of the liquid column. When a uniform delivery has been established, the quantity of water which passes in each unit of time is constant, and is the same for all sections; and the average velocity across any section will be obtained by dividing the volume discharged in a second by the sectional area. Many experiments have been performed with the view of determining this velocity in a certain number of particular cases, and certain empirical rules have thus been arrived at. It is difficult to treat the subject rationally, or to establish results that shall be perfectly general.

168. Fountains.—If the lower end of a water-pipe be connected with a mouth-piece pointing vertically upwards, the liquid on issuing from the opening will rise in a vertical jet to a height depending on the velocity of efflux. If there were no resistance, this height would be that *due to the velocity*, namely $\frac{v^2}{2g}$, according to the formulæ of § 38; but this is not actually the case. The friction of the liquid against itself, and the weight of the particles which fall back upon the rest, counteract the tendency to ascend. The effect of this last cause can be somewhat diminished by slightly inclining the jet.

169. Efflux of a Liquid in contact with Confined Air.—When the surface of the liquid is in contact with a volume of air whose pressure varies, the velocity of efflux is affected by this variation. Let $ABCD$ be a closed vessel filled with a liquid as far as MN , and let the space above contain air at the atmospheric pressure. If a small orifice be opened below, the liquid will begin to flow out; but the air above will become gradually rarefied, until at length its pressure, together with that due to the depth of liquid, will be equal to that of the atmosphere; when this happens the liquid will cease to flow, unless the circumstances are such that air-bubbles can enter. Let us see what will be the height of the liquid column in the vessel when the flow ceases. Let $AC=l$, $AM=h$, and let p be the height of a column of the liquid which will balance the pressure of the atmosphere. The air, which at the beginning of the experiment had a volume $l-h$, and a pressure p , will now have a volume $l-x$, and consequently a pressure $p \frac{l-h}{l-x}$; we have then

$$p \frac{l-h}{l-x} + x = p,$$

whence

$$x = \frac{p+l \pm \sqrt{(p+l)^2 - 4ph}}{2}.$$

The $-$ sign alone can be taken with the radical, since x must be less than l . This case of equilibrium occurs in several well-known experiments.

Pipette.—This is a glass tube (Fig. 171) open at both ends, and terminating below in a small tapering spout. If a certain quantity of water be introduced into the tube, either by aspiration or by direct immersion in water, and if the upper end be closed with the finger, the efflux of the liquid will cease after a few seconds. On admitting the air above, the efflux will begin again, and can again be stopped at pleasure.

The Magic Funnel.—This funnel is double, as is shown in Fig. 172. Near the handle is a small opening by which the space between the two funnels communicates with the external air. Another opening connects this same space with the tube of the inner funnel. If the interval between the two funnels be filled with any liquid,

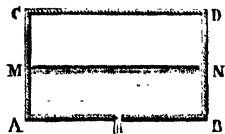


Fig. 170.

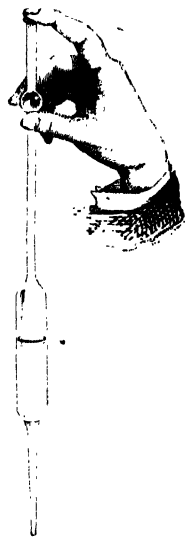


Fig. 171.—Pipette.

this liquid will run out or will cease to flow according as the upper hole is open or closed. The opening and closing of the hole can be



Fig. 172.—Magic Funnel.

easily effected with the thumb of the hand holding the funnel without the knowledge of the spectator. This device has been known from very early times.

The instrument may be used in a still more curious manner. For this purpose the space inside is secretly filled with highly-coloured wine, which is prevented from escaping by closing the opening above.

Water is then poured into the central funnel, and escapes either by itself or mixed with wine, according as the thumb closes or opens

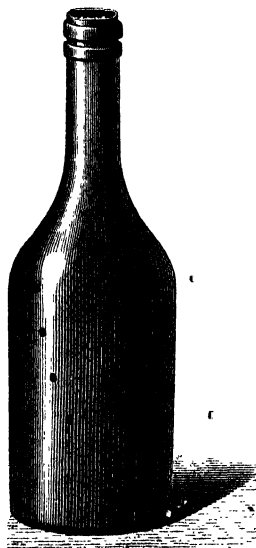


Fig. 173.—Inexhaustible Bottle.

the orifice for the admission of air. In the second case, the water being coloured with the wine, it will appear that wine alone is issuing from the funnel; thus the operator will appear to have the

power of making either water or wine flow from the vessel at his pleasure.

The Inexhaustible Bottle.—The inexhaustible bottle is a toy of the same kind. It is an opaque bottle of sheet-iron or gutta-percha, containing within it five small vials. These communicate with the exterior by five small holes, which can be closed by the five fingers of the hand. Each vial has also a small neck which passes up the large neck of the bottle. The five vials are filled with five different liquids, any one of which can be poured out at pleasure by uncovering the corresponding hole.

170. Intermittent Fountain.—The intermittent fountain is an apparatus analogous to the preceding, except that the interruptions in the efflux are produced automatically by the action of the instrument, without the intervention of the operator. It consists of a globe V, which can be hermetically closed by means of a stopper, and can be put in communication with efflux-tubes *a*, by which the water contained in it can escape. A vertical tube *t* rises nearly to the top of the globe, and terminates below at a short distance from the bottom of the basin B. This basin is pierced with a small opening *o*, by which the water contained in it escapes into the lower basin C. Suppose the globe to be filled with water, and communication with the efflux-tubes to be established, then

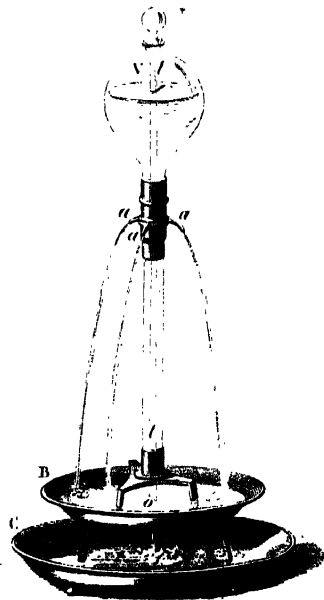


Fig. 174.—Intermittent Fountain.

the liquid will flow into the basin B, and thence into C. But the size of the opening *o* is such that it suffers less water to escape than passes out by the efflux-tubes; the liquid will therefore accumulate in B, and will finally cover the bottom of the tube *t*. Communication between the external air and the upper part of the globe will then be cut off, and the liquid will after a few moments cease to flow.

But as the basin B continues to be emptied by the opening *o*, the liquid in the basin will sink below the end of the tube; air will then enter the globe, the liquid will again begin to issue from the efflux-tubes, and so on.

171. Siphon.—The object of the siphon is the transference of liquid from one vessel to another. It essentially consists of a bent tube (Fig. 175) with branches of unequal length. The short branch dips

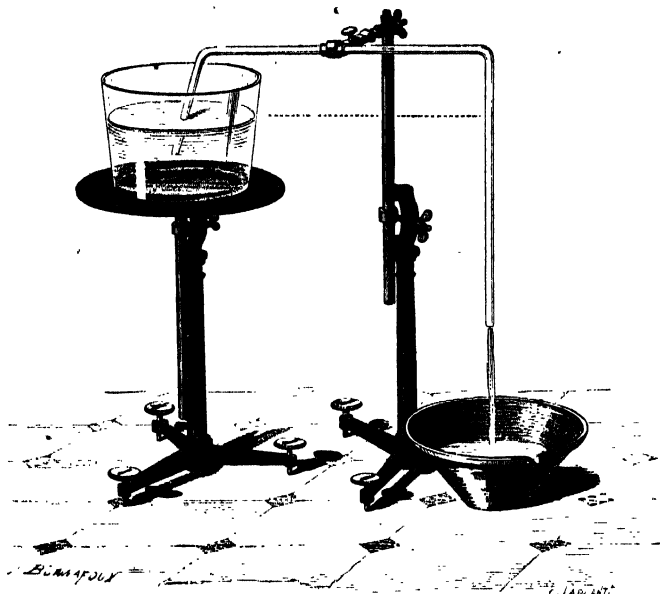


Fig. 175.—Siphon.

into the liquid to be transferred, the other opens directly into the air. If we suppose the siphon full of liquid, it is easy to see that the liquid will flow from the short to the long branch.

For if we consider (Fig. 176) a layer of liquid *M*, at the highest point of the siphon, this layer will be subjected to a pressure from left to right equal to the atmospheric pressure diminished by the height *DC*, or *MI*.

Let this latter height be *h*, and let *H* be the external pressure expressed as the height of an equivalent column of the liquid, then the pressure from left to right will be $H - h$. The pressure from right to left will be $H - EF' = H - h'$. Now as *h'* is greater than *h*, the first pressure will overcome the second, and the layer *M* will consequently move from left to right. But if the height of

the smaller branch be less than H , the liquid cannot separate, for the pressure of the atmosphere would immediately fill up the vacuum which would be formed. Thus the liquid will continue to flow uninterruptedly until the liquid in the vessel AB has fallen below the level of the end of the shorter branch of the siphon.

The force causing the liquid to flow is the pressure represented by a column of liquid $h' - h$, the velocity of efflux is thus equal to $\sqrt{2g(h' - h)}$, friction being neglected.

In the above reasoning we have supposed the external pressure H to be the same at C and at F . This is evidently the case when the pressure is that of the atmosphere. If we suppose the surrounding medium to be of a density such that the variation of pressure for the levels AB and KF cannot be neglected, the expression for the velocity must be modified. Let d be the density of the liquid, d' that of the surrounding medium, the excess of pressure from left to right is represented by the weight of a liquid column of density d , and of height $h' - h$, diminished by the weight of a column of the same height of density d' ; that is, it is given by the expression $(h' - h)d - (h' - h)d' = (h' - h)(d - d')$. Now the height m of the liquid which would produce the same pressure is given by the equation $md = (h' - h)(d - d')$. Thus the velocity of efflux will be

$$V = \sqrt{2gm} = \sqrt{\frac{2g(h' - h)(d - d')}{d}}.$$

In the case (which could scarcely occur in practice), where d' is greater than d , the pressure from left to right will be negative; that is, the excess of pressure will be from right to left. The liquid will then flow from right to left, and with a velocity given by the above formula if $d - d'$ is replaced by $d' - d$.

172. Starting the Siphon.—In order that the siphon should work, it must first be charged with liquid. This is effected in various ways. When the liquid can be taken into the mouth without danger, the charging can be effected (Fig. 177) by sucking at a side-tube attached to the long branch.

This method is inapplicable to liquids which would have an injurious effect upon the mouth. The following method is often employed in the transfer of sulphuric acid from one vessel to another.

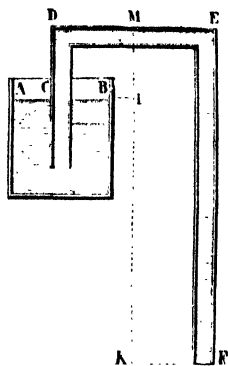


Fig. 176.

The long branch of the siphon (Fig. 178) is first filled with sulphuric acid. This is effected by means of two funnels (which can be

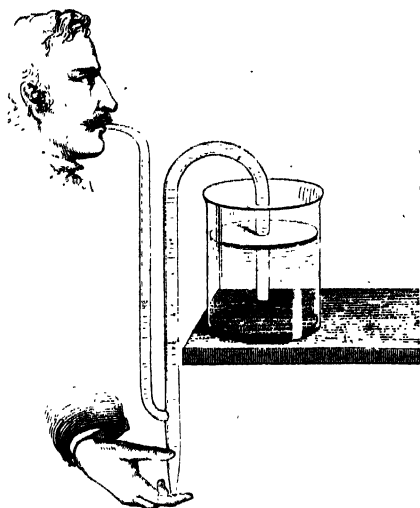


Fig. 177.—Starting the Siphon.

plugged at pleasure) at the bend of the tube. One of these admits the liquid, and the other suffers the air to escape. The two funnels above are then closed, and the tap at the lower end of the tube is opened so as to allow the liquid to escape. The air in the short branch follows the acid, and becomes rarefied; the acid behind it rises, and if it passes the bend, the siphon will be charged; for each portion of the liquid which issues from the tube will draw a corresponding portion from the short to the long branch.

To insure the working of the sulphuric acid siphon, it is not sufficient to have the vertical height of the long branch greater than that

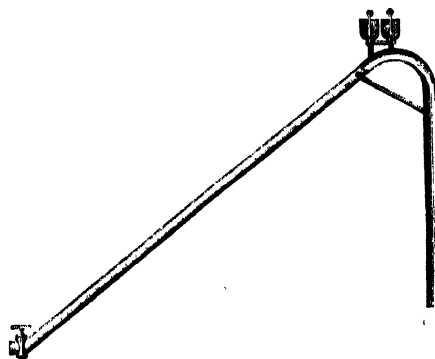


Fig. 178.—Siphon for Sulphuric Acid.

of the short branch; it is farther necessary that it should exceed a certain limit, which depends upon the dimensions of the siphon in each particular case. In order to calculate this limit, we must remark that when the liquid begins to flow, its height diminishes in the long and increases in the short branch; if these two heights should become equal, there would be equilibrium.

We see, then, that in order that the siphon may work, it is necessary that when the liquid rises to the bend of the tube, there should be in the long branch a column of liquid whose vertical height is at least equal to that of the short branch, which we shall denote by h , and the actual length of the short branch from the surface of the liquid in which it dips to the summit of the bend by h' . Then if a be the

inclination of the long branch to the vertical, and L the length of the long branch, which we suppose barely sufficient, the length of the column of liquid remaining in the long branch will be $h \sec \alpha$. The air which at atmospheric pressure H occupied the length h' , now under the pressure $H - h$ occupies a length $L - h \sec \alpha$; hence, by Boyle's law, we have

$$Hh' = (H - h) (L - h \sec \alpha), \text{ whence } L = h \sec \alpha + \frac{Hh'}{H - h}.$$

In this formula H denotes the height of a column of sulphuric acid whose pressure equals that of the atmosphere.

173. Vase of Tantalus.—The siphon may be employed to produce the intermittent flow of a liquid. Suppose, for instance, that we have a vase in which is a bent tube rising to a height n , and with the short branch terminating near the bottom of the vase, while the long branch passes through the bottom. If liquid be poured into the vase, the level will gradually rise in the short branch of the bent tube, and will finally reach and pass the point n , when the siphon will begin to discharge the liquid. If, then, we suppose the liquid to escape by the siphon faster than it is poured into the vessel, the level of the liquid will gradually fall below the termination of the shorter branch. The siphon will then empty itself, and will not recommence its action till the liquid has again risen to the level of the bend.

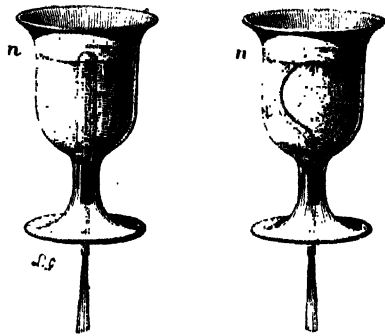


Fig. 179.—Vase of Tantalus.

If the cup is made of metal with the siphon concealed in the thickness of the sides, when a person in lifting it to his lips inclines it to the side in which the siphon is, the siphon will become charged, and will empty the vessel. Hence the name *vase of Tantalus* given to this cup in old treatises on physics. Instead of a bent tube we may employ, as in the first figure, a straight tube covered by a bell-glass left open below; in this case the space between the tube and the bell takes the place of the shorter leg of the siphon.

It is to an action of this kind that natural intermittent springs are generally attributed. Suppose a reservoir (Fig. 180) to communicate with an outlet by a bent tube forming a siphon, and suppose it to

be fed by a stream of water at a slower rate than the siphon is able to discharge it. When the water has reached the bend, the siphon will become charged, and the reservoir will be emptied; it will then be filled again as far as the bend, and so on.

174. Mariotte's Bottle.—This is an apparatus often employed to obtain a continuous flow of water. It consists of a flask whose cork is pierced by a straight tube open at both ends, and with the lower



Fig. 180.—Intermittent Spring.

extremity descending to *a*. An efflux-tube is placed at *b* near the bottom of the flask. Suppose that the flask is full of water, and that the tube is also full to the upper end. If the tube *b* be now opened, the liquid molecules at the orifice will be pressed inwards with a force equal to the atmospheric pressure, but will be pressed outwards with a force exceeding this pressure by the height of the column of water as far as the upper end of the tube. The liquid will therefore flow out; but no vacuum will be produced in the upper part of the flask, for the pressure of the atmosphere will compel the liquid in the tube to replace that which escapes. The level of the liquid in the tube will thus rapidly fall, and the velocity will gradually decrease, as will be seen by the diminished range of the jet. When the liquid reaches the point *a*, the efflux will continue; but then air will enter

the vase in successive bubbles, and will rise to the upper part of the vase, in such quantity that its pressure, together with that of the height of water above the horizontal plane through *a*, will maintain a pressure on this plane equal to that of the atmosphere. From this time the liquid will flow with a constant velocity due to the height of *a* above *b*. Strictly speaking, inasmuch as the air enters, not in a continuous manner, but in successive bubbles, that is, in jerks, the velocity of discharge oscillates about a constant mean value, but the oscillations are in general almost imperceptible. Instead of the ver-

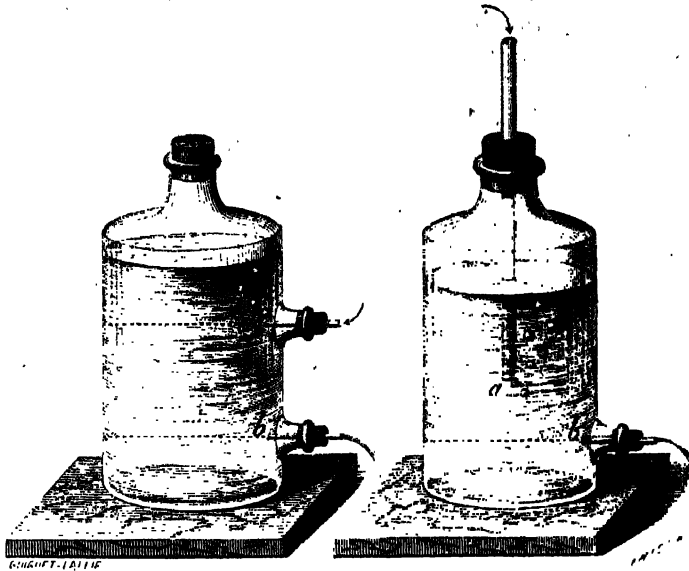


Fig. 181.—Mariotte's Bottle.

tical tube, we may use a vase with two openings at different levels; the liquid escapes by the lower orifice *b*, while air enters by the upper orifice *a*. Mariotte's vase is sometimes used in the laboratory to produce the uniform flow of a gas by employing the water which escapes to expel the gas. We may also draw in gas through the tube of Mariotte's bottle; in this case, the flow of the *water* is uniform, but the flow of the *gas* is continually accelerated, since the space occupied by it in the bottle increases uniformly, but the density of the gas in this space continually increases.

H E A T.

C

CHAPTER XIX.

THERMOMETRY.

175. **Heat—Cold.**—The words *heat* and *cold* express sensations so well known as to need no explanation; but those sensations are modified by subjective causes, and do not furnish an invariable criterion of objective reality. In fact, we may often see one person suffer from heat while another complains of cold. Even for the same person the sensations of heat and cold are comparative. A temperature of 50° Fahr. suddenly occurring amid the heat of summer produces a very decided sensation of cold, whereas the same temperature in winter has exactly the opposite effect. We may mention an old experiment upon this subject, which is at once simple and instructive. If we plunge one hand into water at 32° Fahr., and the other into water at about 100°; and if after having left them some time in this position we immerse them simultaneously in water at 70°, they will experience very different sensations. The hand which was formerly in the cold water now experiences a sensation of heat; that which was in the hot water experiences a sensation of cold, though both are in the same medium. This plainly shows that the sensations of heat and cold are modified by the condition of the observer, and consequently cannot serve as a sure guide in the study of calorific phenomena. Recourse must therefore be had to some more constant standard of reference, and such a standard is furnished by the thermometer.

176. **Temperature.**—If several bodies heated to different degrees are placed in presence of each other, an interchange of heat takes place between them, by which they undergo modifications of opposite kinds; those that are hottest grow cooler, and those that are coldest grow warmer; and after a longer or shorter time these inverse phenomena cease to take place, and the bodies come to a state of mutual

equilibrium. They are then said to be at the same *temperature*. If a source of heat is now brought to act upon them, their temperature is said to *rise*; if they are left to themselves in a colder medium, they all grow cold, and their temperature is said to *fall*. *Two bodies are said to have the same temperature if when they are placed in contact no heat passes from the one to the other.* If when two bodies are placed in contact heat passes from one to the other, that which gives heat to the other is said to have the higher temperature. Heat always tends to pass from bodies of higher to those of lower temperature.

177. Expansion.—At the same time that bodies undergo these changes in temperature, which may be verified by the different impressions which they make upon our organs, they are subjected to other modifications which admit of direct measurement, and which serve as a means of estimating the changes of temperature themselves. These modifications are of different kinds, and we shall have occasion to speak of them all in the course of this work; but that which is especially used as the basis of thermometric measurement is change of volume. In general, when a body is heated, it increases in volume; and, on the other hand, when it is cooled its volume diminishes. The expansion of bodies under the action of heat may be illustrated by the following experiments.

1. *Solid Bodies.*—We take a ring through which a metal sphere

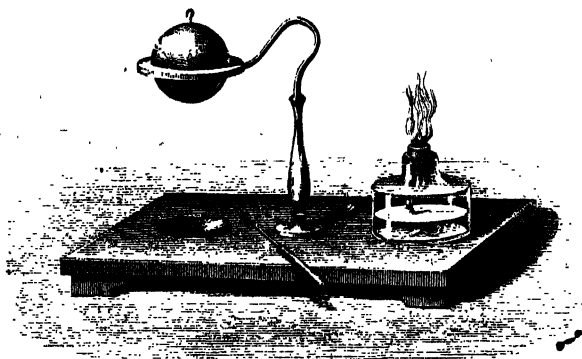


Fig. 182.—Gravesande's Ring.

just passes. This latter is heated by holding it over a spirit-lamp, and it is found that after this operation it will no longer pass through the ring. Its volume has increased. If it is now cooled by immersion in water, it resumes its former volume, and will again pass

through the ring. If, while the sphere was hot, we had heated the ring to about the same degree, the ball would still have been able to pass, their relative dimensions being unaltered. This little apparatus is called *Gravesande's Ring*.

2. *Liquids*.—A liquid, as water for instance, is introduced into the

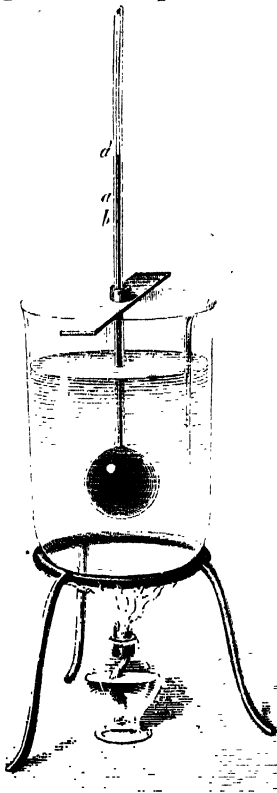


Fig. 183.—Expansion of Liquids.

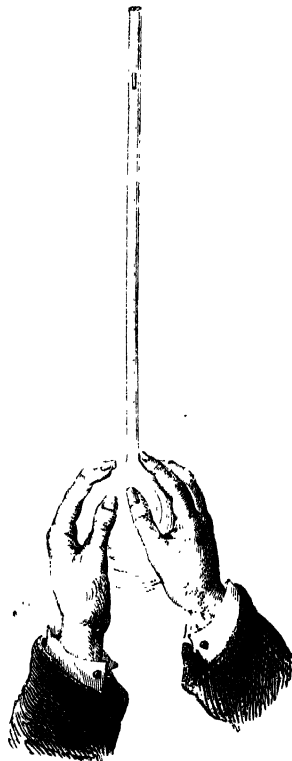


Fig. 184.—Expansion of Gases

apparatus shown in Fig. 183, so as to fill at once the globe and a portion of the tube as far as *a*. The instrument is then immersed in a vessel containing hot water, and at first the extremity of the liquid column descends for an instant to *b*; but when the experiment has continued for some time, the liquid rises to a point *a'* at a considerable height above. This twofold phenomenon is easily explained. The globe, which receives the first impression of heat, increases in volume before any sensible change can take place in the temperature of the liquid. The liquid consequently is unable to fill the entire

capacity of the globe and tube up to the original mark, and thus the extremity of the liquid column is seen to fall. But the liquid receiving in its turn the impression of heat, expands also, and as it passes the original mark, we may conclude that it not only expands, but expands more than the vessel which contains it.

3. *Gases*.—The globe in Fig. 184 contains air, which is separated from the external air by a small liquid index. We have only to warm the globe with the hands and the index will be seen to be pushed quickly upwards, thus showing that gases are exceedingly expandible.

178. General Idea of the Thermometer.—Since the volume of a body is always changed by heat, it follows that when a body is subjected to variations of temperature, it undergoes at the same time corresponding variations of volume. If we suppose that the different volumes successively assumed by the body can easily be measured,

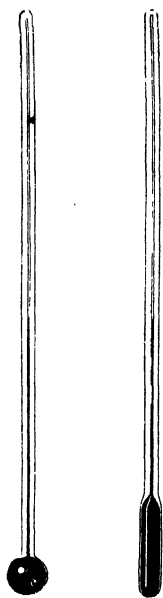


Fig. 185.—Mercurial Thermometers.

we may indicate the temperature by stating the volume. And the body will not only indicate its own temperature by this means, it will also exhibit the temperature of the bodies by which it is surrounded, and which are in equilibrium with it as regards temperature; that is, which do not experience those inverse changes mentioned in § 176. Such is the most general idea of the thermometer, which may be defined as *a body which, under the action of heat, exhibits changes of volume which can be ascertained and measured*.

179. Choice of the Thermometric Substance.—Any substance whatever will serve as a thermometric substance, and in fact there are several kinds of thermometers, founded upon the expansion of different substances. In order, however, that thermometric indications may be comparable with each other, it is necessary to adopt a standard substance or combination of substances, and physicists have by common consent adopted as the standard of reference

the apparent expansion of mercury contained in a glass vessel. The instrument which exhibits this expansion is called the *mercurial thermometer*. It consists essentially, as shown in Fig. 185, of a tube of very small diameter, terminating in a bulb or reservoir of a cylindrical, spherical, or any other form. The reservoir and a portion of

the tube are filled with mercury. If the temperature varies, the level of the liquid will rise or fall in the tube, and the points at which it remains stationary can be easily identified by means of a scale attached to or engraved on the tube.

The choice of mercury as a thermometric substance is extremely suitable. It is a liquid which may easily be procured in a state of purity. It is a very good conductor of heat, and consequently soon comes into equilibrium of temperature with the bodies which it touches. Besides, its *calorific capacity* is very small, so that if it be brought into contact with a heated body, for instance, at whose expense it grows hot, this body experiences in consequence only a very slight change of temperature, which may generally be neglected.

180. Construction of the Mercurial Thermometer.—The construction of a mercurial thermometer is an operation of great delicacy, and comprises several different processes, which we shall successively indicate.

1. *Choice of the Tube.*—The first object is to procure a tube of as uniform bore as possible. In order to ascertain whether this condition is fulfilled, a small column of mercury is introduced into the tube, and its length in different parts of the tube is measured. If these lengths are exactly equal, the tube must be of uniform bore. This is not generally the case, and we have to content ourselves with an approximation to this result; but we must reject tubes in which the differences of length observed are too great. When a suitable tube has been obtained, a reservoir is either blown at one end or attached by melting, the former plan being usually preferable.

When a thermometer of great precision is required, the tube is first *calibrated*; that is, divided into parts of equal volume.

2. *Introduction of the Mercury.*—At the upper extremity of the tube a bulb has been blown, drawn out to a point, at which there is a small opening; this bulb is gently heated, and the point is then immersed in a vessel containing mercury (Fig. 186). The air in the tube growing cold, suffers a diminution at once of volume and of pressure, so that mercury is forced into the bulb by the atmospheric pressure. The end of the point is then closed to prevent the escape of mercurial vapours, and the reservoir and tube, still remaining empty, are heated in a gas or charcoal furnace (Fig. 187), so as to rarefy the contained air. The liquid in the bulb is then heated, and on setting the instrument upright, and allowing it to cool, a portion of the mercury enters the reservoir in consequence of the contraction

of the air. The liquid in the reservoir is then heated to ebullition, the air is expelled from the reservoir and tube by the vapour of mercury, and on placing the instrument in an upright position, the

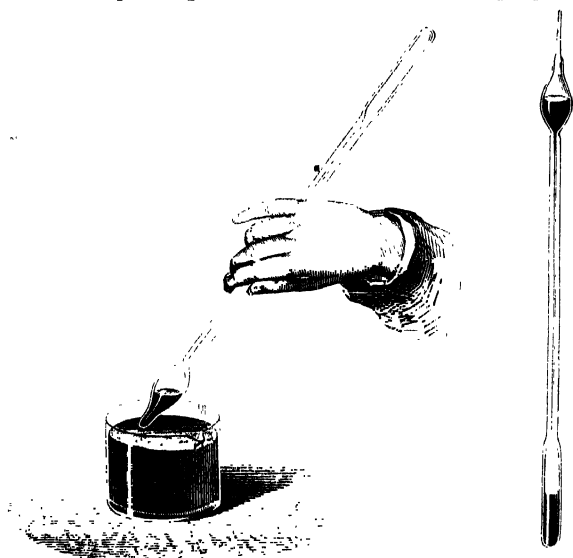


Fig. 186.—Introduction of the Mercury.

mercury during the process of cooling enters the reservoir and completely fills it. If a bubble of air still remains, as is often the case, it may be expelled by repeating the same operation several times. The quantity of mercury to be left is then regulated according to the

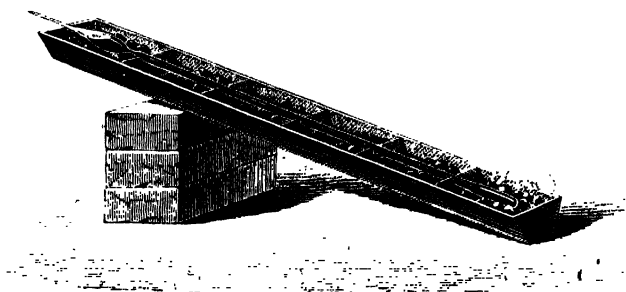


Fig. 187.—Furnace for heating Thermometers.

temperatures which the instrument is intended to indicate; the bulb at the upper end is removed, and the tube, having been drawn out to a point, is hermetically sealed at the moment when the mercury reaches its extremity, so as to leave no air in the interior.

3. *Determination of the Fixed Points.*—The instrument, under these conditions, and with any scale of equal parts marked on the tube, would of course indicate variations of temperature, but these indications would be arbitrary, and two thermometers so constructed would in general give different indications.

In order to insure that the indications of different thermometers shall be comparable, it has been agreed to adopt two standard temperatures, which can easily be reproduced and maintained for a considerable time, and to denote them by fixed numbers. These two temperatures are the freezing-point and boiling-point of water; or to speak more strictly, the temperature of melting ice, and the temperature of the steam given off by water boiling under average atmospheric pressure. It has been observed that if the thermometer be surrounded with melting ice (or melting snow), the mercury, under whatever circumstances the experiment is performed, invariably stops at the same point, and remains stationary there as long as the melting continues. This then is a fixed temperature. On the Centigrade scale it is called zero, on Fahrenheit's scale 32° .

In order to mark this point on a thermometer, it is surrounded by melting ice, which is contained in a perforated vessel, so as to allow the water produced by melting to escape. When the level of the mercury ceases to vary, a mark is made on the tube with a fine diamond at the extremity of the mercurial column. This is frequently called for brevity the *freezing-point*.

It has also been observed that if water be made to boil in an open metallic vessel, under average atmospheric pressure (760 millimetres, or 29.922 inches), and if the thermometer be plunged into the steam, the mercury stands at the same point during the entire time of ebullition, provided that the external pressure does not change. This second fixed temperature is called 100° in the Centigrade scale (whence the name), and 212° on Fahrenheit's scale. In order to mark this second point on the thermometer, an apparatus is employed which was devised by Gay-Lussac, and perfected by Regnault. It consists

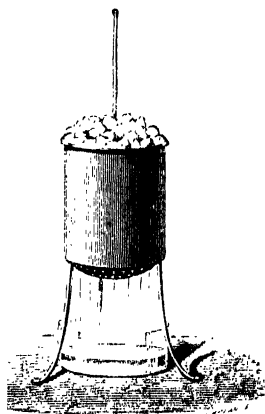


Fig. 188.—Determination of Freezing-point.

of a copper boiler (Fig. 189) containing water which is raised to ebullition by means of a furnace. The steam circulates through a double casing, and escapes by a tube near the bottom. The thermometer is fixed in the interior casing, and when the mercury has

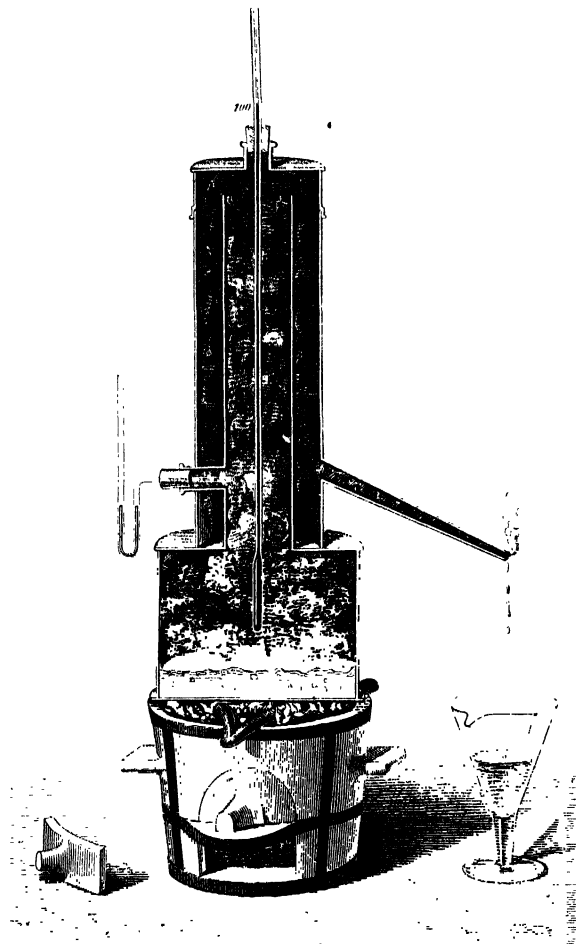


Fig. 189.—Determination of Boiling-point

become stationary, a mark is made at the point at which it stops, which denotes what is commonly called for brevity the *boiling-point*. It now only remains to divide the portion of the instrument between the freezing, and boiling points into equal parts corresponding to single degrees, and to continue the division beyond the fixed points. Below the zero point are marked the numbers 1, 2, 3, &c. These

temperatures are generally expressed with the sign —. Thus the temperature of 17° below zero is written -17° .

A small manometric tube, open at both ends, serves to show, by the equality of level of the mercury in its two branches, that the ebullition is taking place at a pressure equal to that which prevails externally, and consequently that the steam is escaping with sufficient freedom. It frequently happens that the external pressure is not exactly 760 millimetres, in which case the boiling-point should be placed a little above or a little below the point at which the mercury remains stationary, according as the pressure is less or greater than this standard pressure. When the difference on either side is inconsiderable, the position of the boiling-point is calculated by the rule, that a difference of 20.6 millimetres, either above or below the normal pressure, causes a difference of 1° in the temperature of the steam produced. We shall return to this point in Chap. xxvi.

181. Adjustment of the Quantity of Mercury.—In order to avoid complicating the above explanation, we have omitted to consider an operation of great importance, which should precede those which we have just described. This is the determination of the volume which must be given to the reservoir, in order that the instrument may have the required range. When the reservoir is cylindrical, this is very easily effected in the following manner. Suppose we wish the thermometer to indicate temperatures comprised between -20° and 130° Cent., so that the range is to be 150° ; the reservoir is left open at O (Fig. 190),

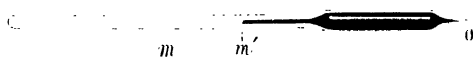


Fig. 190.

and is filled through this opening, which is then hermetically sealed. The instrument is then immersed in two baths whose temperatures differ by 50° , for instance, and the mercury rises through a distance mm' . This length, if the quantity of mercury in the reservoir be exactly sufficient, should be the third part of the length of the stem. But the quantity of mercury in the reservoir is always taken too large at first, so that it has only to be reduced, and thus the space traversed by the liquid is at first too great. Suppose it to be equal to $\frac{3}{4}$ ths of the length of the stem. The degrees will then be too long, in the ratio $\frac{3}{4} : \frac{1}{3} = 2$. That is, the reservoir is $\frac{2}{3}$ of what it should be. We therefore measure off $\frac{1}{4}$ ths of the length of the reservoir, beginning at the end next the stem; this distance is marked by a line, and the end O is then broken and the mercury suffered to escape. The glass

is then melted down to the marked line, and the reservoir is thus brought to the proper dimensions. It only remains to regulate the quantity of mercury admitted, by making it fill the tube at the highest temperature which the instrument is intended to indicate.

If the reservoir were spherical, which is a shape generally ill adapted for delicate thermometers, the foregoing process would be inapplicable, and it would be necessary to determine the proper size by trial.

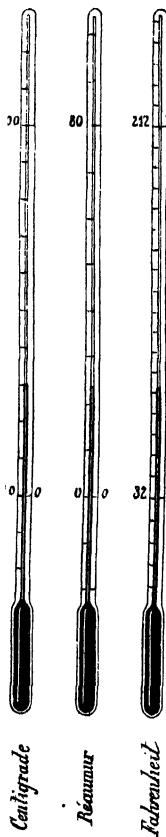


Fig. 191.
Thermometric Scales.

182. Thermometric Scales.—In the *Centigrade* scale the freezing-point is marked 0° , and the boiling-point 100° . In *Réaumur's* scale, which is still sometimes used abroad, the freezing-point is also marked 0° , but the boiling-point is marked 80° . Hence, 5 degrees on the former scale are equal to 4 on the latter, and the reduction of temperatures from one of these scales to the other can be effected by multiplying by $\frac{4}{5}$ or $\frac{5}{4}$.

For example, the temperature 75° Centigrade is the same as 60° Réaumur, since $75 \times \frac{4}{5} = 60$; and the temperature 36° Réaumur is the same as 45° Centigrade, since $36 \times \frac{5}{4} = 45$.

The relation between either of these scales and that of *Fahrenheit* is rather more complicated, inasmuch as Fahrenheit's zero is not at freezing-point, but at 32 of his degrees below it.

As regards intervals of temperature, 180 degrees Fahrenheit are equal to 100 Centigrade, or to 80 Réaumur, and hence, in lower terms, 9 degrees Fahrenheit are equal to 5 Centigrade, or to 4 Réaumur.

The conversion of temperatures themselves (as distinguished from intervals of temperature) will be best explained by a few examples.

Example 1. To find what temperatures on the other two scales are equivalent to the temperature 50° Fahrenheit.

Subtracting 32, we see that this temperature is 18 Fahrenheit degrees above freezing-point, and as this interval is equivalent to $18 \times \frac{5}{9}$, that is 10 Centigrade degrees, or to $18 \times \frac{4}{9}$, that is 8 Réaumur degrees, the equivalent temperatures are respectively 10° Centigrade and 8° Réaumur.

Example 2. To find the degree on Fahrenheit's scale, which is equivalent to the temperature 25° Centigrade.

An interval of 25 Centigrade degrees is equal to $25 \times \frac{9}{5}$, that is 45 Fahrenheit degrees, and the temperature in question is above freezing-point by this amount. The number denoting it on Fahrenheit's scale is therefore $32 + 45$, that is 77° .

The rules for the conversion of the three thermometric scales may be summed up in the following formulæ, in which F, C, and R denote equivalent temperatures expressed in degrees of the three scales:—

$$F = \frac{9}{5}C + 32 = \frac{9}{4}R + 32.$$

$$C = \frac{5}{9}R - \frac{16}{9} \quad (F - 32).$$

$$R = \frac{4}{9}C + \frac{16}{9} \quad (F - 32).$$

It is usual in stating temperatures to indicate the scale referred to by the abbreviations *Fahr.*, *Cent.*, *Réau.*, or more briefly by the initial letters F., C., R.

183. Apparent Expansion of Mercury. Degree of the Mercurial Thermometer.—The indications of the mercurial thermometer depend not upon the real but upon the *apparent* expansion of mercury, that is, upon the difference between the expansion of the mercury and that of the glass in which it is contained.

To understand the physical meaning of a degree, suppose that we have a thermometric tube (Fig. 192) divided into parts of equal capacity, and that by gauging the reservoir we have ascertained its capacity to be equal to N of these parts. Mercury is introduced, and when the instrument is surrounded with melting ice the mercury fills the bulb and n parts of the tube. The instrument is then raised to the temperature of 100° C., and the mercury expands so as to occupy n' parts of the stem. The apparent volumes of the mercury at 0° and 100° Centigrade are therefore $N + n$ and $N + n'$ respectively, and the apparent increase of volume is $n' - n$. The apparent increase per unit volume is therefore $\frac{n' - n}{N + n}$, and as this is the increase for 100° the apparent expansion per degree Centigrade is $\frac{1}{100} \frac{n' - n}{N + n}$. This quantity is found by experiment to be about equal to $\frac{1}{6480}$. Each degree Centigrade represents then a difference of apparent volume of the mercury equal to about $\frac{1}{6480}$ of its volume at freezing-point.

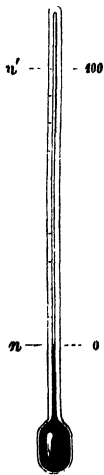


Fig. 192.

184. Comparability of Mercurial Thermometers.—This enables us to examine the important question, whether different thermometers are comparable with each other, that is, whether they will indicate the same temperature under the same conditions. This must evidently be the case where the glass employed for the construction of the tubes is absolutely the same. The agreement will likewise be exact, even when the apparent expansions due to the quality of the glass employed in each case are not the same, provided that they preserve a constant ratio at different temperatures.

Experiment alone can teach us whether this disagreement actually exists, and in what degree. The result of Regnault's investigations upon this point tends to show that up to 300° Cent. this disagreement is almost nothing, or, at any rate, may be neglected with perfect safety; above this point a slight difference is observed, which increases to about 3° or 4° at the temperature of 350° , that is, at the superior limit to the use of the mercurial thermometer.

This defect in the comparability of thermometers, slight as it is, must be attributed to irregularities in the expansion of the glass. The influence of these irregularities would obviously be less sensible with a more expansible material than mercury, and this is the reason that in experiments where great precision is necessary air-thermometers are employed, which at the same time serve to indicate the highest temperatures, whereas mercury cannot be employed beyond 350° , its boiling-point.

185. Displacement of the Zero Point.—A thermometer left to itself after being made, gradually undergoes a contraction of its capacity, leading to a uniform error of excess in its indications. This phenomenon is attributable to molecular change in the glass, which has, so to speak, been tempered in the construction of the instrument, and to atmospheric pressure on the exterior of the bulb, which is unre-sisted by the internal vacuum. The progress of this change ceases at the end of a certain time, about fifteen or eighteen months, and the displacement is always inconsiderable, never amounting to a degree. In precise experiments, however, it is necessary to verify the position of the zero point in the thermometer employed, and, in the observation of temperatures, to take into consideration the slight displacement which may have occurred.

186. Sensibility of the Thermometer.—The power of the instrument to detect very small differences of temperature may be regarded as measured by the length of the degrees, which is proportional to the

capacity of the bulb directly and to the section of the tube inversely. In fact, if l denote the length of one degree, and s the sectional area of the tube, ls will be the volume of one degree, which is $\frac{1}{6480}C$, where C denotes the capacity of the bulb, together with as much of the tube as is below the freezing-point. Hence, $l = \frac{1}{6480} \frac{C}{s}$, which varies directly as C , and inversely as s .

Quickness of action, on the other hand, requires that the bulb be small in at least one of its dimensions, so that no part of the mercury shall be far removed from the exterior, and also that the glass of the bulb be thin.

Quickness of action is important in measuring temperatures which vary rapidly. It should also be observed that, as the thermometer, in coming to the temperature of any body, necessarily causes an inverse change in the temperature of that body, it follows that when the mass of the body to be investigated is very small, the thermometer itself should be of extremely small dimensions, in order that it may not cause a sensible variation in the temperature to be ascertained.



Fig. 193.
Weight Thermometer.

187. Weight Thermometer.—In this thermometer, which often takes the place of the ordinary thermometer in physical investigations, the stem is done away with, and the mercury contained in the reservoir overflows into a small cup, where it is collected; and the temperature is deduced from the weight of the mercury which thus escapes. Let P be the weight of the mercury which fills the reservoir at freezing-point, and π the weight which issues when the instrument is placed in the steamer in order to determine the boiling-point. The apparent expansion between these points is evidently represented by the fraction $\frac{\pi}{P-\pi}$, and, consequently, the value of a degree Centigrade is the one-hundredth part of this fraction, that is $\frac{\pi}{100(P-\pi)}$. Suppose now that the instrument, containing again a weight P of mercury at zero is placed in a bath whose temperature we wish to determine, and that a weight p of mercury flows over. The whole apparent expansion is $\frac{p}{P-p}$, and dividing this by the value of a degree, we obtain the temperature required, which is thus expressed by the formula

$$T = \frac{p}{P-p} \div \frac{\pi}{100(P-\pi)} = 100 \frac{p}{\pi} \frac{P-\pi}{P-p}.$$

188. Alcohol Thermometer.—In the construction of thermometers other liquids may be introduced instead of mercury, and alcohol is very frequently employed for this purpose. But if an alcohol thermometer were constructed so as to agree with a mercurial thermometer at two fixed temperatures, and were graduated by dividing the intervening space into equal parts, and continuing the equal graduations both ways, it would be found to give different readings from a mercurial thermometer, except at or very near the two fixed temperatures. This fact may be expressed by saying, that alcohol does not expand equally for equal increments of temperature as indicated by a mercurial thermometer; or, more symmetrically, by saying that intervals of temperature which are equal as measured by the expansion of mercury, are not equal as measured by the expansion of alcohol.

In practice, alcohol thermometers are graduated by comparison with mercurial thermometers, and the degrees of an alcohol thermometer have consequently unequal volumes in different parts of the scale. The degrees, in fact, increase in length as we ascend on the scale.

Alcohol has the disadvantage of being slower in its action than mercury, on account of its inferior conductivity; but it can be employed for lower temperatures than mercury, as the latter congeals at -39° Cent. (-38° Fahr.), whereas the former has never congealed at any temperature yet attained.

189. Self-registering Thermometers.—It is often important for meteorological purposes to have the means of knowing the highest or the lowest temperature that occurs during a given interval. Instruments intended for this purpose are called maximum and minimum thermometers.

The oldest instrument of this class is *Six's* (Fig. 194 A), which is at once a maximum and a minimum thermometer. It has a large cylindrical bulb C filled with alcohol, which also occupies a portion of the tube. The remainder of the tube is partly filled with mercury, which occupies a portion of the tube shaped like the letter U, one extremity

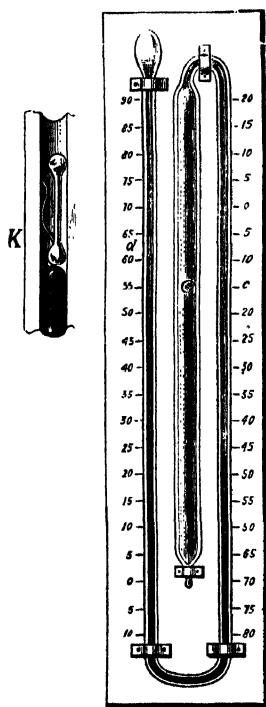


Fig. 194A.—Six's Self-registering Thermometer.

of the mercurial column being in contact with the alcohol already mentioned, while the other extremity is in contact with a second column of alcohol; and beyond this there is a small space occupied only with air, so as to leave room for the expansion of the liquids. When the alcohol in the bulb expands it pushes the mercurial column before it, and when it contracts the mercurial column follows it. The extreme points reached by the two ends of the mercurial column are registered by a pair of light steel indices *c*, *d* (shown on an enlarged scale at K), which are pushed before the ends of the column, and then are held in their places by springs, which are just strong enough to prevent slipping, so that the indices do not follow the mercury in its retreat. One of the indices *d* registers the maximum and the other *c* the minimum temperature which has occurred since the instrument was last set. The setting consists in bringing the indices into contact with the ends of the mercurial column, and is usually effected by means of a magnet. This instrument is now, on account of its complexity, little used. It possesses, however, the advantages of being equally quick (or slow) in its action for maximum and minimum temperatures, which is an important property when these temperatures are made the foundation for the computation of the mean temperature of the interval, and of being better able than most of the self-registering thermometers to bear slight jolts without disturbance of the indices.

► *Rutherford's* self-registering thermometers are frequently mounted together on one frame, as in Fig. 195, but are nevertheless distinct

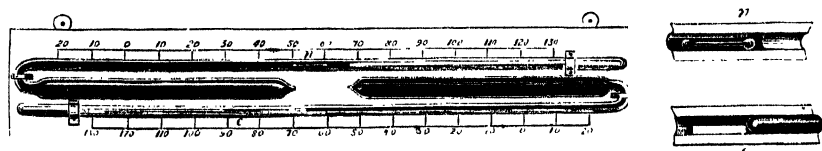


Fig. 195.—Rutherford's Maximum and Minimum Thermometers.

instruments. His *minimum* thermometer, which is the only minimum thermometer in general use, has alcohol for its fluid, and is always placed with its tube horizontal, or nearly so. In the fluid column there is a small index *n* of glass or enamel, shaped like a dumb-bell.

When contraction occurs, the index, being wetted by the liquid, is forced backwards by the contractile force of the superficial film which forms the extremity of the liquid column (§ 97); but when expansion

takes place the index remains stationary in the interior of the liquid. Hence the minimum temperature is indicated by the position of the forward end of the index. The instrument is set by inclining it so as to let the index slide down to the end of the liquid column.

The only way in which this instrument is liable to derangement, is by a portion of the spirit evaporating from the column and becoming condensed in the end of the tube, which usually terminates in a small bulb. When the portion thus detached is large, or when the column of spirit becomes broken into detached portions by rough usage in travelling, "let the thermometer be taken in the hand by the end farthest from the bulb, raised above the head, and then forcibly swung down towards the feet; the object being, on the principle of centrifugal force, to send down the detached portion of spirit till it unites with the column. A few throws or swinging strokes will generally be sufficient; after which the thermometer should be placed in a slanting position, to allow the rest of the spirit still adhering to the sides of the tube to drain down to the column. But another method must be adopted if the portion of spirit in the top of the tube be small. Heat should then be applied slowly and cautiously to the end of the tube where the detached portion of spirit is lodged; this being turned into vapour by the heat will condense on the surface of the unbroken column of spirit. Care should be taken that the heat is not too quickly applied. . . . The best and safest way to apply the requisite amount of heat, is to bring the end of the tube slowly down towards a minute flame from a gas-burner; or if gas is not to be had, a piece of heated metal will serve instead."¹

Rutherford's *maximum* thermometer is a mercurial thermometer, with the stem placed horizontally, and with a steel index *c* in the tube outside the mercurial column. When expansion occurs, the index, not being wetted by the liquid, is forced forwards by the contractile force of the superficial film which forms the extremity of the liquid column (§ 97); but when contraction takes place, the index remains stationary outside the liquid. Hence the maximum temperature is indicated by the position of the backward end of the index. The instrument is set by bringing the index into contact with the end of the liquid column, an operation which is usually effected by means of a magnet.

This thermometer is liable to get out of order after a few years' use, by chemical action upon the surface of the index, which causes it to

¹ Buchan's *Handy Book of Meteorology*, p. 62.

become wetted by the mercury, and thus renders the instrument useless.

Phillips' maximum thermometer (invented by Professor Phillips, the eminent geologist, and made by Casella) is recommended for use in the official *Instructions for Taking Meteorological Observations*, drawn up by Sir Henry James for the use of the Royal Engineers. It is a mercurial thermometer not deprived of air. It has an exceed-

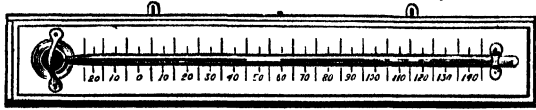


Fig. 194 A.—Phillips' Maximum Thermometer.

ingly fine bore, and the mercurial column is broken by the insertion of a small portion of air. The instrument is set by reducing this portion of air to the smallest dimensions which it can be made to assume, and is placed in a horizontal position. When the mercury expands it pushes forwards this intervening air and the detached column of mercury beyond it; but when contraction takes place the intervening air expands, and the detached column remains unmoved.

"The thread of mercury in these thermometers is easily broken at any point required, by simply raising the bulb end, and allowing the mercury to run into the open cell at the end; and, as it descends, detaching, with a slight jerk, as much of it as may be thought necessary, which should be an inch, or an inch and a half."¹

The detached column is not easily shaken out of its place, and when the bore of the tube is made sufficiently narrow the instrument may even be used in a vertical position, a property which is often of great service.

In Negretti and Zambra's maximum thermometer (Fig. 194), which is employed at the Royal Observatory, Greenwich, there is an

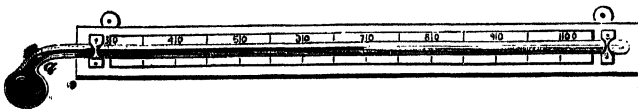


Fig. 194.—Negretti's Maximum Thermometer.

obstruction in the bent part of the tube, near the bulb, which barely leaves room for the mercury to pass when forced up by

¹ *Instructions for Taking Meteorological Observations*. By Colonel Sir Henry James, R.E., Director of the Ordnance Survey.

expansion, and is sufficient to prevent it from returning when the bulb cools.

The objection chiefly urged against this thermom ter is the extreme mobility of the detached column, which renders it very liable to accidental displacement; but in the hands of a skilful observer this is of no moment. Dr. Balfour Stewart (*Elementary Treatise on Heat*, p. 20, 21), says:—"When used, the stem of this instrument ought to be inclined downwards. . . . It does not matter if the column past the obstruction go down to the bottom of the tube; for when the instrument is read, it is gently tilted up until this detached column flows back to the obstruction, where it is arrested, and the end of the column will then denote the maximum temperature. In resetting the instrument, it is necessary to shake the detached column past the obstruction in order to fill up the vacancy left by the contraction of the fluid after the maximum had been reached."

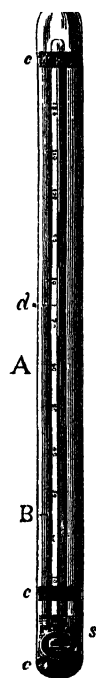


Fig. 194c.
Thomson's
Protected
Thermometer.

MERCURIAL MINIMUM THERMOMETERS.—As it is obviously undesirable that the minimum thermometer employed should be slower in its action than the maximum thermometer used in conjunction with it, several attempts have been made to construct a minimum thermometer in which mercury instead of alcohol shall be the expanding fluid (see a description of Casella's mercurial minimum thermometer, Stewart on *Heat*, p. 22); but no one has yet succeeded in producing such an instrument fit for general use.

DEEP-SEA AND WELL THERMOMETERS.—The instruments which have been most successfully employed for the observation of the temperature of water at great depths are self-registering thermometers either of Six's or Phillips' construction, inclosed in a glass-case, hermetically sealed, containing air and a little alcohol. The glass-case and inclosed air protect the bulb of the thermometer from the immense pressure of the superincumbent water, which by compressing the bulb would force mercury into the tube, and make the reading too high. The instrument represented in Fig. 194c was designed by Sir Wm. Thomson, and is used by the Committee on Underground Temperature appointed by the British Association. A is the protecting case, B the Phillips' thermometer inclosed in it, and supported by three pieces of cork *ccc*. A small quantity of spirit *s* occupies the lower

part of the case; *d* is the air-bubble characteristic of Phillips' thermometer, and serving to separate one portion of the mercurial column from the rest. In the figure this air-bubble is represented as expanded by the descent of the lower portion of mercury, while the upper portion remains suspended by adhesion. This instrument has been found to register correctly even under a pressure of $2\frac{1}{2}$ tons to the square inch.

The use of the spirit *s* is to bring the bulb more quickly to the temperature of the surrounding medium.

Another instrument, designed, like the foregoing, for observations in wells and borings, is *Walferdin's maximum thermometer* (Fig. 196). Its tube terminates above in a fine point opening into a cavity of considerable size, which contains a sufficient quantity of mercury to cover the point when the instrument is inverted. The instrument is set by placing it in this inverted position and warming the bulb until the mercury in the stem reaches the point and becomes connected with the mercury in the cavity. The bulb is then cooled to a temperature lower than that which is to be observed, and during the operation of cooling mercury enters the tube so as always to keep it full. The instrument is then lowered in the erect position into the bore where observations are to be made, and when the temperature of the mercury rises a portion of it overflows from the tube. To ascertain the maximum temperature which has been experienced, the instrument may be immersed in a bath of known temperature, less than that of the boring, and the amount of void space in the upper part of the tube will indicate the excess of the maximum temperature experienced above that of the bath.



Fig. 196.

If the tube is not graduated, the maximum temperature can be ascertained by gradually raising the temperature of the bath till the tube is just full.

If the tube is graduated, the graduations can in strictness only indicate true degrees for some one standard temperature of setting, since the length of a true degree is proportional to the quantity of mercury in the bulb and tube; but a difference of a few degrees in the temperature of setting is immaterial, since 10° Cent. would only alter the length of a degree by about one six-hundredth part.

190. Thermograph.—A much more complete method of obtaining

a register of the indications of a thermometer in the absence of an observer is now adopted in the principal British observatories. It consists in the photographic registration of the height of the mercurial column at every instant during the entire day. A sheet of sensitized paper is mounted on a vertical cylinder just behind the mercurial column, which is also vertical, and is protected from the action of light by a cover of blackened zinc, with the exception of a narrow vertical strip just behind the mercurial column. A strong beam of light from a lamp or gas flame is concentrated by a cylindric lens, so that if the thermometer were empty of mercury a bright vertical line of light would be thrown on the paper. As this beam of light is intercepted by the mercury in the tube, which for this purpose is made broad and flat, only the portion of the paper above the top of the mercurial column receives the light, and is photographically affected. The cylinder is made to revolve slowly by clock-work, and if the mercury stood always at the same height, the boundary between the discoloured and the unaffected parts of the paper would be straight and horizontal, in consequence of the horizontal motion of the paper itself. In reality, the rising and falling of the mercury, combined with the horizontal motion of the paper, causes the line of separation to be curved or wavy, and the height of the curve above a certain datum-line is a measure of the temperature at each instant of the day.¹ The whole apparatus is called a *thermograph*, and apparatus of a similar character is employed for obtaining a continuous photographic record of the indications of the barometer² and magnetic instruments.

191. Metallic Thermometers.—Thermometers have sometimes been constructed of solid metals. Breguet's thermometer, for example (Fig. 197), consists of a helix carrying at its lower end a horizontal needle which traverses a dial. The helix is composed of three metallic strips, of silver, gold, and platinum, soldered together so as to form a single ribbon. The silver, which is the most expansible, is placed in the interior of the helix; the platinum, which is the least

¹ Strictly speaking, the temperatures corresponding to the various points of the curve are not read off by reference to a single datum-line, but to a number of datum-lines which represent the shadows of a set of horizontal wires stretched across the tube of the thermometer at each degree, a broader wire being placed at the decades, and also at 32°, 52°, and 72°.

In order to give long degrees, the bulb of the thermometer is made very large—eight inches long, and $\frac{1}{4}$ of an inch in internal diameter.—(*Greenwich Observations*, 1847.)

² See § 110A.

expansible, on the exterior; and the gold serves to connect them. When the temperature rises, the helix unwinds and produces a deflection of the needle; when the temperature falls, the helix winds up and deflects the needle in the opposite direction.

Fig. 198 represents another dial-thermometer, in which the thermometric portion is a double strip composed of steel and brass, bent into

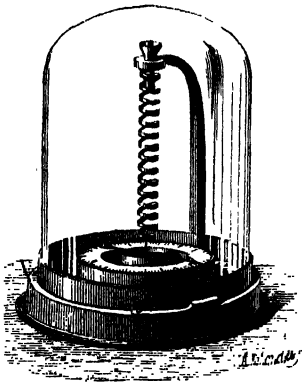


Fig. 197.—Breguet's Thermometer.

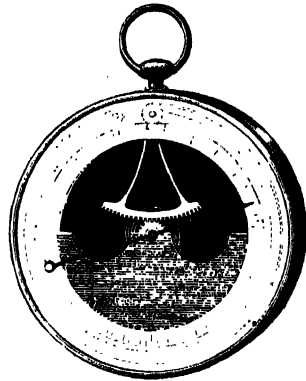


Fig. 198.—Metallic Thermometer.

a circular form. One extremity is fixed, the other is jointed to the shorter arm of a lever, whose longer arm carries a toothed sector. This latter works into a pinion, to which the needle is attached.

It may be remarked that dial-thermometers are very well adapted for indicating maximum and minimum temperatures, it being only necessary to place on opposite sides of the needle a pair of movable indices, which could be pushed in either direction according to the variations of temperature.

Generally speaking, metallic thermometers offer great facilities for automatic registration.

In Secchi's meteorograph, for example, the temperature is indicated and registered by the expansion of a long strip of brass (about 17 metres long) kept constantly stretched by a suitable weight; this expansion is rendered sensible by a system of levers connected with the tracing point. The thermograph of Hasler and Escher consists of a steel and a brass band connected together and rolled into the form of a spiral. The movable extremity of the spiral, by acting upon a projecting arm, produces rotation of a steel axis which carries the tracer.

192. Pyrometers.—Metallic thermometers can generally be em-

ployed for measuring higher temperatures than a mercurial thermometer could bear; but there is great difficulty in constructing any instrument to measure temperatures as high as those of furnaces. Instruments intended for this purpose are called pyrometers.

Wedgwood, the famous potter, invented an apparatus of this kind, consisting of a gauge for measuring the contraction experienced by a

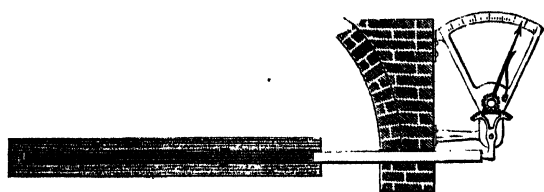


Fig. 199.—Brongniart's Pyrometer.

piece of baked clay when placed in a furnace; and Brongniart introduced into the porcelain manufactory at Sèvres the instrument represented in Fig. 199,

consisting of an iron bar lying in a groove in a porcelain slab, with one end abutting against the bottom of the groove, and the other projecting through the side of the furnace, where it gave motion to an indicator.

Neither of these instruments has, however, been found to furnish consistent indications, and the only instrument that is now relied on for the measurement of very high temperatures is the air-thermometer.

193. Differential Thermometer.—Leslie of Edinburgh invented, in the beginning of the present century, an ingenious instrument which enables us to measure small variations of temperature. A column of sulphuric acid, coloured red, stands in the two branches of a bent tube, the extremities of which terminate in two globes of equal volume (Fig. 200).

When the air contained in the two globes is at the same temperature, whatever that temperature may be, the liquid, if the instrument is in order, stands at the same height in the two branches. This point is marked zero. One of the globes being then maintained at a constant temperature, the other is raised through, for instance, 5 degrees, when the column rises on the side of the colder globe up to a point *a*, and descends on the other side to a point *b*. Suppose the space traversed by the liquid in each branch to be divided into 10 equal parts, each part will be equivalent to a quarter of a degree.

This division is continued upon each branch on both sides of zero.

The differential thermometer is an instrument of great sensibility, and enabled Leslie to make some very delicate investigations on the

subject of the radiation of heat. It is now, however, superseded by the thermo-pile invented by Melloni. This latter instrument will be described in another portion of this work. Rumford's thermoscope (Fig. 201) is analogous to Leslie's differential thermometer. It differs

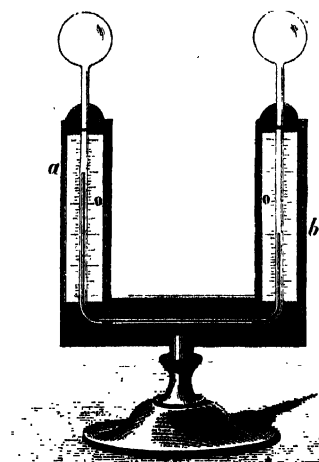


Fig. 200.—Leslie's Differential Thermometer.

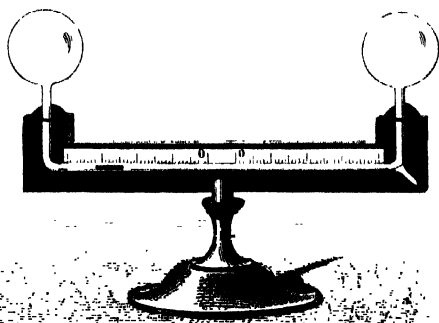


Fig. 201.—Rumford's Thermoscope.

from it in having the horizontal part much longer, and the vertical branches shorter. In the horizontal tube is an alcohol index, which, when the two globes are at the same temperature, occupies exactly the middle.

The tube is divided into equal parts, and the number of divisions traversed by the index is proportional very nearly to the difference of temperature between the two bulbs.

CHAPTER XX.

FORMULÆ RELATING TO EXPANSION.

194. Measure of Expansion, Modulus of Expansion.—When a substance expands, so that its volume changes from V to $V' = V + v$, the ratio $\frac{v}{V}$ is the numerical measure of the expansion per unit volume, and is usually called simply the expansion of the substance.

Another ratio which it is frequently necessary to consider, is $\frac{V'}{V}$, or $1 + \frac{v}{V}$, that is to say, the ratio of the final to the initial volume. This may conveniently be called the *factor of expansion*, or the *expansion factor*. If m denote the expansion, $1 + m$ will be the factor of expansion, and we have

$$V' = V (1 + m). \quad (1)$$

195. Coefficient of Expansion.—It often happens that when the temperature of a body is increased by successive equal amounts, the successive increments of volume are also equal. In this case the body is said to expand uniformly between the extreme temperatures, and if V denote the volume of the body at 0° Cent., and V' its volume at t° , then we have

$$V' = V (1 + K t); \quad (2)$$

where K is a constant number, called the coefficient of expansion per degree Centigrade.

The coefficient of expansion per degree Fahrenheit will be $\frac{5}{9}$ of this value of K , and the coefficient of expansion for the interval from 0° to 100° C. (if the expansion be uniform through this range) will be $100 K$.

196. It is usual to employ the name *coefficient of expansion* to denote the coefficient of expansion per degree, the thermometric scale referred to being stated in the context.

Example. The volume of a glass vessel is 450 cubic inches at 0° C.; find its volume at 80° C., the coefficient of expansion for glass being .00002. By (1) we have

$$V' = 450 (1 + 80 \times .00002) = 450.72 \text{ cubic inches.}$$

197. Cubic, Linear, and Superficial Expansion.—Thus far we have considered only expansion of volume. When a solid body expands, we may consider separately the increase of one of the linear dimensions of the body; this is called the *linear expansion*. Or we may consider the increase in area of any portion of its surface, which is called the *superficial expansion*; or finally, the increase of volume, which is called expansion of volume, or *cubical expansion*. By substituting the words “length” and “surface” for “volume” in § 194 we obtain definitions of linear and superficial expansion as numerically expressed, and we can demonstrate the two following propositions:—

(1.) The cubical expansion is three times the linear expansion.

(2.) The superficial expansion is twice the linear expansion.

Suppose Fig. 202 to represent a cube formed of any substance, and let the length of each edge of the cube be unity at zero; the volume is consequently equal to 1, and the area of any one of the faces is also represented by 1. If the body be heated to any temperature t , each of the edges will increase by a certain quantity l , and the area of each face will become $(1+l)^2 = 1 + 2l + l^2$, while the volume of the cube will become

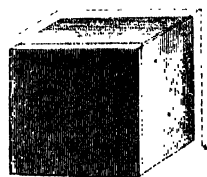


Fig. 202.

$$(1+l)^3 = 1 + 3l + 3l^2 + l^3.$$

But as the quantity l , which represents the linear expansion, is always very small, we may, without sensible error, neglect its second and third powers in comparison with its first power. We thus see that the increase in area of one of the faces of the cube is sensibly equal to $2l$, and that the increase in volume is sensibly equal to $3l$, which proves the propositions. These propositions evidently hold for the coefficients of expansion, so that we may say that *the coefficient of linear expansion is equal to one-third of the coefficient of cubical expansion, and to one-half of the coefficient of superficial expansion.*

The above demonstration supposes the body to remain similar to itself during expansion, which is not the case with bodies of a fibrous

or laminated structure, nor with crystals, except those belonging to the cubic system.

198. Various Formulæ.—From equations (1) and (2) we may find the value of V in terms of V' , thus,

$$V = \frac{V'}{1+m}; \quad (3)$$

$$V = \frac{V'}{1+Kt'}, \quad (4)$$

that is to say, given the volume of a body at a certain temperature, the volume at zero is found by *dividing the given volume by the factor of expansion*.

Formulæ (1), (2), (3), and (4) are particular cases of a more general formula. Let V and V' be the volumes of the same body at temperatures t and t' respectively, U the volume at zero, K the coefficient of expansion, and m and m' the respective expansions between 0 and t , and 0 and t' . We then have, by formulæ (1) and (2),

$$\begin{aligned} V &= U(1+m) = U(1+Kt) \\ V' &= U(1+m') = U(1+Kt'); \end{aligned}$$

whence by division

$$\frac{V}{V'} = \frac{1+m}{1+m'} = \frac{1+Kt}{1+Kt'}, \quad (5)$$

or *the volumes of the same body at different temperatures are proportional to the factors of expansion*.

The above formulæ are evidently applicable also to linear and superficial expansions.

199. Influence of Temperature upon Density.—As the density of a substance is inversely as the volume occupied by unit mass, it follows from last section that *the densities of the same substance at different temperatures are inversely as the factors of expansion*, so that if D , D' , D_0 denote the densities at the temperatures t , t' , 0, then

$$D = \frac{D_0}{1+m} = \frac{D_0}{1+Kt}$$

$$D' = \frac{D_0}{1+m'} = \frac{D_0}{1+Kt'}$$

$$\frac{D}{D'} = \frac{1+m'}{1+m} = \frac{1+Kt'}{1+Kt}.$$

200. Correction of Specific Gravity for Temperature.—Let d be the density of a substance at temperature t° Cent., and d_0 its density at 0° C. Also, let D be the density of water at t° C., and D_4 its density at 4° C. (the temperature of maximum density). Then if the specific

gravity of the substance be computed by comparing its density with that of water at the same temperature, as in the ordinary methods of experimental determination, the specific gravity thus obtained is $\frac{d}{D}$, and has different values according to the temperature at which the determination is made.

The specific gravity as commonly given in tables is the value of $\frac{d_0}{D_4}$; that is to say, is the ratio of the density of the substance at 0° C. to that of water at the temperature of maximum density. The tabular specific gravity is easily derivable from the observed specific gravity $\frac{d}{D}$, if the law of expansion of the substance be known; for if $1+m$ be the factor of expansion of the substance from 0° C. to t° C., and $1+e$ the factor of expansion of water from 4° C. to t° C., then

$$\begin{aligned} d_0 &= d(1+m) \\ D_4 &= D(1+e). \end{aligned}$$

Hence,

$$\frac{d_0}{D_4} = \frac{d}{D} \frac{1+m}{1+e}, \text{ or the correcting factor required is } \frac{1+m}{1+e}.$$

In the case of solid bodies this correction is generally of little importance; but in determining the specific gravities of liquids, especially those which are very expansible by heat, it cannot be neglected.

201. Formulæ for the Expansion of Gases.—The volume of a gas depends both on the temperature and on the pressure to which it is subjected; hence the formulæ of expansion for this class of bodies are somewhat more complicated. Suppose we wish to find the relation between the volumes V and V' of the same mass of gas at temperatures t and t' , and under pressures P and P' respectively. Let U be the volume of the given mass of gas at pressure P and temperature t' , and let α be the coefficient of expansion of the gas.

The two volumes V and U , being under the same pressure, are proportional to the expansion factors corresponding to their temperatures (§ 198), which gives

$$\frac{V}{U} = \frac{1+\alpha t}{1+\alpha t'}.$$

The volumes U and V' , at the same temperature t' , are, by Boyle's law, inversely proportional to the pressures; whence we have

$$\frac{U}{V'} = \frac{P'}{P}.$$

From these two equations we obtain by multiplication

$$\frac{V}{V'} = \frac{P'}{P} \cdot \frac{1+at}{1+at'}, \quad (9)$$

which means that *the volumes assumed by the same mass of gas are, inversely proportional to the pressures, and directly proportional to the expansion factors corresponding to the temperatures.*

From equation (9) we may easily deduce another equation, by remarking that the densities of the same quantity of gas must be inversely as the volumes occupied. Thus we have

$$\frac{D}{D'} = \frac{P'}{P} \cdot \frac{1+at'}{1+at}, \quad (10)$$

which signifies that *the density of a gas varies directly as the pressure, and inversely as the expansion factor corresponding to the temperature.*

CHAPTER XXI.

EXPANSION OF SOLIDS.

202. Laplace and Lavoisier's Experiments.—Laplace and Lavoisier determined the linear expansion of a great number of solids by the following method.

The bar AB (Fig. 203) whose expansion is to be determined, has one end fixed at A, while the other can move freely, pushing before it the lever OB, which is movable about the point O, and carries a telescope whose line of sight is directed to a scale at some distance. It is evident that a displacement BB' will correspond to a considerably greater length CC' on the scale, which will increase with the distance

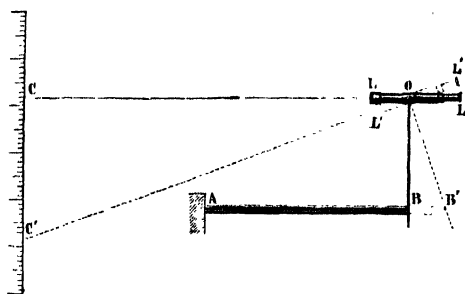


Fig. 203.
Principle of the Method of Laplace and Lavoisier.

of the scale from the telescope. The ratio of CC' to BB' is equal to the ratio of OC to OB, and is the same throughout the whole course of experiments. In order to determine this ratio once for all, we have only to measure the distance CC' on the scale corresponding to a *known* displacement BB'.

The apparatus employed by Laplace and Lavoisier is shown in Fig. 204. The trough C, in which is laid the bar whose expansion is to be determined, is placed between four massive uprights of hewn stone N. One of the extremities of the bar rests against a fixed bar B', firmly joined to two of the uprights; the other extremity pushes the bar B, which produces the rotation of the axis aa' . This axis carries with it in its rotation the telescope LL', which is directed to the

scale. The first step is to surround the bar with melting ice, and then observe the number on the scale marked by the line of sight of the

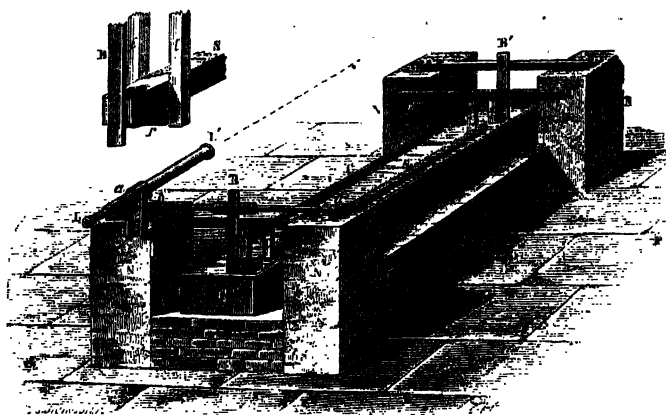


Fig. 204.—Apparatus of Laplace and Lavoisier.

telescope. The temperature of the trough is then raised, and the corresponding increase of length is measured.

Laplace and Lavoisier have discovered by this means that the expansion of solids is sensibly uniform between 0° and 100° C.; above this latter point it varies with the temperature.

The following table contains the most important results obtained by them:—

COEFFICIENTS OF LINEAR EXPANSION.

Gold, Paris standard, annealed, 0.000015153	Soft wrought iron, 0.000012204
" " unannealed, 0.000015515	Round iron, wire drawn, . . 0.000012350
Steel not tempered, 0.000010792	English flint-glass, 0.000008116
Tempered steel reheated to 65° , 0.000012395	Gold, procured by parting, . . 0.000014660
Silver obtained by cupellation, 0.000019075	Platina, 0.000009918
Silver, Paris standard, 0.000019086	Lead, 0.000088483
Copper, 0.000017173	French glass with lead, . . . 0.000008715
Brass, 0.000018782	Sheet zinc, 0.000029416
Malacca tin, 0.000019376	Forged zinc, 0.000031083
Falmouth tin, 0.000021729	

A simpler and probably more accurate method of observing expansions was employed by Ramsden and Roy. It consists in the direct observation of the distance moved by either end of the bar, by means of two microscopes furnished with micrometers, the microscopes themselves being attached to an apparatus which is kept at a constant temperature by means of ice.

203. **Compensating Pendulum.**—The pendulum, as we know, regu-

lates the motion of a clock. Suppose the clock to keep exact time at the temperature of zero; then if the temperature rises the length of the pendulum will increase, and with it the duration of each oscillation, so that the clock will lose. The opposite effect would be produced by a fall of the temperature below zero. We thus see that clocks are liable to go too fast in winter, and too slow in summer, and that we must move the bob of the pendulum from time to time in order to insure their regularity.

The effect of temperature may be notably diminished by means of compensating pendulums, of which there are several different kinds.

1. *Harrison's Gridiron Pendulum.*—This consists of four oblong frames, the uprights of which are alternately of steel F and of brass C (Fig. 205); the brass uprights rest upon the bottom of the steel frames, and to the top of the second brass frame is attached the steel rod carrying the bob. From this arrangement it follows that the effect of the lengthening of the steel rods will be to lower the bob, while that of the lengthening of the brass rods will be to raise it. It will be seen that these two effects may be made completely to neutralize each other; we have only to insure that the whole expansion of the steel shall be equal to that of the brass.¹ If L and L' be the



Fig. 205.
Plan of Gridiron Pendulum.

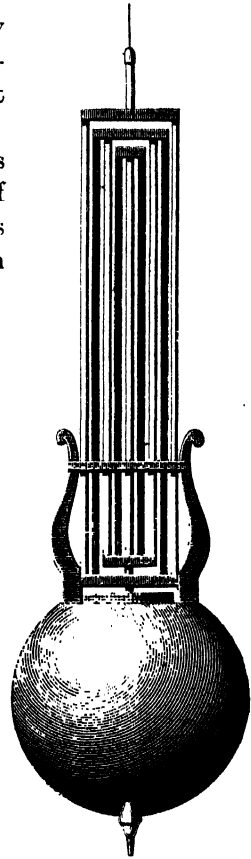
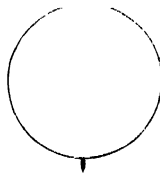


Fig. 206.
Gridiron Pendulum.

¹ As the weight of the frame cannot be altogether neglected, and the change of dimensions produced by expansion affects the moment of inertia, the condition of compensation stated in the text can only be taken as approximate.

sum of the lengths of the steel and brass rods respectively; then this neutralization will be effected if $LKt = L'K't$, or if $LK = L'K'$, where K and K' are the respective coefficients of expansion of steel and brass.

From the table on page 270 we learn that K is about $\frac{2}{3} K'$; thus the sum of the lengths of the brass rods must be about $\frac{3}{2}$ of that of the steel rods. This result shows that we must have at least *two* brass frames, for with only one the compensating effect could not be produced, as the length of the steel rods would in that case be about double that of the brass.

If we wish to have only a single frame of each different metal, we must choose two whose difference of expansion is much more marked; such, for instance, as iron and zinc, which are employed in Jurgesen's pendulum.

In order that the compensation may be complete, the centre of oscillation (§ 46) must remain at the same distance from the centre of suspension. The screw above the bob, shown in Fig. 206, enables us to attain this end by slightly raising or depressing the bob, and is intended to be used once for all to adjust the pendulum to the proper rate.

2. *Graham's Pendulum.*—

This consists of an iron rod carrying at its lower end a frame, in which are fixed one or two glass cylinders containing mercury. When the temperature rises the length-

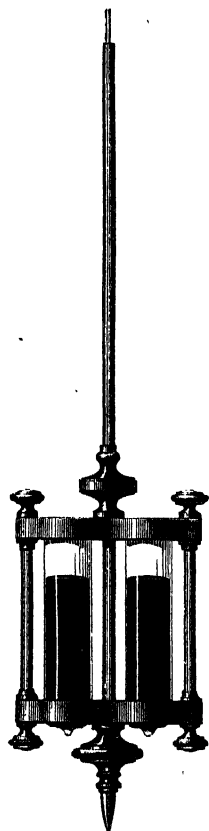


Fig. 207.—Graham's Mercurial Pendulum.

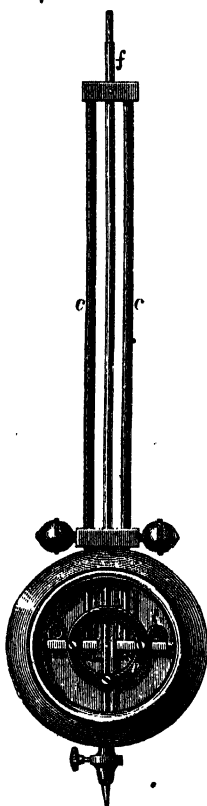


Fig. 208.
Ellicott's Pendulum.

ening of the rod lowers the centre of gravity and centre of oscillation of the whole; but the expansion of the mercury produces the contrary effect; and it will readily be understood that the quantity

of mercury in the cylinders may be such as to produce an approximately perfect compensation.

3. *Ellicott's Pendulum*.—This pendulum, which was invented in England in the last century, is known in France as Brocot's pendulum, and is frequently used in small French clocks. The main rod *f* is of iron. Attached to a cross-bar at the upper part of this rod are two brass rods *cc*, which, by means of the levers *aa* and the pins *tt*, attached to the bob, raise this latter when the temperature rises. The arms of the levers may evidently be so chosen as to maintain the centre of oscillation at an invariable distance from the axis of suspension, by means of the different expansive powers of iron and brass.

204. *Force of Expansion of Solids*.—The *force* of expansion is very considerable, being equal to the force necessary to compress the body to its original dimensions. Thus, for instance, iron when heated from 0° to 100° increases by $\cdot 0012$ of its original length. In order to produce a corresponding change of length in a rod an inch square, a force of about 15 tons would be required. It would be useless to attempt to offer any mechanical resistance to a force so enormous; the only thing that can be done, in the case of structures in which metals are employed, is to arrange the parts in such a manner that the expansion shall not be attended with any evil effects. Thus, in a railway, the rails do not touch each other, a small interval being left to allow room for the variations of length. Iron beams employed in buildings must have the end free to move forward without encountering any obstacles, which they would inevitably overthrow. Sheets of zinc and lead employed in roofing, are so arranged as to be able in a certain extent to overlap each other on expansion.

We may further remark that the expansion of metals, though relatively very small, may practically become very considerable, if the length of metal which expands is sufficiently great. Suppose we take as an instance the length of railway from London to Edinburgh, which is about 400 miles. The extreme variations of temperature from winter to summer are about 50° C., which would produce a variation of length amounting to $400 \times 5280 \times \cdot 00061 = 1288$ feet. The actual variation is very considerable, and if the rails formed a continuous line at a certain temperature, this line would be interrupted or broken in pieces upon a change of temperature occurring in either direction.

205. *Conversion of Heat into Work*.—In conclusion, we may remark

that heat when applied to a bar of metal produces two distinct and separate effects; one shown in the rise of temperature, and the other in the increase of volume. We may reasonably suppose that if the solid body were heated under such conditions as to preclude its expansion, the same quantity of heat would produce a much greater thermometric effect than in the former case. A similar remark applies to liquids and gases, and can be easily verified by experiment in the case of these latter. Here, then, is the first instance of a physical phenomenon of very frequent occurrence, namely, the conversion of heat into work, or reciprocally, of work into heat. Whenever a quantity of heat appears to be lost, the reason is that a corresponding amount of work is produced. If, on the other hand, work is done in compressing a body, so as to reduce it to the volume which it would occupy at a lower temperature, a rise of temperature is necessarily produced.

CHAPTER. XXII.

EXPANSION OF LIQUIDS.

206. Relation between Real and Apparent Expansion.—If a vessel containing a liquid be heated, the level of the liquid rises, in consequence of the excess of the expansion of the liquid over that of the vessel. The observed increase of volume, not corrected for the expansion of the vessel, is called the apparent expansion. It is evidently less than the real expansion, for if the volume of the vessel had remained the same, the level would have risen higher.

The coefficients of real and apparent expansion are connected with the coefficient of expansion of the vessel by a very simple relation.

Let us take the case of a liquid contained in a vessel similar to a thermometer in shape, that is, suppose the tube to be divided into parts of equal capacity, and that we know by previous gauging how many divisions are equivalent to the volume of the reservoir.

Let n_0 denote the number of divisions occupied by the liquid at zero, and n_t the number of divisions occupied at temperature t° .

Then $\frac{n_t}{n_0}$ is the factor of apparent expansion, and $\frac{n_t}{n_0} - 1$ is the apparent expansion.

Let us, for simplicity, take for our unit of volume the volume of a division at zero. Then if K be the expansion of the glass, the volume of a division at t° will be $1 + K$.

The volume of the liquid at t° is n_t of these divisions, and is therefore $n_t (1 + K)$.

But if m be the real expansion of the liquid, the volume at t° is $(1 + m) n_0$, since n_0 is the volume at zero.

Hence we have

$$n_t (1 + K) = (1 + m) n_0;$$

and

$$\frac{n_t}{n_0} = \frac{1 + m}{1 + K}.$$

That is, *the factor of apparent expansion is equal to the factor of real expansion of the liquid divided by that of the vessel.* Denote the factor of apparent expansion by $1 + \Delta$.

Then since from above

$$1 + \Delta = \frac{1 + m}{1 + K},$$

we have

$$1 + m = (1 + \Delta)(1 + K) = 1 + \Delta + K + \Delta K;$$

whence

$$m = \Delta + K + \Delta K;$$

or since ΔK is much smaller than either Δ or K , and may in general be neglected,

$$m = \Delta + K.$$

That is, the real expansion of the liquid is equal to the apparent expansion plus the expansion of the vessel; and consequently, *the coefficient of real expansion is equal to the coefficient of apparent expansion plus the coefficient of expansion of the vessel.*

207. Expansion of Glass.—By means of this relation we can find the coefficient of expansion of any kind of glass; we have only to measure the coefficient of apparent expansion of the mercury in a thermometer made of this glass, and to subtract it from the coefficient of absolute expansion of the metal, which, as we shall see afterwards, is equal to $\frac{1}{5550}$. The coefficient of apparent expansion varies a little according to the quality of the glass employed; if we take it as $\frac{1}{6480}$, which is Dulong and Petit's determination of its mean value, we shall have for the coefficient of expansion of glass

$$K = \frac{1}{5550} - \frac{1}{6480} = \frac{1}{38700}.$$

208. Expansion of any Liquid.—The coefficient of expansion of the glass of which a thermometer is composed being known, we may use the instrument to measure the expansion of any liquid. For this purpose, the liquid whose coefficient of expansion is to be determined is introduced into the thermometer, and the number of divisions n_0 and n_t occupied by the liquid at the temperatures 0° and t° respectively, are observed. Then, if D , Δ , K , be the coefficients of real expansion, of apparent expansion, and of expansion of the glass, each reckoned per degree Centigrade, we have

$$\frac{n_t}{n_0} - 1 = \Delta t, \text{ whence } \Delta \text{ is known; and}$$

$1 + D = (1 + \Delta)(1 + K)$, or $D = \Delta + K$ nearly, whence D is known.

M. Pierre has performed an extensive series of experiments by this method upon a great number of liquids.

The apparatus employed by him is shown in Fig. 209. The thermometer containing the given liquid is fixed beside a mercurial thermometer, which marks the temperature. The reservoir and a small part of the tube are immersed in the bath contained in the cylinder below. The upper parts of the stems are inclosed in a second and smaller cylinder, the water in which is maintained at a sensibly constant temperature indicated by a very delicate thermometer.

From these experiments it appears that the expansions of liquids are in general much greater than those of solids. Further, expansion does not proceed uniformly, as compared with the indications of a mercurial thermometer, but increases very perceptibly as the temperature rises. This is shown by the following table:—

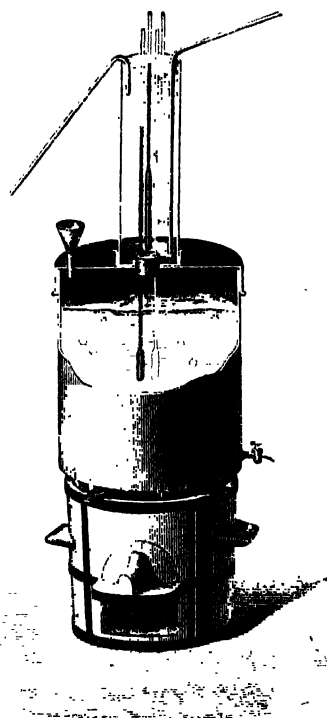


Fig. 209.—Pierre's Apparatus.

	Volume at 0°.	Volume at 10°.	Volume at 40
Water.....	1	1·000146	1·007492
Alcohol.....	1	1·010661	1·044882
Ether	1	1·015408	1·066863
Bisulphide of carbon...	1	1·011554	1·049006
Wood-spirit.....	1	1·012020	1·050509

209. Maximum Density of Water.—By applying the experimental method just described to the case of water, we may easily observe the volume occupied by the same weight of this liquid at different temperatures, and it has thus been found that this volume is least at 4° Centigrade. At this temperature, accordingly, the density of water is a maximum, so that if a quantity of water at this temperature be

either heated or cooled it undergoes an increase of volume. This is a curious and unique exception to the general law of expansion by heat.

This anomaly may be exhibited by means of the apparatus represented in Fig. 210, which consists of two thermometers, one of alcohol, and the other of water. The reservoir of this latter, on account of the smaller expansive power of the liquid, is a long spiral,

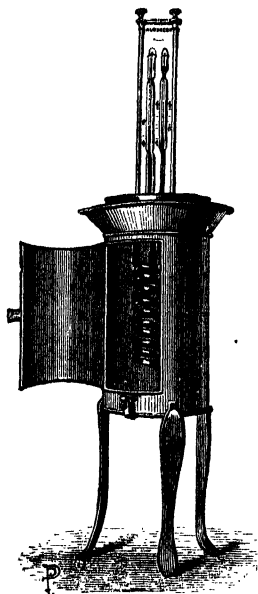


Fig. 210.—Maximum Density of Water.

enveloping that of the alcohol thermometer. Both the reservoirs are contained in a metal box, which is at first filled with melting ice. The two instruments are so placed, that at zero the extremities of the two liquid columns are on the same horizontal line. This being the case, if the ice be now removed, and the apparatus left to itself, or if the process be accelerated by placing a spirit-lamp below the box, the alcohol will immediately be seen to rise, while the water will descend; and the two liquids will thus continue to move in opposite directions until a temperature of about 4° is attained. From this moment the water ceases to descend, and begins to move in the same direction as the alcohol. This experiment, although very well adapted for exhibiting the phenomenon, does not enable us to measure exactly the temperature of maximum density, since it is the apparent, and not the real, expansion of water which

is thus observed. The following experiment, which is due to Hope, is more rigorous.

A glass jar is employed, having two lateral openings, one near the top and the other near the bottom, which admit two thermometers placed horizontally. The tube is filled with water, and its middle is surrounded with a frigorific mixture. The following phenomena will then be observed.

The lower thermometer descends steadily to 4° , and there remains stationary. The upper thermometer at first undergoes very little change, but when the lower one has reached the fixed temperature, the upper one begins to fall, reaches the temperature of zero, and, finally, the water at the surface freezes, if the action of the frigorific

mixture continues for a sufficiently long time. These facts admit of a very simple explanation.

As the water in the middle portion of the tube grows colder its density increases, and it sinks to the bottom. This process goes on till all the water in the lower part of the vessel has attained the temperature of 4° . But when all the water from the centre to the bottom has attained this temperature, any further cooling of the water in the centre fails to produce motion, until needles of ice are formed. These being specifically lighter than water, rise to the surface, and thus produce a circulation which causes the water near the surface to freeze, while that near the bottom remains at the temperature of 4° .

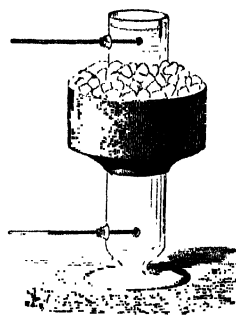


Fig. 211.
Hope's Experiment.

This experiment represents on a small scale what actually takes place during winter in pools of fresh water. The fall of temperature at the surface does not extend to the bottom of the pool, where the water, whatever be the external temperature, seldom falls below 4° . This is a fact of great interest, as exemplifying the close connection of natural phenomena, and the manner in which they contribute to a common end. It is in virtue of this anomaly exhibited by water in its expansion, taken in conjunction with the specific lightness of ice and the low conducting power of water, that the temperature at the bottom of deep pools remains moderate even during the severest cold, and that the lives of aquatic animals are preserved.

210. Saline Solutions.—In the case of saline solutions of different densities, the temperature of maximum density falls along with the freezing-point, and in fact falls more rapidly than this latter, so that for solutions containing a certain proportion of salt the temperature of maximum density is below the freezing-point. In order to show this experimentally, the solution must be placed in such circumstances as to remain liquid at a temperature below its freezing-point. This is a curious example of the continuity of physical laws, and of the restriction which must be applied in physics to the generally correct and logical principle of final causes. For salt water has a point of maximum density, although before that point is reached congelation takes place. In this instance the maximum density plays no part in the designs of nature, and leads to no practical benefit, being simply

a proof of the permanence of a physical law, even when the circumstances on which its utility depended have disappeared.

211. Law of the Expansion of Liquids.—As we have shown above. (§ 208), the expansion of liquids does not advance uniformly with the temperature; whence it follows that the mean coefficient of expansion will vary according to the limiting temperatures between which it is taken.

From a general review of the researches which have been made on this subject, it appears that for a great number of liquids the mean coefficient of expansion increases uniformly with the temperature. If, therefore, Δ be the expansion from 0 to t , we have

$$\frac{\Delta}{t} = a + bt, \text{ whence } \Delta = at + bt^2,$$

a and b being two constants specifying the expansibility of the given liquid.

For some very expansible liquids two constants are not sufficient, and the expansion is represented by the formula

$$\Delta = at + bt^2 + ct^3.$$

We subjoin a few instances of this class taken from the work of M. Pierre:—

Alcohol.....	$\Delta = 0.0010486 t + 0.0000017510 t^2 + 0.00000000134518 t^3$
Ether.....	$\Delta = 0.0015132 t + 0.0000023592 t^2 + 0.000000040051 t^3$
Bisulphide of carbon....	$\Delta = 0.0011398 t + 0.0000013707 t^2 + 0.00000019123 t^3$
Bromine ...	$\Delta = 0.0010382 t + 0.0000017114 t^2 + 0.0000000054471 t^3$

An examination of the formulæ for the different liquids which have been tested, shows that none of them have a point of maximum density; this property remains peculiar to water.

212. Absolute Expansion of Mercury.—The great importance of mercury in physical experiments, especially in connection with the barometer and thermometer, render it necessary to determine very accurately the coefficient of expansion of this liquid. This determination has been effected by Dulong and Petit by a very ingenious method, in which the observation of the heights of liquid columns is substituted for the measurement of volumes, which is always open to some uncertainty.

A and B are two tubes containing mercury, and communicating with each other by a very narrow tube CD. If the temperature of the liquid be uniform, the mercury should stand at the same height

in both branches, according to the fundamental law of liquids in communicating vessels. But if the tube AC, for instance, be kept at 0° Cent., and the temperature of BD be raised, the density of the heated liquid will become less, and a greater height will consequently be required to equilibrate the pressure of the other column.

Suppose the horizontal tube divided by a vertical section, and let h and h' denote the respective heights of the liquid in the cold and in the heated branches above the centre of gravity of this section. As the pressures on both sides must be equal, we must have the equation $hd = h'd'$, where d and d' are the densities of the mercury at zero and at the other given temperature.

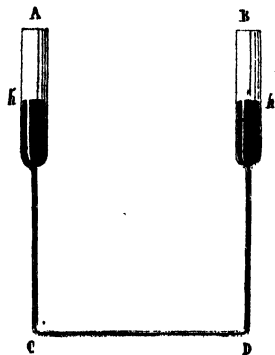


Fig. 212.—Principle of Dulong's Method.

But from what has been said above (§ 199), we have $d' = \frac{d}{1+m}$, where m is the expansion of the liquid from 0° to t° ; and consequently, $hd = \frac{h'd}{1+m}$, whence

$$m = \frac{h' - h}{h}.$$

We thus see that it is sufficient to measure exactly the heights h' and h in order to find m .

We should remark, that the centre of gravity of the liquid section which we have been considering, is the only point of it which is in equilibrium; the upper part is subject to a greater pressure on the side of the heated liquid, and the lower part on the side of the cold; there is thus a tendency to the production of two opposite currents, which, however, is almost completely destroyed by the smallness of the bore of the connecting tube. Even supposing these currents actually to exist, the inverse effects produced by them would sensibly compensate each other, compared with the heights of the liquid in the two branches.

The following is the method by which Dulong and Petit have carried out the above principle.

The connecting tube, between the two branches A and B of the apparatus (Fig. 213), rests upon a T-shaped iron support, carrying two spirit-levels at right angles to each other for insuring horizontality. One of the branches B is inclosed in a cylinder containing

melting ice; the other A is surrounded by a copper cylinder filled with oil, and heated by a furnace connected with the apparatus. In making an observation the first step is to arrange the apparatus so

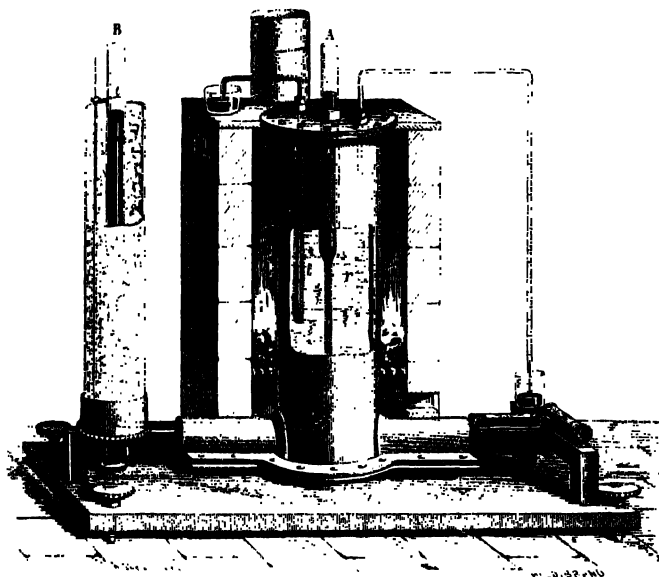


Fig. 213.—Apparatus of Dulong and Petit.

that, when the oil is heated to the temperature required, the mercury in the tube A may just be seen above the top of the cylinder, so as to be sighted with the telescope of a cathetometer; this may be effected by adding or taking away a small quantity of oil. The extremity of the column B is near or near which gives the difference of the heights h' and h , which remains peculiar to water, to determine the absolute height h . **Expansion of Mercury.**—The

For this purpose the experiments, especially in t above the surface of the T-shaped sup^{er} thermometer, render it necessary to be measured. From this height half the external of expansion of t horizontal tube is to be subtracted, and it only remains by Dulong and the distance of the end of the column B from the main observation of t of the experiment.

The temperature of the oil is given by the weight-thermometer t , and by the air-thermometer r , which latter we shall explain hereafter.

By means of this method Dulong and Petit ascertained that the expansion of mercury is sensibly uniform, as compared with the

indications of an air-thermometer, between 0° and 100° C. Above this point it goes on increasing like other liquids, but not in any marked degree. Thus, the mean coefficient between 0° and 100° is $\frac{1}{5550}$. Between 0° and 200° it becomes $\frac{1}{5425}$, and $\frac{1}{5300}$ between 0° and 300° .

Regnault, without altering the principle of the experiments of Dulong and Petit, introduced several improvements into their apparatus, and added greatly to the length of the tubes A and B, thereby rendering the apparatus more sensitive. The results obtained by him do not differ very materially from those of Dulong and Petit; thus, for instance, he makes the coefficient between 0° and 100° equal to $\frac{1}{5509}$.

His experiments show that the mean coefficient between 0° and 50° is $\frac{1}{5547}$, a number almost identical with $\frac{1}{5550}$.

213. Expansion of Iron and Platinum.—The coefficient of absolute expansion of mercury being known, that of glass is deduced from it in the manner already indicated (§ 207). Dulong and Petit have deduced from it also the coefficients of expansion of iron and platinum, neither of which metals are attacked by mercury. The method employed is the following.



Fig. 214.—Expansion of Iron and Platinum.

The metal in question was introduced, in the shape of a cylindrical bar, into the reservoir of a weight-thermometer. Let W be the weight of the metal introduced, and D its density at zero. The process is the same as in using the weight-thermometer; that is, after having filled the reservoir with mercury at 0° C., we observe the weight w of the metal which issues at a given temperature t . The volume at 0° C. of the mercury which has issued, is $\frac{w}{d}$, d being the density of mercury at zero; the volume at t° is therefore $\frac{w}{d} (1 + mt)$, m being the coefficient of expansion of mercury. This volume evidently represents the expansion of the metal, *plus* that of the mercury, *minus* that of the glass. If then M denote the weight of mercury that fills the apparatus at 0° C., and if K be the coefficient

of cubical expansion of glass, and α the expansion of unit volume of the given metal, we have the equation

$$\frac{w}{d}(1+mt) = \frac{W}{D}\alpha + \frac{M}{d}mt - \left(\frac{W}{D} + \frac{M}{d}\right)\kappa t,$$

whence we can find α .

214. Convection of Heat in Liquids.—When different parts of a liquid are heated to different temperatures, corresponding differences of density arise, leading usually to the formation of currents which tend to produce equality of temperature as far as their presence extends. To this phenomenon, which is altogether distinct from conduction, the name of *convection* is given.

Thus, for instance, if we apply heat to the bottom of a vessel containing water, the parts immediately subjected to the action of the heat expand and rise to the surface; they are replaced by colder layers, which in their turn are heated and ascend; and thus the process continues indefinitely. The two currents can be very well shown by throwing some oak saw-dust into the water. By the movements of this substance, which has nearly the same density as water, it will be seen that the ascending current occupies the centre of the vessel, while the descending current passes along the sides.

215. Heating of Buildings by Hot Water.—This is a simple application of the principle just stated. One of the most common arrangements for this purpose is shown in Fig. 215; it is called the low-pressure system, because the temperature never exceeds 100° C. The boiler C is heated by a fire below it, and the products of combustion escape through the chimney AB. At the top of the house is a reservoir D, communicating with the boiler by a tube. From this reservoir the liquid flows into another reservoir E in the story immediately below, thence into another reservoir F, and so on. Finally, the last of these reservoirs communicates with the bottom of the boiler. The boiler, tubes, and reservoirs are all completely filled with water, with the exception of a small space left above in order to give room for the expansion of the liquid. An ascending current flows through the left-hand tube, and the circulation continues with remarkable regularity, so long as the temperature of the water in the boiler remains constant.

216. Currents in the Sea.—In the production of these currents convection plays an important, though perhaps not the principal part. In fact, the sea is an enormous mass of liquid whose temperature varies from point to point. Equilibrium is consequently impossible,

and the different parts must therefore be in a state of continual motion with regard to each other. The waters of the tropical seas should, by reason of their excess of temperature, have a higher level than those of the polar seas; and the result is, a continual kind of overflowing of the waters about the equator, and consequently a vast current setting towards the poles. But to this current evidently corresponds a lower current of cold water flowing towards the equator, there to become heated, to overflow again, and so on. One of the most remarkable of oceanic currents is that which is known as the *Gulf Stream*. This current of warm water forms a kind of immense river in the midst of the sea, differing in the temperature, saltness, and colour of its waters from the medium in which it flows. Its origin is in the Gulf of Mexico, whence it issues through the straits between the Bahamas and Florida, turns to the north-west, and splits into two branches, one of which goes to warm the coasts of Ireland and Norway, the other gradually turns southwards, traverses the Atlantic from north to south, and finally loses itself in the regions of the equator.

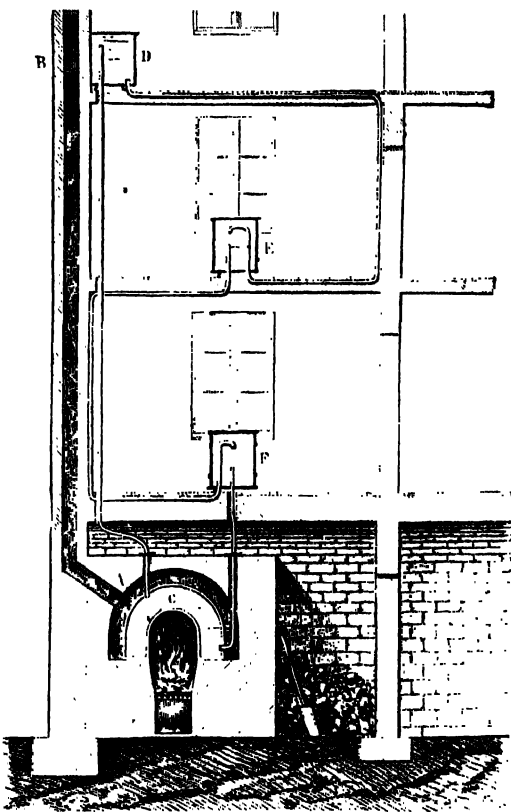


Fig. 215.—Heating by Hot Water.

"The Gulf Stream is a river in the ocean; in the severest droughts it never fails, and in the mightiest floods it never overflows; its banks and its bottom are of cold water, while its current is of warm; it takes its rise in the Gulf of Mexico, and empties into Arctic seas. There is on earth no other such majestic flow of waters. Its current

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is more rapid than the Mississippi or the Amazon, and its volume more than a thousand times greater. Its waters, as far out from the Gulf as the Carolina coasts, are of indigo blue. They are so distinctly marked that their line of junction with the common sea-water may be traced by the eye. Often one-half of the vessel may be perceived floating in Gulf Stream water, while the other half is in common water of the sea, so sharp is the line."—(Maury, *Physical Geography of the Sea*.)

Another cause of oceanic currents is to be found in the winds, which again are themselves examples of convective currents in the atmosphere. In the case of the Gulf Stream, it would appear that an accumulation of water is produced in the Gulf of Mexico by the trade-wind which blows steadily towards it over the South Atlantic. The elevation of level occasioned by this accumulation is probably to be regarded as the principal cause of the Gulf Stream. We shall discuss the origin of winds in a later chapter (Chap. xxxiv.)

CHAPTER XXIII.

EXPANSION OF GASES.

217. Experiments of Gay-Lussac.—Gay-Lussac conducted a series of researches on the expansion of gases, the results of which were long regarded as classical. He employed a thermometer with a large reservoir A, containing the gas to be operated on; an index of mercury *mn* separated the gas from the external air, while leaving it full liberty to expand. The gas had previously been dried by pass-

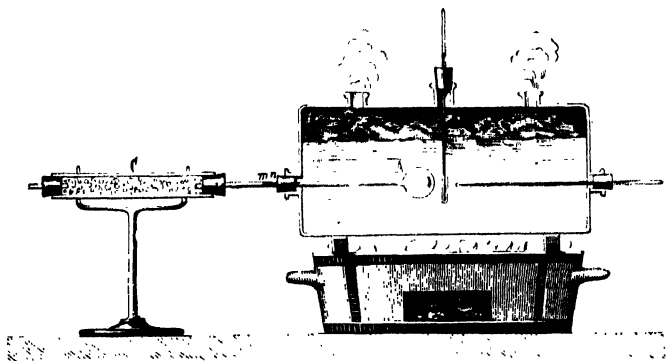


Fig. 216.—Gay-Lussac's Apparatus.

ing it through a tube containing chloride of calcium, or some other desiccating substance. The thermometer was fixed in a vessel which was first filled with melting ice, and when the gas had thus been brought to 0° C., the tube was so adjusted that the index coincided with the opening through which the thermometer passed.

The tube being divided into parts of equal capacity, and the reservoir having been previously gauged, the volume V is known which is occupied by the gas at an external pressure H indicated by a barometer; the apparatus is then raised to a given temperature T by

means of the furnace below the vessel, and the stem of the thermometer is moved until the index reaches the edge of the opening; at this new temperature the gas occupies a volume V' expressed in divisions of the tube: at the same time the pressure may have varied; suppose it to have become H' . From these data it is easy to deduce the expansion of unit volume of the gas from 0° to T at constant pressure. If D denote this expansion, the volume of the gas at T at the original pressure would be $V(1+D)$. But the gas occupies a volume V' at the temperature T and pressure H' . At the pressure H the volume would therefore be $V' \frac{H'}{H}$. But the divisions of the thermometer have expanded in the ratio $1+KT$, K being the coefficient of expansion of glass: the true expression therefore for the new volume of the gas at the pressure H is $\frac{V'(1+KT)H'}{H}$; whence we have the equation

$$V(1+D) = \frac{V'(1+KT)H'}{H},$$

from which we can find the value of D , and consequently that of the mean coefficient of expansion $\frac{D}{T}$. By means of this method Gay-Lussac arrived at the following results:—

1st.—All gases expand by the same amount between the same limits of temperature.

2d.—The coefficient of expansion is independent of the pressure. He also found that the coefficient of expansion of air between 0° and 100° was .00375.

These laws, which, together with Boyle's law, were long regarded as defining the fundamental properties of the gaseous state, are not rigorously exact, but are subject to restrictions similar to those which apply to Boyle's law. The absolute value of the co-efficient of expansion of air as laid down by Gay-Lussac is very sensibly erroneous. The true value has been determined, by subsequent experiments conducted with greater precision, to be .003665, or $\frac{1}{273}$.

218. Regnault's Experiments.—The apparatus employed by Gay-Lussac had one serious imperfection. The mercurial index did not constitute a sufficient barrier between the gas under investigation and the external air; so that a portion of the gas was able to escape, while at the same time some of the external air became mixed with the gas; either of which circumstances would impair the accuracy of the experiment. It also appears that the means employed by Gay-Lussac

for desiccating the gas were insufficient. However this may be, the subject of the expansion of gases has been taken up by several physicists, as Pouillet, Rudberg, Magnus, and Regnault, who have performed a number of experiments on this subject, of undoubted accuracy, the result of which has been slightly to modify the conclusions arrived at by Gay-Lussac.

We shall confine ourselves to describing one of the methods employed by Regnault.

The apparatus consists of a glass ball with a narrow neck, containing the gas. This is placed in a boiler (Fig. 217), containing water,

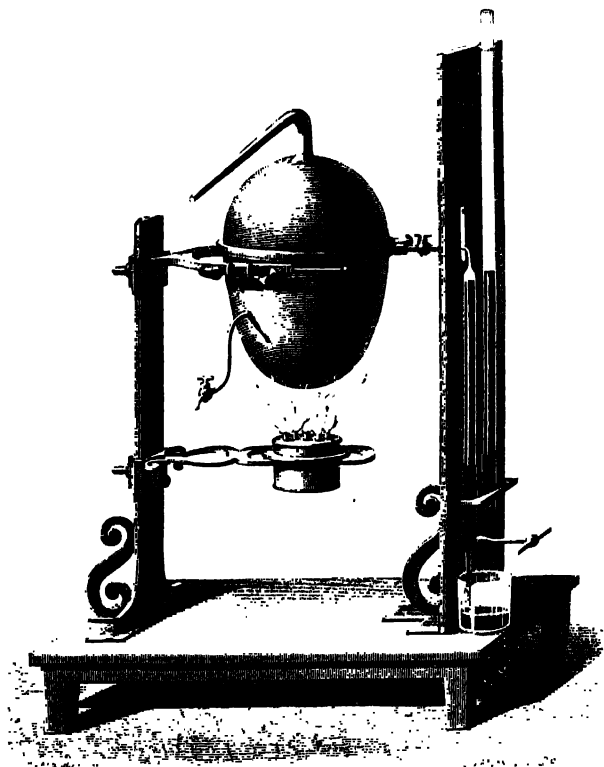


Fig. 217.—Regnault's Apparatus.

which can afterwards be raised to ebullition. A T-shaped tube, with three branches, establishes communication between the neck of the globe and a system, consisting of two tubes, containing mercury, and forming in fact a mercurial manometer, and also between the globe and a series of desiccating tubes, not shown in the figure,

which are themselves in communication with a small air-pump. On the first branch of the manometer, near the capillary portion of the tube, is a reference mark.

The following is the mode of determining by means of this apparatus the coefficient of expansion between 0° and 100° C. The first step is to exhaust the globe a certain number of times, each time refilling it with air, or with the gas under investigation, which has been dried by passing it through the desiccating tubes. The drying may be rendered more complete and rapid by raising the temperature of the globe. This series of operations is, as Regnault has shown, absolutely necessary in order to remove from the surface of the glass the last traces of moisture, which are exceedingly tenacious.

The gas having been admitted for the last time, the globe is surrounded with melting ice, and is left to itself. The gas contracts, and a fresh portion enters the globe, having first been perfectly dried by passing through the desiccating tubes. The apparatus is thus filled with gas, and communication is established for a few seconds with the external air, so that the gas is at atmospheric pressure. Mercury is then poured into the manometer so as to bring the level of the fluid, which is the same in both branches, up to the reference mark. The branch of the T which established communication between the globe and the desiccating tubes is then hermetically sealed by directing the flame of a blowpipe upon it.

The effect of this operation has thus been to isolate a quantity of gas at the atmospheric pressure H . This quantity consists of—

(1.) A volume V at the temperature of 0° C., V being the known volume of the globe.

(2.) A volume v , which is very small, extending from the neck of the globe to the mark on the manometric tube. This volume v is at the surrounding temperature t ; if brought to zero, it would become $\frac{v}{1+at}$, a being the coefficient of expansion of the gas. In the first part of the experiment, therefore, we have a quantity of gas which, when reduced to the temperature of zero and pressure H , would have the volume

$$V + \frac{v}{1+at}.$$

The ice surrounding the globe is now removed, the boiler is filled with water, which is heated to ebullition; the volume and pressure of the gas increase, the mercury consequently falls in the first branch of the manometer, and rises in the other. When equilibrium of tem-

perature has been established, mercury is poured into the open branch, so as to raise that in the other branch to the mark. There is then found to be a difference of level h : the external pressure may have changed at the same time, and become H' ; the gas is consequently subjected to a pressure $H' + h$. Its volume consists of two parts:—

(1.) The volume $V(1+KT)$ of the globe, K denoting the coefficient of expansion of the glass, and T the temperature of the boiling water at the moment of the experiment; this volume when reduced to zero becomes

$$\frac{V(1+KT)}{1+\alpha T}.$$

(2.) The volume v of the tube as far as the mark, which volume at zero becomes $\frac{v}{1+\alpha t'}$, where t' is the surrounding temperature. Thus, in the second part of the experiment, the given quantity of gas under the pressure $H' + h$, would, at the temperature of zero, occupy the volume

$$\frac{V(1+KT)}{1+\alpha T} + \frac{v}{1+\alpha t'}.$$

We have thus, by expressing that the volumes are inversely as the pressures,

$$(H' + h) \left\{ \frac{V(1+KT)}{1+\alpha T} + \frac{v}{1+\alpha t'} \right\} = \left(V + \frac{v}{1+\alpha t} \right) H,$$

whence

$$1 + \alpha T = \frac{V(1+KT)}{\left(V + \frac{v}{1+\alpha t} \right) \frac{H}{H' + h} - \frac{v}{1+\alpha t'}}.$$

To solve this equation we have recourse to a method frequently employed in physics, which is called the *method of successive approximations*; v being a very small quantity, is at first supposed to be zero; on this supposition the value of α is easily obtained. This value is then substituted in the correcting terms $\frac{v}{1+\alpha t}$, $\frac{v}{1+\alpha t'}$, whence the real value of αT is deduced. Now T is the temperature at which the water boils, and is always known, as we shall see hereafter, if the external pressure is known.

In the experiment just described, the volume of the gas remains sensibly the same, and the effect of heat is shown by an increase of pressure. We might have proceeded differently, and caused the gas to expand under a constant pressure.

We shall not stop to describe the modified form of the apparatus

which is adapted to this other mode of experiment; we shall only remark that the results obtained by the two processes do not exactly agree, as will be seen from the following table:—

COEFFICIENTS OF EXPANSION PER DEGREE CENTIGRADE.

	From experiments at constant volume.	From experiments at constant pressure.
Air.....	0·003665	0·003670
Nitrogen.....	0·003668	
Hydrogen.....	0·003667	0·003661
Carbonic oxide.....	0·003667	0·003669
Carbonic acid.....	0·003688	0·003710
Nitrous oxide.....	0·003676	0·003720
Cyanogen.....	0·003829	0·003877
Sulphurous acid.....	0·003845	0·003903

This table shows that each gas has its own coefficient of expansion, as we have already seen that each has its own coefficient of compressibility. Non-liquefiable gases, however, have nearly the same coefficient of expansion, a result which accounts for the conclusion arrived at by Gay-Lussac.

As regards the differences between the two sets of numbers, it is to be noted that the second set alone represent expansion as directly observed. The first set directly measure the increase of pressure which occurs when expansion is prevented. If Boyle's law were rigorously true, the two sets of results ought to be identical. In point of fact, it will be remarked that, except in the case of hydrogen, the numbers in the second column are larger than those in the first, indicating that the product of volume and pressure diminishes slightly as the pressure increases.

We may add that the coefficient of expansion increases very sensibly with the pressure; thus, between the pressures of one and of three atmospheres the coefficient of expansion of air varies from 0·00367 to 0·00369. This increase is still more marked in the case of liquefiable gases.

The coefficient of expansion per degree Fahrenheit is $\frac{5}{9}$ of the coefficient per degree Centigrade. For air or any non-liquefiable gas this may be taken as 0·002036 or $\frac{1}{494}$.

219. Air-thermometer.—We have already stated, in connection with the mercurial thermometer, that the name *degree* (Centigrade) is given to the hundredth part of the apparent expansion of the mercury in the glass. As the different kinds of glass employed in the construction of these instruments have not the same law of

expansion, it follows, as we have remarked, that mercurial thermometers are not rigorously comparable with each other, particularly above 100° .

The expansion of air being much more considerable than that of mercury, the variations caused by differences in the glass will be relatively less; for this reason (as well as for others which will be stated hereafter) the air-thermometer is employed to measure temperature in experiments of precision.

Any apparatus that will measure the expansion of air may serve as an air-thermometer; we have only to consider T as the unknown quantity in the expression $1 + \alpha T$, writing for α the value of the coefficient of expansion given in last section.

M. Pouillet proposed to employ air in pyrometric investigations, and constructed an instrument to which he gave the name of air-pyrometer, and with which he performed some interesting experiments. Pouillet's pyrometer was very similar to Regnault's apparatus described above (§ 218); but the reservoir and part of the tube were of platinum, so as to be able to resist high temperatures. The indications of this instrument, however, could not be entirely relied on, since platinum has the property of condensing air upon its surface, and partially giving it out at high temperatures. At such temperatures, also, platinum becomes quite permeable to some of the gases of the furnace. For pyrometric purposes it is better to inclose the air in a porcelain vessel, as has been done by Deville and Troost.¹

219A. Absolute Temperature. Absolute Zero.—If the air-thermometer be made the standard of temperature, equal differences of temperature will correspond to equal differences in the volume occupied by a given mass of air at constant pressure, the difference amounting to $\frac{1}{273}$ of the volume at 0° C. for each degree Centigrade, or to $\frac{1}{459}$ of the volume at 32° F. for each degree Fahrenheit.

¹ The following account of a pyrometer constructed by Regnault, on the principle of the air-thermometer, is given in Dr. B. Stewart's treatise on heat. "There is a kind of flask, either cylindrical or spherical, which may be either of cast or wrought iron, of platinum or of porcelain: the mouth is closed by a plate containing a small aperture. From 15 to 20 grammes of mercury are added to this flask, which is then placed in that part of the furnace the temperature of which we desire to know. The mercury soon boils, its vapour expels the air by the orifice, and the excess of mercurial vapour goes off by the same means. When the apparatus has acquired the temperature of the furnace the flask is withdrawn and made to cool rapidly, and the mercury which remains in the flask is weighed. It may be weighed directly, or if it contains impurity, it is dissolved in acid, and estimated as a precipitate. This weight is that of the vapour of mercury which filled the flask at the temperature of the furnace, and the volume of the flask, as well as the density of mercurial vapour being known, this temperature may thus be determined."

The lowest temperature that could thus be expressed is evidently -273° C. or -459° F., since at this temperature the given mass of air would be reduced to a mathematical point. This is often called the *absolute zero* of temperature, and temperatures reckoned from it are called *absolute temperatures*.

If C and F denote temperatures Cent. and Fahr. respectively, $273 + C$ and $459 + F$ will be the corresponding expressions for absolute temperature.

The statement of the relations between the volume, pressure and temperature of a given mass of air or other gas can be somewhat simplified by the employment of this term. These relations (assuming the correctness of Boyle's and Gay-Lussac's laws) are as follows:—

1. The volume varies directly as the absolute temperature, when the pressure is constant.

2. The pressure varies directly as the absolute temperature, when the volume is constant.

3. For all variations, the expression $\frac{VP}{T}$ remains constant in value, V denoting volume, P pressure, and T absolute temperature.

The subject of this section will be further discussed in Chap. xxxii.

220. Density of Gases.—The volume of gases, owing to their great expansibility and compressibility, is subject to enormous variations. Hence, in stating the *absolute density* of a gas, it is very important to specify the temperature and pressure at which we suppose it to be taken.

In stating the *specific gravity* or *relative density* of a gas as compared with dry atmospheric air, which is always adopted as the standard substance, it is to be understood that the gas is at the same temperature and pressure as the air with which it is compared; and, in consequence of the inexactness of the laws of Boyle and Gay-Lussac, it is further necessary, for purposes of accuracy, to specify the temperature and pressure at which the comparison is made. These are usually the temperature 0° C., and the pressure of 760 millimetres. The specific gravity or relative density of a gas is therefore defined as *the ratio of the weight of any volume of the gas at the temperature 0° C., and the pressure of 760 millimetres, to the weight of the same volume of dry air at the same temperature and pressure.*

This ratio being known, we can deduce from it the weight of any volume of the gas in question; by employing as a factor the weight

of unit volume of air (§ 100). Thus, for instance, the ratio of the density of oxygen to that of air being 1.1056, and the weight of a litre of air at 0° C. and 760 millimetres being 1.293 gramme, we conclude that the weight of a litre of oxygen at this temperature and pressure is $1.293 \times 1.1056 = 1.429$ gramme.

221. Measurement of the Density of a Gas.—The densities of gases have been the subject of numerous investigations; we shall here mention only the ingenious and exact method employed by Regnault. The gas is inclosed in a globe, of about 12 litres' capacity, and furnished with a stop-cock. This communicates with a three-way tube, furnished with stop-cocks *a* and *b* (Fig. 218), and through it with an air-pump on one side, and a manometer on the other. The globe is exhausted several times, and each time the gas is dried on its way to the globe by passing through a number of tubes containing pieces of pumice-stone moistened with sulphuric acid. When all moisture is supposed to be removed, the globe is surrounded with melting ice,

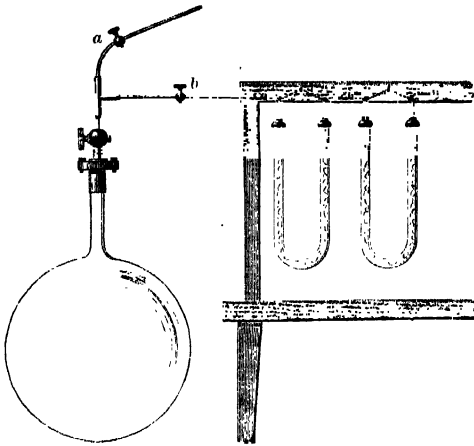


Fig. 218.—Measurement of Density of Gases.

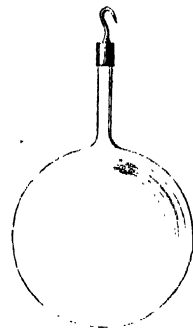


Fig. 219.—Compensating Globe.

and is allowed to fill with gas at the pressure of the atmosphere. When equilibrium of temperature has been established, the globe is taken out, carefully dried, and suspended from one of the scales of a balance. From the other scale is suspended a globe of the same glass, and of the same external volume (Fig. 219). The equality of the volumes is tested by weighing each in water, and noting the upward pressure of the liquid in each case. Weights are now added until equilibrium is established; and it can be proved that this equilibrium will be rigorously maintained, whatever be the variations of

external pressure and temperature, because these variations will produce the same effect upon both globes. It is this introduction of a *compensating globe* which gives Regnault's method its great precision;¹ for since all external causes that could disturb equilibrium are balanced, the observed differences in weight can result only from variations in the gas inside.

The globe is then again surrounded with ice, communication with the air-pump and the manometer is established, and a partial vacuum is produced as far as a certain limit h . On the globe being again suspended from the scale, the equilibrium of the balance is disturbed, and the weight w , necessary to re-establish it, is evidently equal to the weight of a volume of dry gas at 0° , and at the pressure $H-h$, H being the external pressure. Hence it follows that the weight of dry gas which would completely fill the globe at the temperature of 0° , and at the pressure of 760 millimetres, is

$$w \frac{760}{H-h}.$$

The same experiment, when performed with air, would give as the weight of the same volume of this gas under the same conditions

$$w' \frac{760}{H'-h'}.$$

The relative density of the given gas is therefore

$$w \frac{760}{H-h} \div w' \frac{760}{H'-h'} = \frac{w}{w'} \cdot \frac{H'-h'}{H-h}.$$

222. Weight of a Litre of Air.—The preceding experiments give the weight of dry air which fills a given globe at 0° , and at a pressure of 760 millimetres. In order to know the weight of a litre of air, we have only to observe the weight of water which fills the same globe at a given temperature. Let m be the difference of weight of the globe when filled with water and with dry air; then the weight of the water contained in the globe is evidently m *plus* the weight of the dry air, which is previously known, and which we shall denote by a . Let x be the volume of the globe, and e the expansion of the water from 4° C. to the temperature of weighing. Then, if the gramme and cubic centimetre be the units of weight and volume, we have the equation

$$x \frac{1}{1+e} = m + a,$$

¹ The same device had previously been employed by Dr. Prout in determining the weight of air.

which gives the volume of the globe at a known temperature, whence the volume at zero may be deduced.

Various minute precautions are necessary in order to fill the globe with water completely free from air, and in order to insure that the temperature shall be the same throughout the whole of the considerable volume employed in the experiment. The first condition is especially difficult to fulfil; in order to attain it, Regnault first expelled the air from the globe by introducing a small quantity of water, and then exhausting the globe, the process being aided by a slight elevation of temperature; on the other hand, water, from which the air had been expelled by boiling, was forced by the pressure of steam into a tube leading to the stop-cock of the exhausted globe, so as to be nowhere exposed to the atmosphere. The difficulties of this process were skilfully overcome by Regnault, and he finally arrived at the following result.

In Paris, at a height of 60 metres above the level of the sea, a litre of dry air, at the temperature of 0° C., and a pressure of 760 millimetres, weighs 1.2932 gramme. A pressure of 760 millimetres of mercury has not the same effective value at different parts of the globe, on account of the variations in the intensity of gravity, whence it follows that the weight of the litre of air, defined by the preceding conditions, varies proportionally to the value of g . (See note to § 100.)

The following table gives the densities of several gases at 0° C., and under the pressure of 760 millimetres at Paris:—

Name of Gas.	Relative Density.	Absolute Density in grammes per litre
Air	1	1.2932
Oxygen	1.10563	1.4298
Hydrogen06926	.8957
Nitrogen97137	1.25615
Chlorine	2.4216	3.1328
Carbonic oxide9569	1.2344
Carbonic acid	1.52901	1.9774
Protoxide of nitrogen	1.5269	1.9697
Binoxide of nitrogen	1.0388	1.3434
Sulphurous acid	2.1930	2.7289
Cyanogen	1.8064	2.3302
Marsh-gas559	.727
Olefiant gas985	1.274
Ammonia5967	.7697

223. Draught of Chimneys.—The expansion of air by heat produces the upward current in chimneys, and an approximate expression for the velocity of this current may be obtained by the applica-

tion of Torricelli's theorem on the efflux of fluids from orifices (Chap. xviii.).

Suppose the chimney to be cylindrical and of height h . Let the air within it be at the uniform temperature t' Centigrade, and the external air at the uniform temperature t . According to Torricelli's theorem, the square of the linear velocity of efflux is equal to the product of $2g$ into the head of fluid, the term *head of fluid* being employed to denote the *pressure* producing efflux, *expressed in terms of depth of the fluid*.

In the present case this head is the difference between h , which is the height of air within the chimney, and the height which a column of the external air of original height h would have if expanded upwards, by raising its temperature from t to t' . This latter height is $h \frac{1+at'}{1+at}$; a denoting the coefficient of expansion .00366; and the head is

$$h \frac{1+at'}{1+at} - h = \frac{ha.(t' - t)}{1+at}.$$

Hence, denoting by v the velocity of the current up the chimney, we have

$$v^2 = \frac{2gha.(t' - t)}{1+at}.$$

This investigation, though it gives a result in excess of the truth, from neglecting to take account of friction and eddies, is sufficient to explain the principal circumstances on which the strength of draught

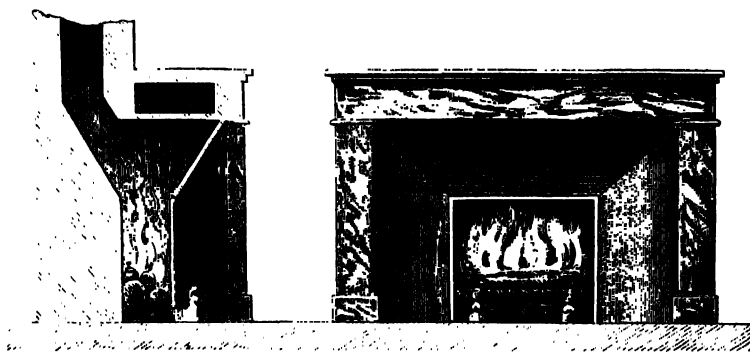


Fig. 220.—Rumford's Fire-place.

depends. It shows that the draught increases with the height h of the chimney, and also with the difference $t' - t$ between the internal and external temperatures.

The draught is not so good when a fire is first lighted as after it has been burning for some time, because a cold chimney chills the air within it. On the other hand, if the fire is so regulated as to keep the room at the same temperature in all weathers, the draught will be strongest when the weather is coldest.

The opening at the lower end of the chimney should not be too wide nor too high above the fire, as the air from the room would then enter it in large quantity, without being first warmed by passing through the fire. These defects prevailed to a great extent in old chimneys. Rumford was the first to attempt rational improvements. He reduced the opening of the chimney and the depth of the fire-place, and added polished plates inclined at an angle, which serve both to guide the air to the fire and to reflect heat into the room (Fig. 220).

The blower (Fig. 221) produces its well-known effects by compelling all air to pass through the fire before entering the chimney. This at once improves the draught of the chimney by raising the temperature of the air within it, and quickens combustion by increasing the supply of oxygen to the fuel.

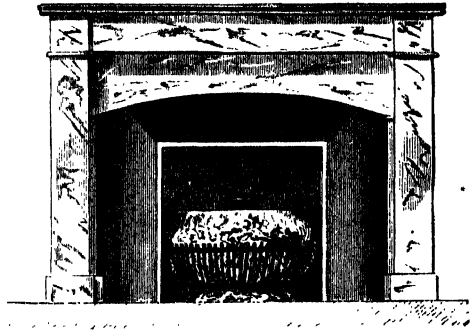


Fig. 221.—Fire-place with Blower.

224. Stoves.—The heating of rooms by open fire-places is effected almost entirely by radiation, and much even of the radiant heat is wasted. This mode of heating then, though agreeable and healthful, is far from economical. Stoves have a great advantage in point of economy, for the heat absorbed by their sides is in great measure given out to the room, whereas in an ordinary fire-place the greater part of this heat is lost. Open fire-places have, however, the advantage as regards ventilation; the large opening at the foot of the chimney, to which the air of the room has free access, causes a large body of air from the room to ascend the chimney, its place being supplied by fresh air entering through the chinks of the doors and windows, or any other openings which may exist.

Stoves are also liable to the objection of making the air of the room too dry, not, of course, by removing water, but by raising the tem-

perature of the air too much above the dew-point (Chap. xxviii.). The same thing occurs with open fire-places in frosty weather, at which time the dew-point is unusually low. This evil can be remedied by placing a vessel of water on the stove. The reason why it is more liable to occur with stoves than with open fire-places, is mainly that the former raise the air in the room to a higher temperature than the latter, the defect of air-temperature being in the latter case compensated by the intensity of the direct radiation from the glowing fuel.

Fire-clay, from its low conducting power, is very serviceable both for the backs of fire-places and for the lining of stoves. In the former

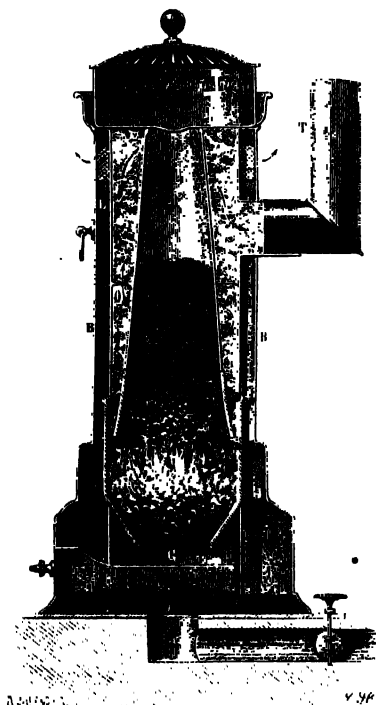


Fig. 222.—Ventilating Stove.

situation it prevents the wasteful escape of heat backwards into the chimney, and keeps the back of the fire nearly as hot as the centre. As a lining to stoves, it impedes the lateral escape of heat, thus answering the double purpose of preventing the sides of the stove from overheating, and at the same time of keeping up the temperature of the fire, and thereby promoting complete combustion. Its use must, however, be confined to that portion of the stove which serves as the fire-box, as it would otherwise prevent the heat from being given out to the apartment.

The stove represented in Fig. 222¹ belongs to the class of what are called in France *calorifères*, and in England *ventilating stoves*, being constructed with a view to promot-

ing the circulation and renewal of the air of the apartment. G is the fire-box, over which is the feeder U, containing unburned fuel, and tightly closed at top by a lid, which is removed only when fresh fuel is to be introduced. The ash-pan F has a door pierced with holes for admitting air to support com-

¹ With the exception of the ventilating arrangement, this stove is identical with what is known in this country as Walker's self-feeding stove.

bustion. The flame and smoke issue at the edge of the fire-box, and after circulating round the chamber O which surrounds the feeder, enter the pipe T which leads to the chimney. The chamber O is surrounded by another inclosure L, through which fresh air passes, entering below at A, and escaping into the room through perforations in the upper part of the stove as indicated by the arrows. The amount of fresh air thus admitted can be regulated by the throttle-valve P.

CHAPTER XXIV.

FUSION AND SOLIDIFICATION.

225. Fusion.—Many solid bodies, when raised to a sufficiently high temperature, become liquid. This change of state is called *melting* or *fusion*, and the temperature at which it occurs (called the melting-point, or temperature of fusion) is constant for each substance, with the exception of the variations—which in ordinary circumstances are insignificant—due to differences of pressure (§ 237). The melting-points of several substances are given in the following table:—

TABLE OF MELTING-POINTS, IN DEGREES CENTIGRADE.

Mercury,	- 39	Tin,	230
Ice,	0	Bismuth,	562
Butter,	33	Lead,	320
Lard,	33	Zinc,	360
Spermaceti,	49	Antimony,	432
Stearine,	55	Bronze,	900
Yellow Wax,	62	Pure Silver,	1000
White Wax,	68	Copper,	1150
Stearic Acid,	70	Coined Gold,	1180
Phosphorus,	44	Pure Gold,	1250
Potassium,	63	Cast Iron,	1050 to 1250
Sodium,	95	Steel,	1300 to 1400
Iodine,	107	Wrought Iron,	1500 to 1600
Sulphur,	110	Platinum,	2000

Some bodies, such as charcoal, have hitherto resisted all attempts to reduce them to the liquid state; but this is to be attributed only to the insufficiency of the means which we are able to employ.

It is probable that, by proper variations of temperature and pressure, all simple substances, and all compound substances which would not be decomposed, could be compelled to assume the three forms, solid, liquid, and gaseous.

The passage from the solid to the liquid state is generally abrupt;

but this is not always the case. Glass, for instance, before reaching a state of perfect liquefaction, passes through a series of intermediate stages in which it is of a viscous consistency, and can be easily drawn out into exceedingly fine threads, or moulded into different shapes.

226. Constant Temperature during Fusion.—During the entire time of fusion the temperature remains constant. Thus if a vessel containing ice be placed on the fire, the ice will melt more quickly as the fire is hotter; but if the mixture of ice and water be constantly stirred, a thermometer placed in it will indicate the temperature zero without variation so long as any ice remains unmelted; it is only after all the ice has become liquid that a rise of temperature will be observed.

In the same way, if sulphur be heated in a glass vessel, the temperature indicated by a thermometer placed in the vessel will rise gradually until it reaches about 110° , when a portion of the sulphur will be seen to become liquid, and if the vessel be shaken during the time of fusion, until the whole of the sulphur is liquefied, the temperature will be observed to remain steadily at this point.

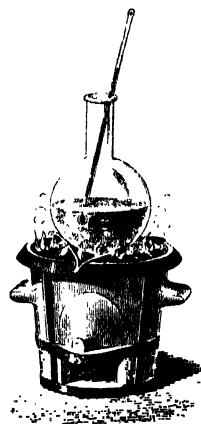


Fig. 223.—Fusion of Sulphur.

227. Latent Heat of Fusion.—This constancy of temperature is very remarkable, and leads to some important conclusions. In fact, as the action of the fire continues the same throughout the entire time of fusion, while the thermometer remains stationary, all the heat supplied after liquefaction has begun, *appears* to be lost. Hence we conclude, that in order that a body may pass from the solid to the liquid state, it must absorb a certain quantity of heat which produces no thermometric effect. Black, who was the first to investigate this subject, gave to the heat thus absorbed the name of *latent heat*, by which it is still usually designated. A similar absorption of heat without thermometric effect occurs when a boiling liquid is converted into vapour. Hence it is necessary to distinguish between the *latent heat of fusion* and the *latent heat of vaporization*.¹ Latent heat then may be defined as the heat absorbed in virtue of change of

¹ The former is often called the latent heat of the liquid, and the latter of the vapour. Thus we speak of the latent heat of water (which becomes latent in the melting of ice), and of the latent heat of steam (which becomes latent in the vaporization of water).

state from solid to liquid, or from liquid to gaseous. Modern philosophy teaches that the processes of fusion and vaporization involve the performance of work by heat in opposition to molecular force, and that an amount of heat disappears (or becomes latent) which is the exact equivalent of the work performed.

228. Heat of Fusion of Ice.—The latent heat of fusion is different for different substances. Its amount for ice (commonly called the latent heat of water) can be approximately determined by the following experiment, which is due to Black, who was the first to make accurate observations on this subject.

Take a pound of ice at 0° C. and a pound of water at 79° C.; let the water be poured over the ice, and the mixture rapidly stirred. The ice will melt, and two pounds of water at 0° will be obtained. This interesting experiment shows that all the heat necessary to raise a pound of water from 0° to 79° has been absorbed in melting a pound of ice; and it is thus directly proved that the heat required to melt a pound of ice is exactly the same as that required to raise the temperature of a pound of water from 0° to 79° .

The name *unit of heat* is given to the amount of heat necessary to raise unit mass of water one degree. It will be different according to the unit of mass and scale of temperature adopted. The pound-centigrade unit is the heat required to raise a pound of water through one degree Centigrade, and we see from above that 79 of these units are required for the melting of a pound of ice.¹

On comparing the heat of fusion of ice with that of some other bodies as given in the table § 348, it will be seen that its amount is notably greater for ice than for any of the other substances. Ice is in this sense the most difficult to melt, and water the most difficult to freeze of all substances, a fact which is of immense importance in the economy of nature, as tending to retard the processes both of freezing and thawing. Even as it is, the effects of a sudden thaw are often very disastrous, and yet, for every pound of ice melted, as much heat is required as would raise the water produced through 79° C. or 142° F.

229. Solution.—The reduction of a body from the solid to the liquid state may be effected by other means than by the direct action of heat; it may be produced by the action of a liquid. This is what occurs when, for instance, a grain of salt or of sugar is placed in

¹ The statements in this paragraph, including the definition of a unit of heat, are only approximate. The subject will be resumed in Chap. xxxi.

water; the body is said to *melt* or *dissolve* in the water. Solution, like fusion, is accompanied by the disappearance of heat consequent on the change from the solid to the liquid state. For example, when nitrate of ammonia is rapidly dissolved in water, a fall of from 20° to 25° Cent. is observed.

Unlike fusion, it is attached to no definite temperature, but occurs with more or less freedom over a wide range. Rise of temperature usually favours it; but there are some strongly marked exceptions.

230. Freezing Mixtures.—The absorption of heat which accompanies the liquefaction of solids is the basis of the action of freezing mixtures. In all such mixtures there is at least one solid ingredient which, by the action of the rest, is reduced to the liquid state, thus occasioning a fall of temperature proportional to the latent heat of its liquefaction.

The mixture most commonly employed in the laboratory is one of snow and salt, in the proportion of two parts of the former to one of the latter. This mixture assumes a temperature of about -18°C. (0°F.), and furnished Fahrenheit with the zero of his scale. In this instance there is a double absorption of heat caused by the simultaneous melting of the snow and dissolving of the salt.

We may obtain a freezing mixture without the use of snow or ice. Such mixtures are often employed for the artificial freezing of water. Various kinds of apparatus have been invented for this purpose, one of which is shown in Fig. 224. It consists of a metal cylinder, con-

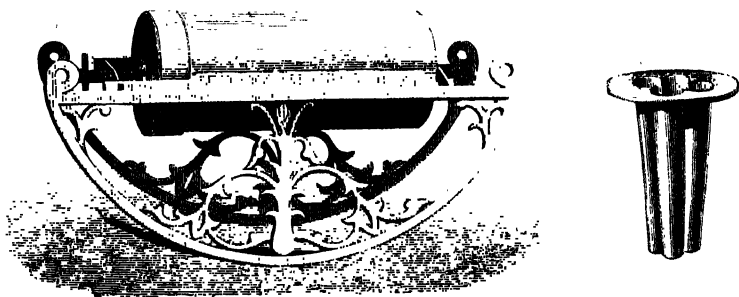


Fig. 224.—Freezing Rocker.

taining the freezing mixture (hydrochloric acid and sulphate of soda). In the cylinder is placed a mould formed of two concentric vessels with the water between them, an arrangement which has the advantage of increasing the surface of contact. The whole is set upon a cradle, the rocking of which greatly assists the operation. We sub-

join a table of the most important freezing mixtures, with the proportions corresponding to the maximum effect. These proportions are evidently of importance, since the amount of heat absorbed in a given time depends upon the quantity of solid matter that is reduced to the liquid state during that time. There may also be chemical action between the materials of which the mixture is composed. This is always attended with generation of heat, so that in this case the actual result depends upon the difference of two opposite effects. Thus the mixture of four parts of sulphuric acid with one of ice causes a rise of temperature of about 50° or 60° , while four parts of ice and one of sulphuric acid produce a cold of from -15° to -20° C.

TABLE.

	Proportions by Weight.	Fall of Temperature in Centigrade degrees.
Snow or Powdered Ice,	2	} from 0° to -21° .
Bay-salt,	1	
Snow,	3	} from 0° to -48° .
Crystallized Chloride of Calcium,	4	
Nitrate of Ammonia,	1	} from $+10^{\circ}$ to -15° .
Water,	1	
Sal-ammoniac,	5	} from $+10^{\circ}$ to -15° .
Nitrate of Potash,	5	
Sulphate of Soda,	8	
Water,	16	
Sulphate of Soda,	8	} from $+10^{\circ}$ to -17° .
Hydrochloric Acid,	5	

231. Solidification or Congelation.—Congelation is the inverse of fusion; that is to say, it is the passage of a substance from the liquid to the solid state. The faculty of undergoing this transformation may be regarded as common to all liquids, although some, for example, alcohol and bisulphide of carbon, have never yet been solidified.

The temperature of fusion is the highest temperature at which congelation can occur, and is frequently called the *temperature of congelation* (or the *freezing-point*); but it is possible to preserve substances in the liquid state at lower temperatures. Liquids thus cooled below their so-called freezing-points have, however, if we may so say, a *tendency to freeze, which is only kept in check by the difficulty of making a commencement*. If freezing once begins, or if ever so small a piece of the same substance in the frozen state be allowed to come in contact with the liquid, congelation will quickly extend until there is none of the liquid left at a temperature below that of fusion. The condition of a liquid cooled below its freezing-

point has been aptly compared to that of a row of bricks set on end in such a manner that if the first be overturned, it will cause all the rest to fall, each one overturning its successor.

The contact of its own solid infallibly produces congelation in a liquid in this condition, and the same effect may often be produced by the contact of some other solid, especially of a crystal, or by giving a slight jar to the containing vessel.

Despretz has cooled water to -20° C. in fine capillary tubes, without freezing, and Dufour has obtained a similar result by suspending globules of water in a liquid of the same specific gravity with which it would not mix.

232. Heat set free in Congelation.—At the moment when congelation takes place, the thermometer immediately rises to the temperature of the melting-point. This may be easily shown by experiment. A small glass vessel is taken, containing water, in which a mercurial thermometer is plunged. By means of a frigorific mixture the temperature is easily lowered to -10° or -12° , without the water freezing; a slight shock is then given to the glass, congelation takes place, and the mercury rises to 0° .

The heat thus produced is the equivalent of the work done by the molecular forces of the body in the passage from the liquid to the solid state. The quantity of heat thus arising is evidently the same as that which disappears in fusion, since they are the equivalents of the same amount of work performed in opposite directions. The production of this heat may be experimentally shown in another way. If we heat a quantity of lead to its melting-point (320°), and when the metal is just beginning to melt, plunge it into water, a certain rise of temperature will be observed. If we repeat the same experiment, allowing the lead time to melt completely, the temperature being still 320° , a much more considerable increase in the temperature of the water will be produced, the reason being that the lead in solidifying in contact with the water gives out its latent heat.

233. Crystallization.—When the passage from the liquid to the solid state is a gradual one, it frequently happens that the molecules group themselves in such a manner as to present regular geometric forms. This process is called crystallization, and the regular bodies thus formed are called crystals. The particular crystalline form assumed depends upon the substance, and often affords a means of recognizing it. The forms, therefore, in which bodies crystallize are among their most important characteristics, and are to some

extent analogous to the shapes of animals and plants in the organic world.

In order to make a body crystallize in solidifying, the following method is employed. Suppose the given body to be bismuth; the first step is to melt it, and then leave it to itself for a time. The metal naturally begins to solidify first at the surface and at the sides, where it is most directly exposed to cooling influences from without; accordingly, when the outer layer of the metal is solidified, the interior is still in the liquid state. If the upper crust be now removed, and the liquid bismuth poured off, the sides of the vessel will be seen to be covered with a number of beautiful crystals.

If the metal were allowed to stand too long, the entire mass would become solid, the different crystals would unite, and no regularity of structure would be observable. This is the case with a great number of solids; ice is one very remarkable instance.

234. Flowers of Ice.—The tendency of ice to assume a crystalline form is seen in the fern-leaf patterns which appear on the windows in winter, caused by the congealing of moisture on them, and still more distinctly in the symmetrical forms of snow-flakes (see Chap. xxviii.) In a block of ice, however, this crystalline structure does not show itself, owing to the closeness with which the crystals fit into each other, so that a mass of this substance appears almost completely *amorphous*. Tyndall, however, in a very interesting experiment, has succeeded in gradually *decrystallizing* ice, if we may use the

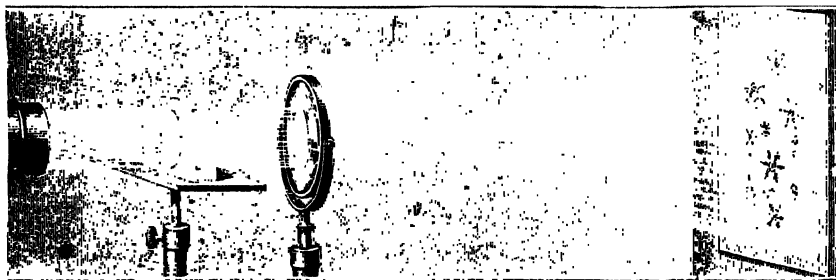


Fig. 225.—Flowers of Ice projected on a Screen.

expression, and thus exhibiting the crystalline elements of which it is composed. The experiment consists in causing a pencil of solar rays to fall perpendicular to the surfaces of congelation on a sheet of ice, such as is naturally formed upon the surface of water in winter. A lens placed behind the ice (Fig. 225) serves to project upon a screen

the image of what is found in the interior of the block. The successive appearances observed upon the screen are shown in Fig. 226. A small luminous circle is first seen, from which branch out rays, resembling the petals of a flower whose pistil is the circle. Frequent changes also occur in the shape of the branches themselves, which are often cut so as to resemble fern-leaves, like those seen upon the windows during frost. In this experiment the solar heat, instead of uniformly melting the mass of ice, which it would certainly do if the mass were amorphous, acts successively upon the different crystals of which it is built up, affecting them in the reverse order of their formation. There are thus produced a number of spaces of regular shape, containing water, and producing comparatively dark images upon

the screen. In the centre of each there is generally a bright spot, which corresponds to an empty space, depending on the fact that the water occupies a smaller volume than the ice from which it has been produced.

235. Supersaturation.—The proportion of solid matter which a liquid

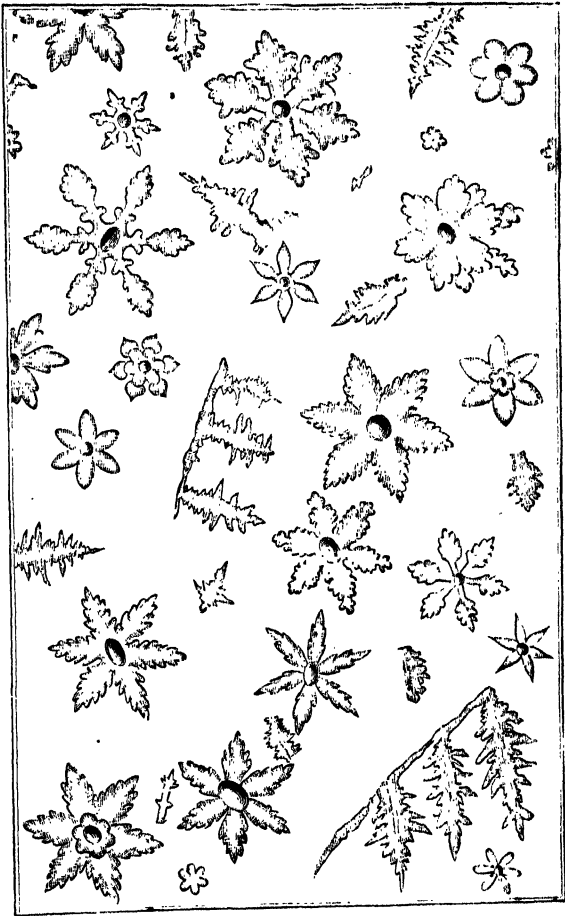


Fig. 226.—Flowers of Ice.

can hold in solution varies according to the temperature; as a general rule, though not by any means in all cases, it increases as the temperature rises. Hence it follows, that if a saturated solution be left to itself, the effect of evaporation or cooling will be gradually to diminish the quantity of matter which can be held in solution. A portion of the dissolved substance will accordingly pass into the solid state, assuming generally a crystalline form. This is an exceedingly common method of obtaining crystals, and is known as the *humid way*.

In connection with this process a phenomenon occurs which is precisely analogous to the cooling of a liquid below its freezing-point. It may be exemplified by the following experiment.

A tube drawn out at one end (Fig. 227) is filled with a warm concentrated solution of sulphate of soda. The solution is boiled, and

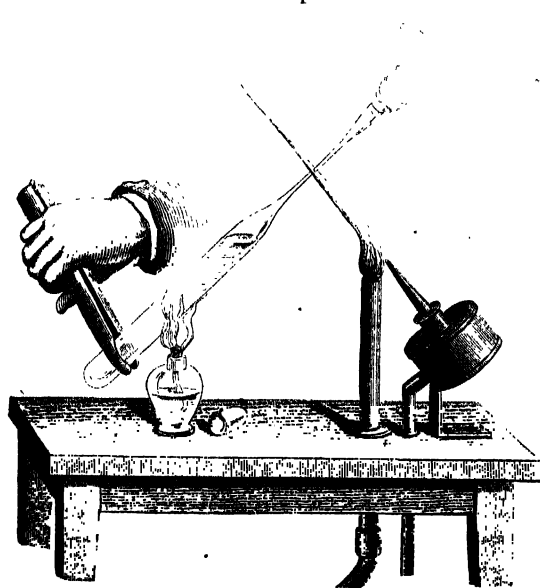


Fig. 227.—Preparation of Supersaturated Solution of Sulphate of Soda.

while ebullition is proceeding freely, the tube is hermetically sealed; by this means the tube is exhausted of air. The solution when left to itself cools without the solid being precipitated, although the liquid is *supersaturated*. But if the end of the tube be broken off, and the air allowed to enter, crystallization immediately commences at the surface, and is quickly propagated through the whole length of the tube;

at the same time, as we should expect, a considerable rise of temperature is observed. If the phenomenon does not at once occur on the admission of the air, it can be produced with certainty by throwing a small piece of the solid sulphate into the solution.

236. Change of Volume at the Moment of Congelation. Expansive Force of Ice.—In passing from the liquid to the solid state, bodies generally undergo a diminution of volume; there are, however,

exceptions, such as ice, bismuth, silver, and cast-iron. It is this property which renders this latter substance so well adapted for the purposes of moulding, as it enables the metal to penetrate completely into every part of the mould. The expansion of ice is considerable, amounting to about $\frac{1}{4}$; its production is attended by enormous mechanical force, just as in the analogous case of expansion by heat.

Its effect in bursting water-pipes is well known. The following experiment illustrates this expansive force. A tube of forged iron (Fig. 228) is filled with water, and tightly closed by a screw-stopper.



Fig. 228.—Bursting of Iron Tube by Expansion of Water in Freezing.

The tube is then surrounded with a freezing mixture of snow and salt. After some time the water congeals, a loud report is often heard, and the tube is found to be rent.

The following experiment, performed by Major Williams at Quebec, is still more striking. He filled a 12-inch shell with water and closed



Fig. 229.—Experiment of Major Williams.

it with a wooden stopper, driven in with a mallet. The shell was then exposed to the air, the temperature being -28°C . (-18°F .) The water froze, and the bung was projected to a distance of more than 100 yards, while a cylinder of ice of about 8 inches in length was protruded from the hole. In another experiment the shell split in halves, and a sheet of ice issued from the rent (Fig. 229).

It is the expansion and consequent lightness of ice which enables

it to float upon the surface of water, and thus afford a protection to animal life below.

237. Effect of Pressure on the Melting-point.—Professor James Thomson was led by theoretical considerations to the conclusion that, in the case of a substance which, like water, expands in solidifying, the freezing (or melting) point must of necessity be lowered by pressure, and that a mixture of ice and ice-cold water would fall in temperature on the application of pressure. His reasoning¹ consisted in showing that it would otherwise be possible (theoretically at least) to construct a machine which should be a perpetual source of work without supply; that is, what is commonly called a perpetual suction.

The matter was shortly afterwards put to the test of experiment by Professor (now Sir) W. Thomson, who compressed, in an *Ærsted's* piezometer, a mixture of ice and water, in which was inserted a very delicate thermometer protected from pressure in the same manner as the instrument represented in Fig. 194c (§189). The thermometer showed a regular fall of temperature as pressure was applied, followed by a return to 0°C. on removing the pressure. Pressures of 8.1 and 16.8 atmospheres (in excess of atmospheric pressure) lowered the freezing-point by .106 and .232 of a degree Fahr. respectively as indicated by the thermometer, results which agree almost exactly with Prof. J. Thomson's prediction of .0075 of a degree Cent., or .0135 of a degree Fahr. per atmosphere.

Mousson has since succeeded in reducing the melting-point several degrees by means of enormous pressure. He employed two forms of apparatus, by the first of which he melted ice at the temperature of -5°C. , and kept the water thus produced for a considerable time at this temperature. This apparatus had windows (consisting of blocks of glass) in its sides, through which the melting of the ice was seen. His second form of apparatus, which bore a general resemblance to the first, is represented in the annexed figure. It consisted of a steel prism with a cylindrical bore, having one of its extremities closed by a conical stopper strongly screwed in, the rest of the bore being traversed by a screw-piston of steel. The apparatus was inverted, and nearly filled with water recently boiled, into which a piece of copper was dropped, to serve as an index. The apparatus,

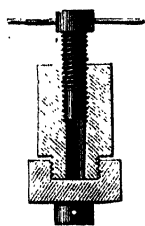


Fig. 230.
Mousson's
Apparatus.

¹ *Transactions Royal Society, Edinburgh.* January, 1849.—*Cambridge and Dublin Math. Journal.* November, 1850.

still remaining in the inverted position, was surrounded by a freezing mixture, by means of which the water was reduced to ice at the temperature of -18° C. The stopper was then screwed into its place, and the apparatus placed in the erect position. The piston was then screwed down upon the ice with great force, the pressure exerted being estimated in some of the experiments at several thousand atmospheres. The pressure was then relaxed, and, on removing the stopper, the copper index was found to have fallen to the bottom of the bore, showing that the ice had been liquefied.

Experiments conducted by Bunsen and Hopkins have shown that wax, spermaceti, sulphur, stearin, and paraffin—substances which, unlike ice, expand in melting—have their melting points *raised* by pressure, a result which had been predicted by Professor W. Thomson.

237A. Effect of Stress in general upon Melting and Solution.—In the experiments above described, the pressure applied was hydrostatical, and was therefore equal in all directions. But a solid may be exposed to pressure in one direction only, or to pull in one or more directions, or it may be subjected to shearing, twisting, or bending forces, all these being included under the general name of *stress*.

Reasoning, based on the general laws of energy, leads to the conclusion that stress of any kind other than hydrostatic, applied to a solid, must lower its melting-point. To quote Professor J. Thomson (*Proc. Roy. Soc.* Dec. 1861), "Any stresses whatever, tending to change the form of a piece of ice in ice-cold water, must impart to the ice a tendency to melt away, and to give out its cold, which will tend to generate, from the surrounding water, an equivalent quantity of ice free from the applied stresses," and "stresses tending to change the form of any crystals in the saturated solutions from which they have been crystallized must give them a tendency to dissolve away, and to generate, in substitution for themselves, other crystals free from the applied stresses or any equivalent stresses."¹ This conclusion he verified by experiments on crystals of common salt. He at the same time suggested, as an important subject for investigation, the effect

¹ Professor Thomson draws these inferences from the following principle, which he assumes (we think justly) as a physical axiom:—If any substance or system of substances be in a condition in which it is free to change its state [as ice, for example, in contact with water at 0° C., is free to melt], and if mechanical work be applied to it as potential energy in such a way that the occurrence of the change of state will make it lose that mechanical work from the condition of potential energy, without receiving other potential energy as an equivalent; *then the substance or system will pass into the changed state.*

of hydrostatic pressure on the crystallization of solutions, a subject which was afterwards taken up experimentally by Sorby, who obtained effects analogous to those above indicated as occurring in connection with the melting of ice and wax.

238. Regelation of Ice.—Faraday in 1850 called attention to the fact that pieces of moist ice placed in contact with one another will freeze together even in a warm atmosphere. This phenomenon, to which Tyndall has given the name of *regelation*, admits of ready explanation by the principles just enunciated. Capillary action at the boundaries of the film of water which connects the pieces placed in contact, produces an effect equivalent to attraction between them, just as two plates of clean glass with a film of water between them seem to adhere. Ice being wetted by water, the boundary of the connecting film is concave, and this concavity implies a diminution of pressure in the interior. The film, therefore, exerts upon the ice a pressure less than atmospheric; and as the remote sides of the blocks are exposed to atmospheric pressure, there is a resultant force urging them together and producing stress at the small surface of contact. Melting of the ice therefore occurs at the places of contact, and the cold thus evolved freezes the adjacent portions of the water film, which, being at less than atmospheric pressure, will begin to freeze at a temperature a little above the ordinary freezing-point.

As regards the amount of the force urging the pieces together, if two flat pieces of ice be supported with their faces vertical, and if they be united by a film from whose lower edge water trickles away, the hydrostatic pressure at any point within this film is less than atmospheric by an amount represented, in weight of water, by the height of this point above the part from which water trickles. If, for simplicity, we suppose the film circular, the plates will be pressed together with a force equal to the weight of a cylinder of water whose base is the film and whose height is the radius.

239. Apparent Plasticity of Ice. Motion of Glaciers.—A glacier may be described in general terms as a mass of ice deriving its origin from mountain snows, and extending from the snow-fields along channels in the mountain sides to the valleys beneath.

The first accurate observations on the movements of glaciers were made in 1842, by the late Professor (afterwards Principal) J. D. Forbes, who established the fact that glaciers descend along their beds with a motion resembling that of a pailful of mortar poured into a sloping trough; the surface moving faster than the bottom

and the centre faster than the sides. He summed up his view by saying, "A glacier is an imperfect fluid, or a viscous body which is urged down slopes of a certain inclination by the mutual pressure of its parts."

This apparent viscosity is explained by the principles of § 237A. According to these principles the ice should melt away at the places where stress is most severe, an equivalent quantity of ice being formed elsewhere. The ice would thus gradually yield to the applied forces, and might be moulded into new forms, without undergoing rupture. Breaches of continuity might be produced in places where the stress consisted mainly of a pull, for the pull would lower the freezing-point, and thus indirectly as well as directly tend to produce ruptures, in the form of fissures transverse to the direction of most intense pull. The effect of violent compression in any direction would, on the other hand, be, not to crack the ice, but to melt a portion of its interior sufficient to relieve the pressure in the particular part affected, and to transfer the excess of material to neighbouring parts, which must in their turn give way in the same gradual manner.

In connection with this explanation it is to be observed that the temperature of a glacier is always about $0^{\circ}\text{C}.$, and that its structure is eminently porous and permeated with ice-cold water. These are conditions eminently favourable (the former, but not the latter, being essential) to the production of changes of form depending on the lowering of the melting-point by stresses.

This explanation is due to Professor J. Thomson¹ (*British Association Report*, 1857). Professor Tyndall had previously attempted to account for the phenomena of glacier motion by supposing that the ice is fractured by the forces to which it is subjected, and that the broken pieces, after being pushed into their new positions, are united by regelation. In support of this view he performed several very interesting and novel experiments on the moulding of ice by pressure, such as striking medals of ice with a die, and producing a clear transparent cake of ice by powerfully compressing broken pieces in a boxwood mould (Fig. 231).

Interesting experiments on the plasticity of ice may be performed by filling an iron shell with water and placing it in a freezing mix-

¹ If it should be objected that the lowering of the melting-point by stress is too insignificant to produce the vast effects here attributed to it, the answer is that, when ice and water are present together, the slightest difference is sufficient to determine which portion of the water shall freeze, or which portion of the ice shall melt. In default of a more powerful cause, those portions of ice which are most stressed will melt first.

ture, leaving the aperture open. As the water freezes, a cylinder of ice will be gradually protruded. This experiment is due to Mr. Christie. Professor Forbes obtained a similar result by using a very

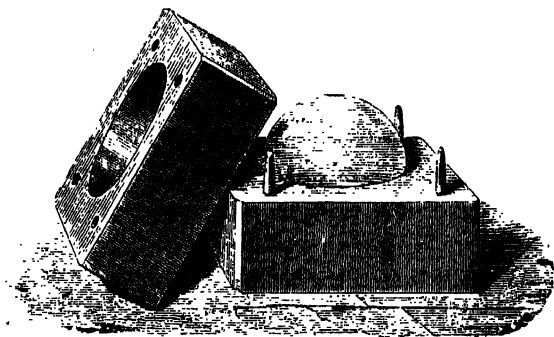


Fig. 231.—Ice Moulded by Pressure.

strong glass jar; and by smearing the interior, just below the neck, with colouring matter, he demonstrated that the external layer of ice which was first formed, slid along the glass as the freezing proceeded, until it was at length protruded beyond the mouth.

In the experiments of Major Williams, described in § 236, it is probable that much of the water remained unfrozen until its pressure was relieved by the bursting of the shells.

CHAPTER XXV.

EVAPORATION.

240. Transformation into the State of Vapour.—The majority of liquids, when left to themselves in contact with the atmosphere, gradually pass into the state of vapour and disappear. This phenomenon occurs much more rapidly with some liquids than with others, and those which evaporate most readily are said to be the most volatile. Thus, if a drop of ether be let fall upon any substance, it disappears almost instantaneously; alcohol also evaporates very quickly, but water requires a much longer time for a similar transformation. The change is in all cases accelerated by an increase of temperature; in fact, when we *dry* a body before the fire, we are simply availing ourselves of this property of heat to hasten the evaporation of the moisture of the body. Evaporation may also take place from solids. Thus camphor, iodine, and several other substances pass directly from the solid to the gaseous state, and we shall see hereafter that the vapour of ice can be detected at temperatures far below the freezing-point.

Evaporation, unlike fusion, occurs over a very wide range of temperature. There appears, however, to be a temperature for each substance, below which evaporation, if it exist at all, cannot be detected. This is the case with mercury at 0°C ., and with sulphuric acid at ordinary atmospheric temperatures.

241. Vapour, Gas.—The words *gas* and *vapour* have no essential difference of meaning. A vapour is the gas into which a liquid is changed by evaporation. Every gas is probably the vapour of a certain liquid. The word vapour is especially applied to the gaseous condition of bodies which are usually met with in the liquid or solid state, as water, sulphur, &c.; while the word gas generally denotes a body which, under ordinary conditions, is never found in any state

but the gaseous. There are a few gases which experimenters have hitherto been unable to obtain under any other form. These are oxygen, hydrogen, nitrogen, nitric oxide, carbonic oxide, and marsh gas; they are sometimes called *permanent gases*.

242. Elastic Force of Vapours. Maximum Tension.—The characteristic property of gases is their expansibility or elastic force.¹ This may be exemplified in the case of vapours by the following experiment.

A glass globe A (Fig. 232) is fitted with a metal cap provided with two openings, one of which can be made to communicate with a mercurial manometer, while the other is furnished with a stop-cock R. The globe is first exhausted of air by establishing communication through R with an air-pump. The mercury rises in the left-hand and falls in the right-hand branch of the manometer; the final difference of level in the two branches differing from the height of the barometer only by the very small quantity representing the tension of the air left behind by the machine. The stop-cock R is then closed, and a second stop-cock R' surmounted by a funnel is fixed above it. The hole in this second stop-cock, instead of going quite through the metal, extends only half-way, so as merely to form a cavity. This cavity serves to introduce a liquid into the globe, without any communication taking place between the globe and the external air. For this purpose we have only to fill the funnel with a liquid, to open the cock R, and to turn that at R' backwards and forwards several times. It will be found, that after the introduction of a small quantity of liquid into the globe, the mercurial column begins to descend in the left branch of the manometer, thus indicating an increase of elastic force. This elastic force goes on increasing as a greater quantity of liquid is introduced into the globe; and as no liquid is visible in the globe, we must infer that it evaporates as fast as it is introduced, and that the fall of the mercurial column is caused by the elastic force of the vapour thus formed.

This increase of pressure, however, does not go on indefinitely. After a time the difference of level in the two branches of the manometer ceases to increase, and a little of the unevaporated liquid may be seen in the globe, which increases in quantity as more liquid is introduced. From this important experiment we conclude that there is a limit to the quantity of vapour which can be formed at a given temperature in an empty space. When this limit is reached, the

¹ The names *pressure*, *tension*, and *elastic force*, are used interchangeably.

space is said to be *saturated*, and the vapour then contained in it is at *maximum tension*, and at *maximum density*. It evidently follows from this that if a quantity of vapour at less than its maximum tension be inclosed in a given space, and then compressed at constant

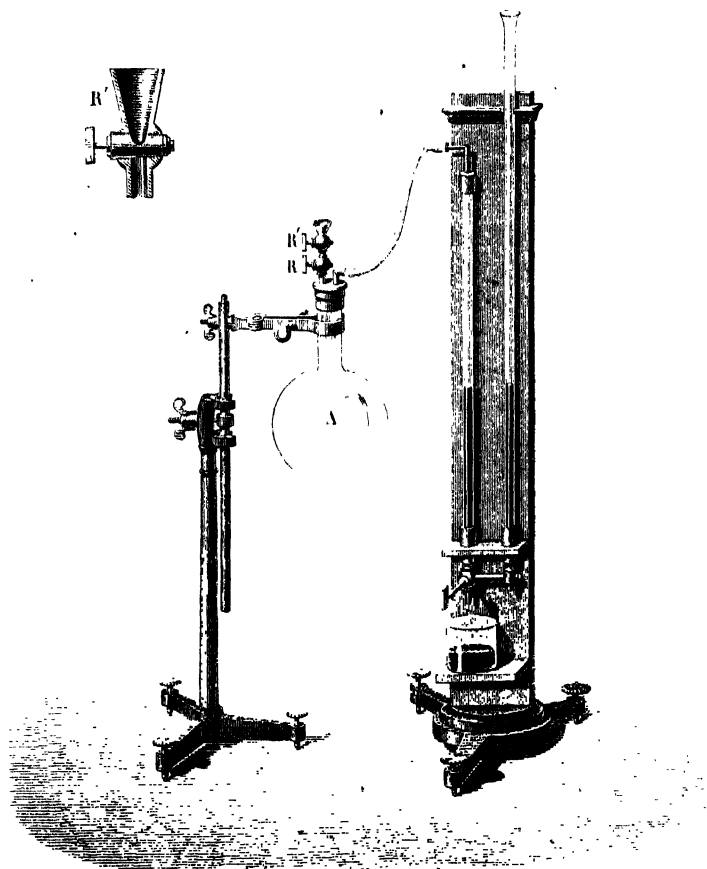


Fig. 232.—Apparatus for studying the Formation of Vapours.

temperature, its tension and density will increase at first, but that after a time a point will be reached when further compression, instead of increasing the density of the vapour, will only cause some of it to pass into the liquid state. This last result may be directly verified by the following experiment. A barometric tube *ab* (Fig. 233) is filled with mercury, with the exception of a small space, into which a few drops of ether are introduced, care having first been

taken to expel any bubbles of air which may have remained adhering to the mercury. The tube is then inverted in the deep bowl MN, when the ether ascends to the surface of the mercury, is there converted into vapour, and produces a sensible depression of the

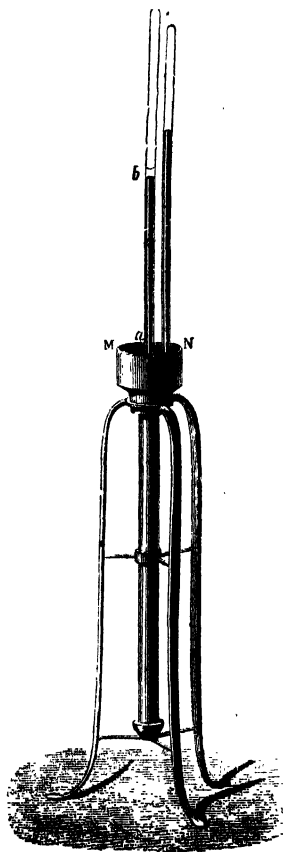


Fig. 233.—Maximum Tension of Vapours.

mercurial column. If the quantity of ether be sufficiently small, and if the tube be kept sufficiently high, no liquid will be perceived in the space above the mercury; this space, in fact, is not saturated. The tension of the vapour which occupies it is given by the difference between the height of the column in the tube and of a barometer placed beside it. If the tube be gradually lowered, this difference will at first be seen to increase, that is, the tension of the vapour of ether increases; but if we continue the process, a portion of liquid ether will be observed to collect above the mercury, and after this, if we lower the tube any further, the height of the mercury in it remains invariable. The only effect is to increase the quantity of liquid deposited from the vapour.¹

243. Influence of Temperature on the Maximum Tension.—Returning now to the apparatus represented in Fig. 232, suppose that some of the liquid remains unevaporated in the bottom of the globe, and let the globe be subjected to an increase of temperature. An increase of elastic force will at once be indicated by the manometer, while the quantity of liquid will be diminished. The maximum tension of

a vapour, therefore, and also its maximum density, increase with the temperature; and consequently, in order to saturate a given space, a quantity of vapour is required which increases with the temperature. In a subsequent chapter we shall give the results of experi-

¹ Strictly speaking, there will be a slight additional depression of the mercurial column due to the weight of the liquid thus deposited on its summit; but this effect will generally be very small, as the vapour occupies much more space than the liquid which it yields.

ments on the maximum tension of aqueous vapour at different temperatures, and it will be seen that the increase is exceedingly rapid.

Fig. 234 is a graphical representation of the rate at which the maximum density of a vapour increases with the temperature. Lengths are laid off on the base-line AB, to represent temperatures from -20° to $+35^{\circ}$ C, and ordinates are erected at every fifth de-

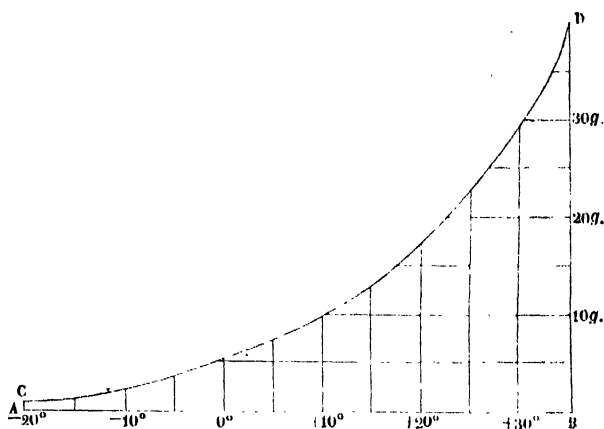


Fig. 234.—Saturation at different Temperatures.

gree, proportional to the weights of vapour required to saturate the same space at different temperatures. The curve CD, drawn through the extremities of these ordinates, is the curve of vapour-density as a function of temperature. The figures on the right hand indicate the number of grammes of vapour required to saturate a cubic metre.

244. Mixture of Gas and Vapour.—The experiments with the apparatus of Fig. 232 may be repeated after filling the globe with dry air, or any other dry gas, and the results finally obtained will be the same as with the exhausted globe. If, as before, we introduce successive small quantities of a liquid, it will be converted into vapour, and the pressure will go on increasing till saturation is attained; the elastic force of vapour will then be found to be exactly the same as in the case of the vacuous globe, and the quantity of liquid evaporated will also be the same.

There is, however, one important difference. In the vacuum the complete evaporation of the liquid is almost instantaneous; in a gas, on the other hand, the evaporation and consequent increase of pressure proceed with comparative slowness; and the difference between

the two cases is more marked in proportion as the pressure of the gas is greater.

We may lay down, then, the two following laws for the mixture of a vapour with a gas:—

1. *The weight of vapour which will enter a given space is the same whether this space be empty or filled with gas, provided plenty of time be allowed.*

2. *When a gas is saturated with vapour, the actual tension of the mixture is the sum of the tensions due to the gas and vapour separately; that is to say, it is equal to the pressure which the gas would exert if it alone occupied the whole space, plus the maximum tension of vapour for the temperature of the mixture.*

This second law evidently comes under the general rule for determining the pressure of a mixture of gases (§ 127); and the same rule applies to a mixture of gas and vapour when the quantity of the latter falls short of saturation. Each element in a mixture of gases and vapours exerts the same pressure on the walls of the containing vessel as it would exert if the other elements were removed.

It is doubtful, however, whether these laws are rigorously true. It would rather appear from some of Regnault's experiments, that the quantity of vapour taken up in a given space is slightly, though almost insensibly, diminished, as the density of the gas which occupies the space is increased.

245. Liquefaction of Gases.—When vapour exists in the state of saturation, any diminution in the volume must, if the temperature is preserved constant, involve the liquefaction of as much of the vapour as would occupy the difference of volumes; and the vapour which remains will still be at the original density and tension. A vapour existing by itself may therefore be completely liquefied by subjecting it to a pressure exceeding, by ever so slight an amount, the maximum tension corresponding to the temperature, provided that the containing vessel is prevented from rising in temperature.

Again, if a vapour at saturation be subjected to a fall of temperature, while its volume remains unchanged, a portion of it must be liquefied corresponding to the difference between the density of saturation at the higher and at the lower temperature. This operation will obviously diminish the tension, since this will now be the maximum tension corresponding to the lower instead of to the higher temperature.

There are therefore two distinct means of liquefying a vapour—

increase of pressure, and lowering of temperature. They are employed sometimes separately, and sometimes in conjunction.

Fig. 235 represents the apparatus usually employed for obtaining sulphurous acid in the liquid state. The gas, which is generated in a glass globe, passes first into a washing-bottle, then through a drying-tube, and finally into a tube surrounded with a freezing mixture of snow and salt.

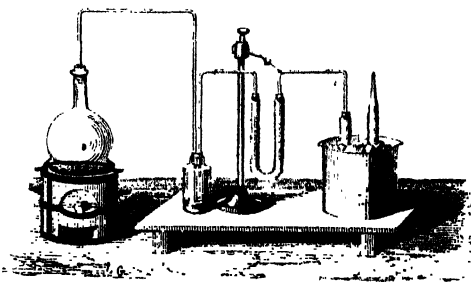


Fig. 235.—Liquefaction of Sulphurous Acid.

Pouillet's apparatus, described in § 120, serves to liquefy most gases by means of compression

In order to ascertain the pressures at which liquefaction takes place, or, in other words, the maximum tensions of gases, one of the tubes in that apparatus is replaced by a shorter tube, containing atmospheric air, and serving as a manometer.

By this means Pouillet has found that, at the temperature of 10°C ., sulphurous acid is liquefied by a pressure of $2\frac{1}{2}$ atmospheres, nitrous oxide by a pressure of 43, and carbonic acid by a pressure of 45 atmospheres.

246. Faraday's Method.—Faraday, who was the first to conduct methodical experiments on the liquefaction of gases, employed, in the first instance, the simple apparatus represented in Fig. 236. It

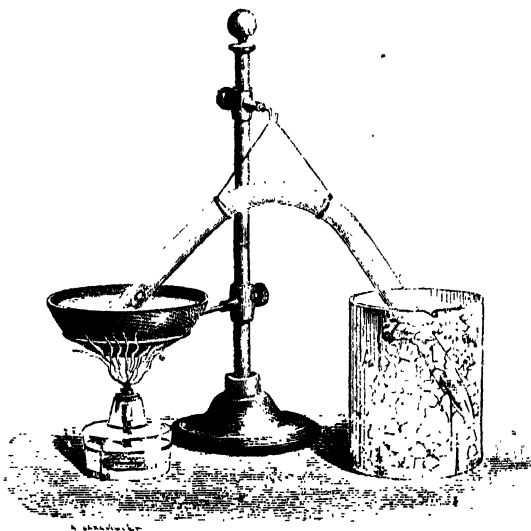


Fig. 236.—Faraday's Apparatus.

consists of a very strong bent glass tube, one end of which contains ingredients which evolve the gas on the application of heat, while

the other is immersed in a freezing mixture. The pressure produced by the evolution of the gas in large quantity in a confined space, combines with the cold of the freezing mixture to produce liquefaction of the gas, and the liquid accordingly collects in the cold end of the tube.

Thilorier, about the year 1834, invented the apparatus represented in Fig. 237, which is based on this method of Faraday, and is intended for liquefying carbonic acid gas. This operation requires the enormous pressure of about fifty atmospheres at ordinary temperatures. If a slight rise of temperature occur from the chemical actions attending the production of the gas, a pressure of 75 or 80 atmospheres may not improbably be required. Hence great care is necessary in testing the strength of the metal employed in the construction of the apparatus. It was formerly made of cast iron, and

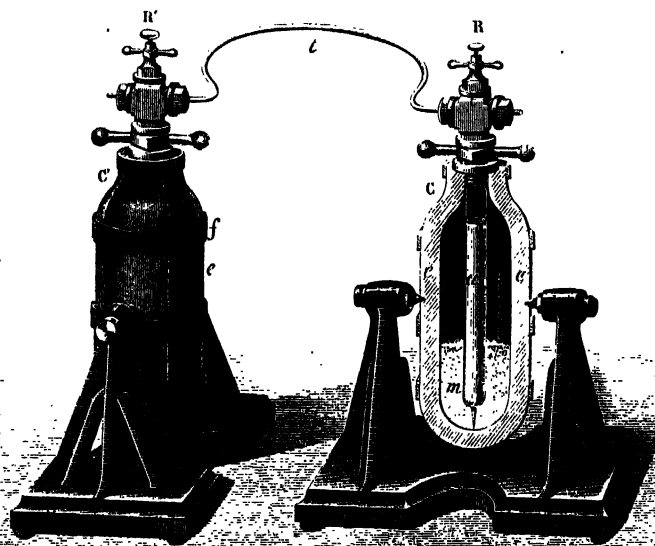


Fig. 237.—Thilorier's Apparatus.

strengthened by wrought-iron hoops; but the construction has since been changed on account of a terrible explosion, which cost the life of one of the operators. At present the vessels are formed of three parts; the inner one of lead, the next *e*, which completely envelops this, of copper, and finally, the hoops *ff* of wrought iron (Fig. 237), which bind the whole together. The apparatus consists of two dis-

tinct reservoirs. In the generator C is placed bicarbonate of soda, and a vertical tube *a*, open at top, containing sulphuric acid. By imparting an oscillatory movement to the vessel about the two pivots which support it near the middle, the sulphuric acid is gradually spilt, and the carbonic acid is evolved, and becomes liquid in the interior. The generator is then connected with the condenser C' by the tube *t*, and the stop-cocks R and R' are opened. As soon as the two vessels are in communication, the liquid carbonic acid passes into the condenser, which is at a lower temperature than the generator, and represents the cold branch of Faraday's apparatus. The generator can then be disconnected and recharged, and thus several pints of liquid carbonic acid may be obtained.

In the foregoing methods, the pressure which produces liquefaction is furnished by the evolution of the gas itself.

In some other forms of apparatus the pressure is obtained by the use of one or more compression-pumps, which force the gas from the vessel in which it is generated into a second vessel, which is kept cool either by ice or a freezing mixture. The apparatus of this kind which is most extensively used is that devised by Bianchi. It consists of a compression-pump driven by a crank furnished with a fly-wheel, and turned by hand.

Faraday, in his later experiments, employed two pumps, the first having a piston of an inch, and the second of only half an inch diameter. The first pump in the earlier stage of the operation forced the gas through the second into the receiver. In the later stage the second pump was also worked, so as to force the gas already condensed to 10, 15, or 20 atmospheres into the receiver at a much higher pressure. The receiver was a tube of green bottle-glass, and was immersed in a very intense freezing mixture, consisting of solid carbonic acid and ether, the cooling effect being sometimes increased by exhausting the air and vapour from the vessel containing the freezing mixture, so as to promote more rapid evaporation.

246A. Continuity of the Liquid and Gaseous States.—Remarkable results were obtained by Cagniard de la Tour¹ by heating volatile liquids (alcohol, petroleum, and sulphuric ether) in closed tubes of great strength, and of capacity about double the volume of the inclosed liquid. At certain temperatures (36°C. for alcohol, and 42° for ether) the liquid suddenly disappeared, becoming apparently converted into vapour.

¹ *Ann. de Chim.* II. xxi.

Drion,¹ by similar experiments upon hydrochloric ether, hyponitric acid, and sulphurous acid, showed—

1. That the coefficients of apparent expansion of these liquids increase rapidly with the temperature.

2. That they become equal to the coefficient of expansion of air, at temperatures much lower than those at which total conversion into vapour occurs.

3. That they may even become double and more than double the coefficient of expansion of air; for example, at 130° C. the coefficient of expansion of sulphurous acid was .009571.

Thilorier had previously shown that the expansion of liquid carbonic acid between the temperatures 0° and 30° C. is four times as great as that of air.

Drion further observed, that when the temperature was raised very gradually to the point of total vaporization, the free surface lost its definition, and was replaced by a nebulous zone without definite edges and destitute of reflecting power. This zone increased in size both upwards and downwards, but at the same time became less visible, until the tube appeared completely empty. The same appearances were reproduced in inverse order on gradually cooling the tube.

When the liquid was contained in a capillary tube, or when a capillary tube was partly immersed in it, the curvature of the meniscus and the capillary elevation decreased as the temperature rose, until at length, just before the occurrence of total vaporization, the surface became plane, and the level was the same within as without the tube.

Dr. Andrews, by a series of elaborate experiments on carbonic acid, with the aid of an apparatus which permitted the pressure and temperature to be altered independently of each other, has shown that at temperatures above 31° C. this gas cannot be liquefied, but, when subjected to intense pressure, becomes reduced to a condition in which, though homogeneous, it is neither a liquid nor a gas. When in this condition, lowering of temperature under constant pressure will reduce it to a liquid, and diminution of pressure at constant temperature will reduce it to a gas; but in neither case can any breach of continuity be detected in the transition.

On the other hand, at temperatures below 31°, the substance remains completely gaseous until the pressure reaches a certain limit depending on the temperature, and any pressure exceeding this limit

¹ *Ann. de Chim.*, **III.** lvi.

causes liquefaction to commence and to continue till the whole of the gas is liquefied, the boundary between the liquefied and unliquefied portions being always sharply defined.

The temperature 31°C ., or more exactly 30.92°C . (87.7°F .), may therefore be called the critical temperature for carbonic acid; and it is probable that every other substance, whether usually occurring in the gaseous or in the liquid form, has in like manner its own critical temperature. Dr. Andrews found that nitrous oxide, hydrochloric acid, ammonia, sulphuric ether, and sulphuret of carbon, all exhibited critical temperatures, which, in the case of some of these substances, were above 100°C .

It is probable that, in the experiments of Cagniard de la Tour and Drion, the so-called total conversion into vapour was really conversion into the intermediate condition.

The continuous conversion of a gas into a liquid may be effected by first compressing it at a temperature above its critical temperature, until it is reduced to the volume which it will occupy when liquefied, and then cooling it below the critical point.

The continuous conversion of a liquid into a gas may be obtained by first raising it above the critical temperature while kept under pressure sufficient to prevent ebullition, and afterwards allowing it to expand.

When a substance is a little above its critical temperature, and occupies a volume which would, at a lower temperature, be compatible with partial liquefaction, very great changes of volume are produced by very slight changes of pressure.

On the other hand, when a substance is at a temperature a little below its critical point, and is partially liquefied, a slight increase of temperature leads to a gradual obliteration of the surface of demarcation between the liquid and the gas; and when the whole has thus been reduced to a homogeneous fluid, it can be made to exhibit an appearance of moving or flickering striæ throughout its entire mass by slightly lowering the temperature, or suddenly diminishing the pressure.

The apparatus employed in these remarkable experiments, which are described in the Bakerian Lecture (*Phil. Trans.* 1869), is shown in Fig. 237A, where *cc* are two capillary glass tubes of great strength, one of them containing the carbonic acid or other gas to be experimented on, the other containing air to serve as a manometer. These are connected with strong copper tubes *dd*, of larger diameter, con-

taining water, and communicating with each other through *ab*, the water being separated from the gases by a column of mercury occupying the lower portion of each capillary tube.

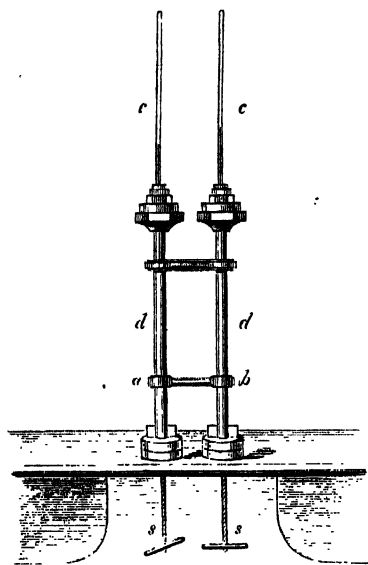


Fig. 237 A.—Andrews' Apparatus.

The steel screws *ss* are the instruments for applying pressure. By screwing either of them forward into the water, the contents of both tubes are compressed, and the only use of having two is to give a wider range of compression. A rectangular brass case (not shown in the figure), closed before and behind with plate-glass, surrounds each capillary tube, and allows it to be maintained at any required temperature by the flow of a stream of water.

247. Latent Heat of Vaporization.

Cold produced by Evaporation.—The passage from the liquid to the gaseous state is accompanied by the disappearance of a large quantity of

heat. Whenever a liquid evaporates without the application of heat, a depression of temperature occurs. Thus, for instance, if any portion of the skin be kept moist with alcohol or ether, a decided sensation of cold is felt. Water produces the same effect in a smaller degree, because it evaporates less rapidly.

The heat which thus disappears in virtue of the passage of a liquid into the gaseous condition, is called the *latent heat of vaporization*. Its amount varies according to the temperature at which the change is effected, and it is exactly restored when the vapour returns to the liquid form, provided that both changes have been effected at the same temperature. Its amount for vapour of water at the temperature 100°C . is 536° ; that is to say, the quantity of heat which disappears in the

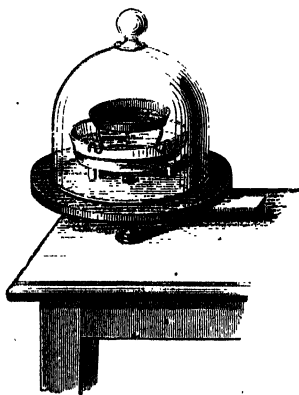


Fig. 238.—Leslie's Experiment.

evaporation of a pound of water at this temperature, and which reappears in the condensation of a pound of steam at the same temperature, would be sufficient to raise the temperature of 536 pounds of water from 0° to 1° .

The latent heat of vaporization plays an important part in the heating of buildings by steam. A pound of steam at 100° , in

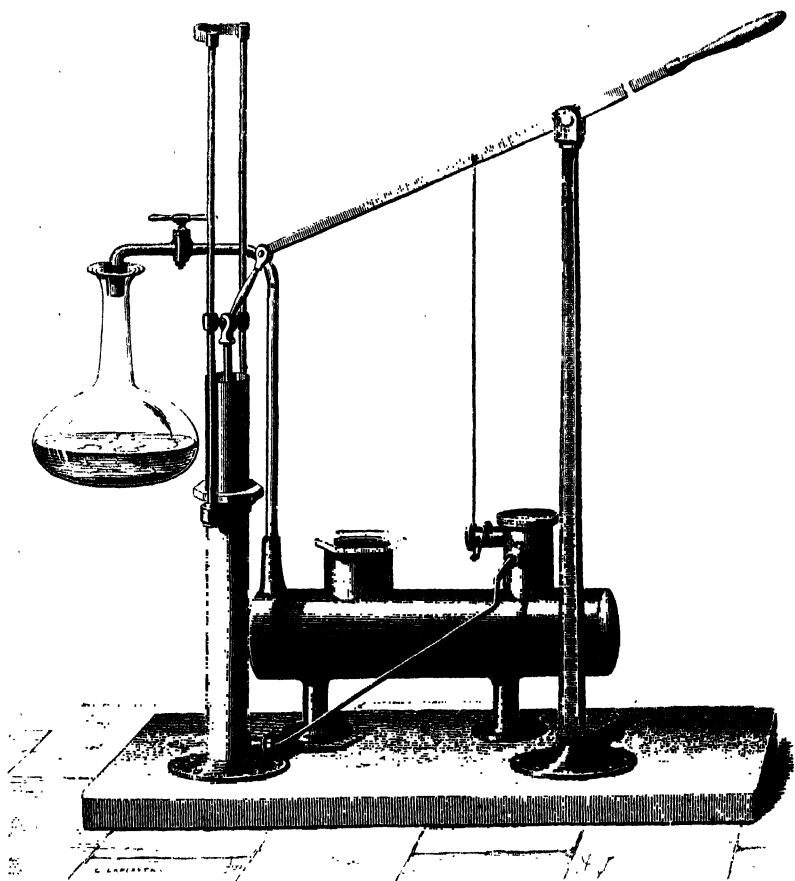


Fig. 239.—Carré's Apparatus.

becoming reduced to water at 30° , gives out as much heat as about $8\frac{1}{2}$ lbs. of water at 100° in cooling down to the same temperature.

248. Leslie's Experiment.—Water can be easily frozen by the cold resulting from its own evaporation, as was first shown by Leslie in a celebrated experiment. A small capsule (Fig. 238) of copper is

taken, containing a little water, and is placed above a vessel containing strong sulphuric acid. The whole is placed under the receiver of an air-pump, which is then exhausted. The water evaporates with great rapidity, the vapour being absorbed by the sulphuric acid as fast as it is formed, and ice soon begins to appear on the surface. The experiment is, however, rather difficult to perform successfully. This arises from various causes.

In the first place, the vapour of water which occupies the upper part of the receiver is only imperfectly absorbed; and, in the second place, as the upper layer of the acid becomes diluted by absorbing the vapour, its affinity for water rapidly diminishes.

These obstacles have been removed by an apparatus invented by M. Carré, which enables us to obtain a considerable mass of ice in a few minutes. It consists (Fig. 239) of a leaden reservoir containing sulphuric acid. At one extremity is a vertical tube, the end of which is bent over and connected with a flask containing water. The other extremity of the reservoir communicates with an air-pump, to the handle of which is fitted a metallic rod, which drives an agitator immersed in the acid. By this means the surface of the acid is continually renewed, absorption takes place with regularity, and the water is rapidly frozen.

249. Cryophorus.—Wollaston's cryophorus (Fig. 240) consists of a bent tube with a bulb at each end. It is partly filled with water, and hermetically sealed while the liquid is in ebullition, thus expelling the air.

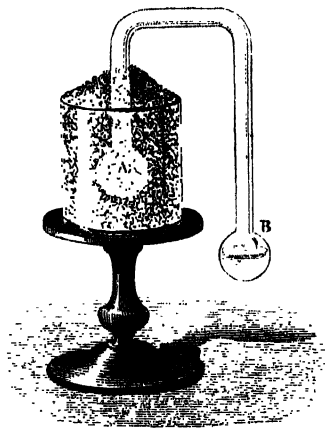


Fig. 240.—Cryophorus.

When an experiment is to be made, all the liquid is passed into the bulb B, and the bulb A is plunged into a freezing mixture, or into pounded ice. The cold condenses the vapour in A, and thus produces rapid evaporation of the water in B. In a short time needles of ice appear on the surface of the liquid.

250. Freezing of Water by the Evaporation of Ether.—Water is poured into a glass tube dipped into ether, which is contained in a glass vessel for the purpose (Fig. 241). By means of a pair of bellows a current of air is made to pass through

the ether; evaporation is quickly produced, and at the end of a few minutes the water in the tube is frozen.



Fig. 241.—Freezing of Water by Evaporation of Ether.

If, instead of promoting evaporation of the ether by means of a current of air, the vessel were placed under the exhausted receiver of an air-pump, a much greater fall of temperature would be obtained, and even mercury might easily be frozen. This experiment, however, is injurious to the pump, owing to the solvent action of the ether on the oil with which the valves and other moving parts are lubricated.

251. Freezing of Mercury by means of Sulphurous Acid.—Mercury may be frozen by means of liquid sulphurous acid, which is much more volatile than ether. In order to escape the suffocating action of the gas, the experiment is performed in the following manner:—

Into a glass vessel (Fig. 242) are poured successively mercury and liquid sulphurous acid. The vessel is closed by an india-rubber stopper, in which two glass tubes are fitted. One of these dips to the bottom of the sulphurous acid, and is connected at its outer end with a bladder full of



Fig. 242.
Freezing of Mercury
by Evaporation of
Sulphurous Acid.

air. Air is passed through the liquid by compressing the bladder, and escapes, charged with vapour; through the second opening, which is fitted with an india-rubber tube leading to the open air. Evaporation proceeds with great rapidity, and the mercury soon freezes.

252. Carré's Apparatus.—The apparatus invented some years ago by M. Carré for making ice is another instance of the application of cold produced by evaporation. It consists (Figs. 243 and 244) of two parts, a boiler and a cooler. The boiler is of wrought iron, and is so constructed as to give a very large heating surface. It is three-

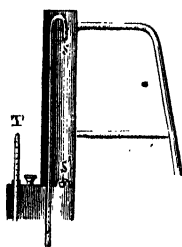
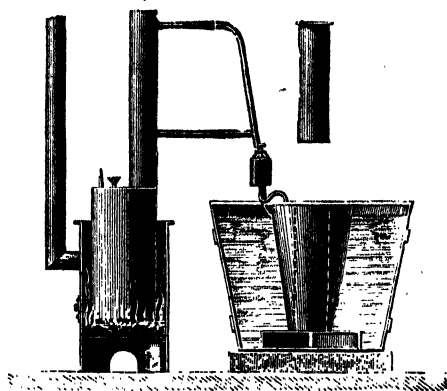


Fig. 243.



Carré's Apparatus.

Fig. 244.

quarters filled with a saturated solution of ammonia, which contains from six to seven hundred times its volume of gas. The cooler is of an annular form, and in the central space is placed a vessel containing the water to be frozen. In the sides of the cooler are a number of small cells, the object of which is to increase the surface of contact of the metal with the liquid.

In the first part of the experiment, which is represented in the figure, the boiler is placed upon a fire, and the temperature raised to 130° , while the cooler is surrounded with cold water. Ammoniacal gas is given off, passes into the cooler by the valve & opening upwards, and is condensed in the numerous cells above mentioned. This first part of the operation, in the small machines for domestic use, occupies about three-quarters of an hour. In the second part of the operation, the cylindrical vessel containing the water to be frozen is placed in the central space; the cooler is surrounded with an envelope of felt, which is a very bad conductor of heat, and the boiler is im-

mersed in cold water. The water in the boiler, as it cools, is able again to receive and dissolve the gas, which enters by the valve *s* of the bent siphon-shaped tube. The liquid ammonia in the cooler accordingly evaporates with great rapidity, producing a fall of temperature which freezes the water in the inclosed vessel.

253. Solidification of Carbonic Acid.—When a small orifice is opened in a vessel containing liquid carbonic acid, evaporation proceeds so rapidly that the cold resulting from it freezes a portion of the vapour, which takes the form of fine snow, and may be collected in considerable quantity.

This carbonic acid snow, which was first obtained by Thilorier, is readily dissolved by ether, and forms with it one of the most intense freezing mixtures known. By immersing tubes containing liquefied gases in this mixture, Faraday succeeded in reducing several of them, including carbonic acid, cyanogen, and nitrous oxide, to the form of clear transparent ice, the fall of temperature being aided, in some of his experiments, by employing an air-pump to promote more rapid evaporation of carbonic acid from the mixture. By the latter process he was enabled to obtain a temperature of -166° F. (-110° C.) as indicated by an alcohol thermometer, the alcohol itself being reduced to the consistence of oil. Despretz, by means of the cold produced by a mixture of solid carbonic acid, liquid nitrous oxide, and ether, rendered alcohol so viscid that it did not run out when the vessel which contained it was inverted.

CHAPTER XXVI.

EBULLITION.

254. Ebullition.—When a liquid contained in an open vessel is subjected to a continual increase of temperature, it is gradually changed into vapour, which is dissipated in the surrounding atmosphere. This action is at first confined to the surface; but after a certain time

bubbles of vapour are formed in the interior of the liquid, which rise to the top, and set the entire mass in motion with more or less vehemence, accompanied by a characteristic noise; this is what is meant by *ebullition* or *boiling*.



Fig. 245.—Ebullition.

If we observe the gradual progress of this phenomenon,—for example, in a glass vessel containing water, we shall perceive that, after a certain time, very minute bubbles are given off; these are bubbles of dissolved air. Soon after, at the bottom of the vessel, and at those parts of the sides which are most immediately exposed to the action of the fire, larger bubbles of vapour are formed, which decrease in volume as they ascend, and disappear before reaching the surface. This stage is accompanied by a peculiar sound, indicative of

approaching ebullition, and the liquid is said to be *singing*. The sound is probably caused by the collapsing of the bubbles as they are condensed by the colder water through which they pass. Finally, the bubbles increase in number, growing larger as they ascend, until they burst at the surface, which is thus kept in a state of agitation; the liquid is then said to boil.

255. Laws of Ebullition.—1. *At the ordinary pressure, ebullition commences at a temperature which (roughly speaking) is definite for each liquid.*

This law is analogous to that of fusion (§ 225). It follows from this that the boiling-point of any liquid is a *specific* element, serving to determine its nature.

The following table gives the boiling-points of several liquids at the pressure of 760 millimetres:—

Sulphurous acid,	- 10° C.	Spirits of turpentine,	+ 130° C.
Hydrochloric ether,	+ 11°	Phosphorus,	290°
Common ether,	37°	Concentrated sulphuric acid, . .	325°
Alcohol,	79°	Mercury,	353°
Distilled water,	100°	Sulphur,	440°

2. *The temperature, in ordinary circumstances, remains constant during ebullition.* If a thermometer be introduced into the glass vessel of Fig. 245, the temperature will be observed to rise gradually during the different stages preceding ebullition; but, when active ebullition has once commenced, no further variation of temperature will be observed. This phenomenon points to the same conclusion as the cold produced by evaporation.

Since, notwithstanding the continuous action of the fire, the temperature remains constant, the conclusion is inevitable, that all the heat produced is employed in doing the work necessary to change the liquid into vapour. The constancy of temperature during ebullition explains the fact that vessels of pewter, tin, or any other easily fusible metal, may be safely exposed to the action of even a very hot fire, provided that they contain water, since the liquid remains at a temperature of about 100°, and its contact prevents the vessel from over-heating.

3. *The elastic force of the vapour given off during ebullition is equal to the pressure of the external air.*

This important proposition may be experimentally demonstrated in the following manner:—

We takê a bent tube A, open at the longer extremity, and closed at the shorter. The short branch is filled with mercury, all but a small space containing water; in the long branch the mercury stands a little higher than the bend. Water is now boiled in a glass vessel, and, during ebullition, the bent tube is plunged into the steam. The water occupying the upper part of the short branch is partially converted into steam, the mercury falls, and it *assumes the same*

level in both branches. Thus the pressure exerted by the atmosphere at the open extremity of the tube is exactly equal to that exerted by the vapour formed by water in ebullition.

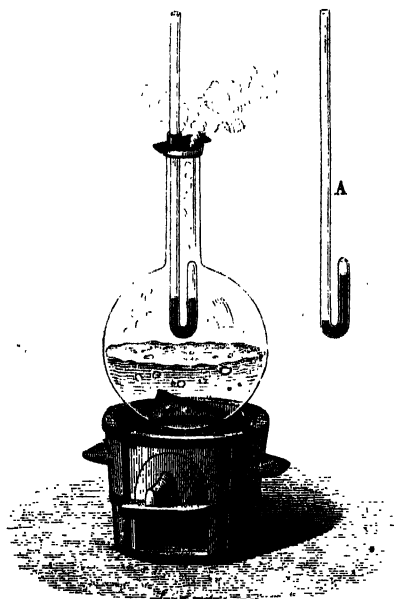


Fig. 246.—Tension of Vapour during Ebullition.

256. Theory of Ebullition.—This latter circumstance supplies the true physical definition of ebullition. *A liquid is in ebullition when it gives off vapour of the same tension as the atmosphere above it.*

The necessity of this equality of tension is easily explained. If a bubble of vapour exists in the interior of a liquid as at *m* (Fig. 247), it is subject to a pressure exceeding atmospheric by the weight of the liquid above it. As the bubble rises, the latter element of pressure becomes less, and the tension of the vapour composing the bubble accordingly diminishes, until it is reduced to

atmospheric pressure on reaching the surface.

The boiling-point of a liquid is therefore necessarily fixed, since it is the temperature at which the tension of the vapour at saturation is equal to that of the atmosphere. It must be remarked, however, that this temperature varies in the different layers of the liquid, and that it increases with the depth below the surface. Accordingly, in determining the second fixed point of the thermometer, we have stated that the instrument should be plunged into the steam, and not into the water.

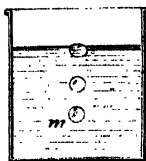


Fig. 247.

257. Effect of Pressure upon the Boiling-point.—It evidently follows from the foregoing considerations that the boiling-point of a liquid must vary with the pressure on the surface; and experiment shows that this is the case. Water, for instance, boils at 100° under the external pressure of 760 millimetres; but if the pressure decreases, ebullition occurs at a lower temperature. Under the receiver of an air-pump, water may be made to boil at any tem-

perature between 0° and 100° . In Carré's apparatus (Fig. 239) the water in the glass bottle is observed to enter into active ebullition a few moments before the appearance of the ice. The reason, therefore, why boiling water has come to be associated in our minds with a fixed temperature is that the variations of atmospheric pressure are comparatively small.

At Paris, for instance, the external pressure varies between 720 and 790 millimetres (28.3 and 31.1 inches), and the boiling-point, in consequence, varies from 98.5° to 101.1° .

258. Franklin's Experiment.—The boiling of water at a temperature lower than 100° may be shown by the following experiment:—

A little water is boiled in a flask for a sufficient time to expel most of the air contained in it. The flask is then removed from the source of heat, and is at the same time securely corked. To render the exclusion of air still more certain, it may be inverted with the corked end immersed in water which has been boiled. Ebullition ceases almost immediately; but if cold water be now poured over the vessel, or, better still, if ice be applied to it, the liquid again begins to boil, and continues to do so for a considerable time. This fact may easily be explained: the contact of the cold water or the ice lowers the temperature and tension of the steam which presses upon the surface of the liquid, and the decrease of tension causes the renewal of ebullition.



Fig. 248.—Franklin's Experiment.

260. Determination of Heights by Boiling-point.—Just as we can determine the boiling-point of water when the external pressure is given, so, if the boiling-point be known, we can determine the external pressure. In either case we have simply to refer to a table of maximum tensions of aqueous vapour at different temperatures.

As the barometer is essentially unsuitable for portability, Wollaston proposed to substitute the observation of boiling-points as a means

of determining pressures. For this purpose he employed a thermometer with a large bulb and with a scale extending only a few degrees above and below 100° . He called this instrument the barometric thermometer.

Regnault has constructed a small instrument for the same purpose, which he calls the *hypsometer*. It consists of a little boiler heated

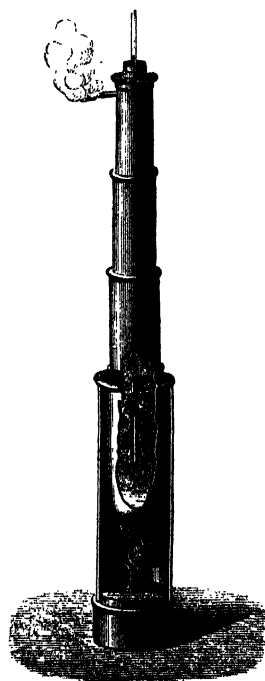


Fig. 250.—Hypsometer.

by a spirit-lamp, and terminating in a telescope tube with an opening at the side through which the steam escapes. A thermometer dips into the steam, and projects through the top of the tube so as to allow the temperature of ebullition to be read:

This temperature at once gives the atmospheric pressure by reference to a table of vapour-tensions, and the subsequent computations for determining the height are the same as when the barometer is employed (§ 112).

When only an approximate result is desired, it may be assumed that the height above sea-level is sensibly proportional to the difference between the observed boiling-point and 100° C., and Soret's formula¹ may be employed, viz.:

$$h = 295 (100 - t),$$

where h is expressed in metres and t in degrees Centigrade.

Thus, at Quito, where the boiling-point of water is about 90.1° , the height above sea-level would be $9.9 \times 295 = 2920$ metres, which agrees nearly with the true height 2808 metres.

At Madrid, at the mean pressure, the boiling-point is 97.8° , which gives $2.2 \times 295 = 649$ metres; the actual height being 610 metres.

261. Papin's Digester.—While a decrease of pressure lowers the boiling-point, an increase of pressure raises it. Accordingly, by putting the boiler in communication with a reservoir containing air at the pressure of several atmospheres, we can raise the boiling-point to 110° , 115° , or 120° ; a result often of great utility in the arts. But in

¹ If h be expressed in feet, and t in degrees Fahrenheit, the formula becomes

$$h = 538 (212 - t).$$

order that the liquid may actually enter into ebullition, the space above the liquid must be sufficiently large and cool to allow of the condensation of the steam. In a confined vessel, water may be raised to a higher temperature than would be possible in the open air, but it will not boil. This is the case in the apparatus invented by the celebrated Papin, and called after him *Papin's digester*. It is a bronze vessel of great strength, covered with a lid secured by a powerful screw. It is employed for raising water to very high temperatures, and thus obtaining effects which would not be possible with water at 100°; such for example as dissolving the gelatine contained in bones.

It is to be observed that the tension of the steam increases rapidly with the temperature, and may finally acquire an enormous power. Thus, at 200°, the pressure is that of 16 atmospheres, that is about 240 pounds on the square inch.

In order to obviate the risk of explosion, Papin introduced a device for preventing the pressure from exceeding a definite limit. This invention has since been applied to the boilers of steam-engines, and is well known as the *safety-valve*. It consists of an opening, closed by a conical valve or stopper, which is pressed down by a lever loaded with a weight. Suppose the area of the lower end of the stopper to be 1 square inch, and that the pressure is not to exceed 10 atmospheres, corresponding to a temperature of 180°. The

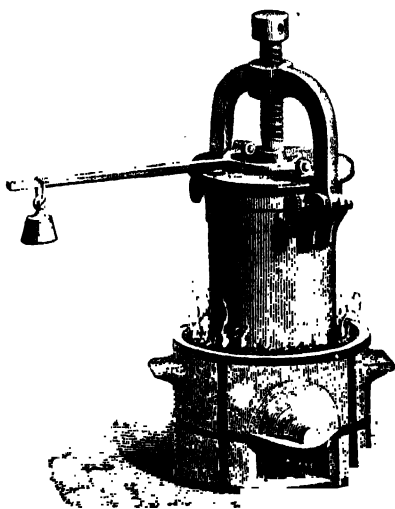


Fig. 251.—Papin's Digester.

magnitude and position of the weight are so arranged that the pressure on the hole is 10 times 15 pounds. If the tension of the steam exceed 10 atmospheres, the lever will be raised, the steam will escape, and the pressure will thus be relieved.

When the tension of the steam contained in the digester has become considerable, if the lever be raised, so as to permit some steam to escape, it rushes out with a loud noise, and produces a cloud in

the air. On placing the hand in this cloud, scarcely any sensation of heat is experienced, whereas, on performing the same experiment with steam at the ordinary pressure, the hand would certainly be scalded. This apparently paradoxical result is completely in accordance with the principles which have already been stated more than once. The steam formed at 100° , being at atmospheric pressure, preserves its pressure and temperature on issuing into the air. On the other hand, the steam generated in Papin's digester has a pressure greatly exceeding that of the atmosphere, and accordingly expands rapidly upon its exit, and thus performs work in forcing back the external air. The performance of this work is accompanied by the loss of an equivalent quantity of heat, and the temperature of the jet is consequently considerably lowered.

262. Boiling-point of Saline Solutions.—When water holds saline matters in solution, the boiling-point rises as the proportion of saline matter in the water increases. Thus with sea-salt the boiling-point can be raised from 100° to 108° .

When the solution is not saturated, the boiling-point is not fixed, but rises gradually as the mixture becomes concentrated; but at a certain stage the salt begins to be precipitated, and the temperature then remains invariable. This is to be considered the normal boiling-point of the saturated solution. Supersaturation, however, sometimes occurs, the temperature gradually rising above the normal boiling-point without any deposition of the salt, until all at once precipitation begins, and the thermometer falls several degrees.

The steam emitted by saline solutions consists of pure water, and it is frequently asserted to have the same temperature as the steam of pure water boiling under the same pressure; but the experiments of Magnus and others have shown that this is not the case. Magnus, for example,¹ found that when a solution of chloride of calcium was boiling at 107° , a thermometer in the steam indicated $105\frac{1}{2}^{\circ}$, and when by concentration the boiling-point had risen to 116° , the thermometer in the steam indicated $111\cdot2^{\circ}$.

These and other observations seem to indicate that the steam emitted by a saline solution when boiling, is in the condition in which the steam of pure boiling water would be, if heated, under atmospheric pressure, to the temperature of the boiling solution. It can therefore be cooled down to the boiling-point of pure water without undergoing any liquefaction. When cooled to this point, it becomes

¹ Poggendorff's *Annalen* cxii. p. 415.

saturated, and precisely resembles the steam of pure water boiling under the same pressure. When saturated steam loses heat, it does not cool, but undergoes partial liquefaction, and it does not become completely liquefied till it has lost as much heat as would have cooled more than a thousand times its weight of superheated¹ steam one degree Centigrade.

262 A. Boiling-point of Liquid Mixtures.—A mixture of two liquids which have an attraction for each other, and will dissolve each other freely in all proportions—for example, water and alcohol—has a boiling-point intermediate between those of its constituents. But a mechanical mixture of two liquids between which no solvent action takes place—for example, water and sulphide of carbon—has a boiling-point lower than either of its constituents. If steam of water is passed into liquid sulphide of carbon, or if sulphide of carbon vapour is passed into water, a mixture is obtained which boils at 42.6°C ., being four degrees lower than the boiling-point of sulphide of carbon alone. This apparent anomaly is a direct consequence of the laws of vapours stated in § 244; for the boiling-point of such a mixture is the temperature at which the sum of the vapour-tensions of the two independent ingredients is equal to one atmosphere.

263. Influence of Dissolved Air upon the Boiling-point.—The presence of air in the midst of the liquid mass is a necessary condition of regularity of ebullition, and of its production at the normal temperature; this is shown by several convincing experiments.

1. Donny's Experiment.—We take a glass tube bent twice, and terminated at one of its extremities by a series of bulbs. The first step is to wash it carefully with alcohol and ether, finally leaving in it some diluted sulphuric acid. These operations are for the purpose of removing the solid particles adhering to the sides, which always detain portions of air. Water is then introduced and boiled long enough to expel the air dissolved in it, and while ebullition is proceeding, the end of the apparatus is hermetically sealed. The other extremity is now plunged in a strong solution of chloride of calcium, which has a very high boiling-point, and the tube is so placed that all the water shall lie in this extremity; it will then be found that the temperature may be raised to 135° without producing ebullition.

¹ That is steam heated above the temperature of saturation. Philosophically speaking, superheated steam is merely nonsaturated steam; but the name is never used except where the temperature exceeds the atmospheric boiling-point.

At about this temperature bubbles of steam are seen to be formed, and the entire liquid mass is thrown forward with great violence.

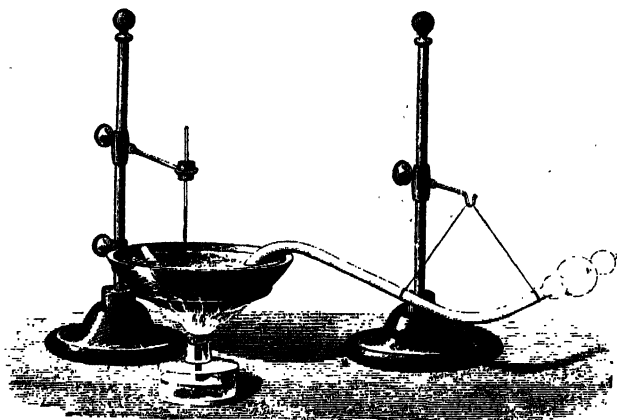


Fig. 252.—Donny's Experiment.

The bulbs at the end of the tube are intended to diminish the shock thus produced.

2. *Dufour's Experiment.*—This experiment is still more decisive. A mixture of linseed-oil and oil of cloves, whose respective densities are about .93 and 1.01, is so prepared that, for temperatures near 100° , the density of the whole is nearly that of water. This mixture is placed in a cubical box of sheet-iron, with two holes opposite each other, which are filled with glass, so as to enable the observer to perceive what is passing within. The box is placed in a metallic envelope, which permits of its being heated laterally. When the temperature of 120° has been reached, a large drop of water is allowed to fall into the mixture, which, on reaching the bottom of the box, is partially converted into vapour, and breaks up into a number of smaller drops, some of which take up a position between the two windows, so as to be visible to the observer. The temperature may now be raised to 140° , 150° , or even 180° , without producing evaporation of any of these drops. Now the maximum tension of steam at 180° is equal to 10 atmospheres, and yet we have the remarkable phenomenon of a drop of water remaining liquid at this temperature under no other pressure than that of the external air increased by an inch or two of oil. The reason is that the air necessary to evaporation is not supplied. If the drops be touched with a rod of metal, or, better still, of wood, they are immediately converted into vapour

with great violence, accompanied by a peculiar noise. This is explained by the fact that the rods used always carry a certain quantity of condensed air upon their surface, and by means of this air the evaporation is produced. The truth of this explanation is proved by the fact, that when the rods have been used a certain number of times, they lose their power of provoking ebullition, owing, no doubt, to the exhaustion of the air which was adhering to their surfaces.

3. *Production of Ebullition by the formation of Bubbles of Gas in the midst of a Liquid.*—A retort is carefully washed with sulphuric acid, and then charged with water slightly acidulated, from which the air has been expelled by repeated boiling. The retort communicates with a manometer and with an air-pump. The air is exhausted until a pressure of only 150 millimetres is attained, corresponding to 60° as boiling-point. Dufour has shown that under these conditions the temperature may be gradually raised to 75° without producing ebullition. But if, while things are in this condition, a current of electricity is sent through the liquid by means of two platinum wires previously immersed in it, the bubbles of oxygen and hydrogen which are evolved at the wires immediately produce violent ebullition, and a portion of the liquid is projected explosively, as in Donny's experiment.

From these experiments we may conclude that liquid, when not in contact with gas, has a difficulty in *making a beginning* of vaporization, and may hence remain in the liquid state even at temperatures at which vaporization would upon the whole involve a fall of potential energy.

That vapour (as well as air) can furnish the means of overcoming this difficulty, is established by the fact noted by Professor G. C. Foster,¹ that when a liquid has been boiling for some time in a retort, it sometimes ceases to exhibit the movements characteristic of ebullition, although the amount of vapour evolved at the surface, as measured by the amount of liquid condensed in the receiver, continues undiminished. In these circumstances, it would appear that the superficial layer of liquid, which is in contact with its own vapour, is the only part that is free to vaporize.

The preceding remarks explain the reluctance of water to boil in glass vessels carefully washed, and the peculiar formation, in these circumstances,² of large bubbles of steam, causing what is called *boil-*

¹ Watts's *Dictionary of Chemistry*, art. "Heat," p. 88.

ing by bumping. In the case of sulphuric acid, the phenomenon is much more marked; if this liquid be boiled in a glass vessel, enormous bubbles are formed at the sides, which, on account of the viscous

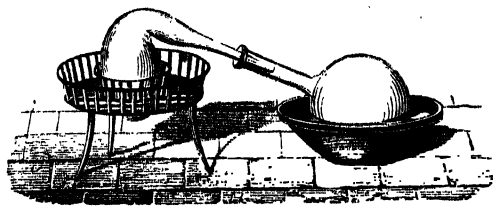


Fig. 253.—Apparatus for boiling Sulphuric Acid.

nature of the liquid, raise the mass of the liquid above them, and then let it fall back with such violence as sometimes to break the vessel. This inconvenience may be avoided by using an annular brazier (Fig. 253),

by means of which the upper part only of the liquid is heated.

The ebullition of ether and alcohol presents some similar features, probably because these liquids dissolve the fatty particles on the surface of the glass, and thus adhere to the sides very strongly.

264. Spheroidal State.—This is the name given to a peculiar condition which is assumed by liquids when exposed to the action of very hot metals.

If we take a smooth plate of iron or silver, and let fall a drop of water upon it, the drop will evaporate more rapidly as the tempera-

ture of the plate is increased up to a certain point. When the temperature of the plate exceeds this limit, which, for water, appears to be about 150° , the drop assumes a spheroidal form, rolls about like a ball or spins on its axis, and frequently exhibits a beautiful rippling, as represented in the figure. While in this condition, it evaporates much more slowly than when the plate was at a lower temperature. This latter circumstance is important, and is easily verified by experi-

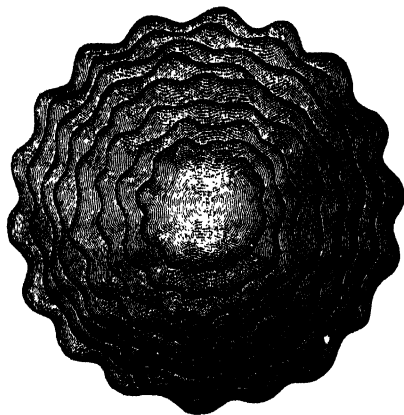


Fig. 254.—Globule in the Spheroidal State.

ment. If the plate be allowed to cool, a moment arrives when the globule of water flattens out, and boils rapidly away with a hissing noise.

These phenomena have been long known, and were studied by

Leidenfrost and Klaproth; but the subject has recently been more completely investigated by Boutigny. All liquids are probably capable of assuming the spheroidal state. Among those which have been tested are alcohol, ether, liquid sulphurous acid, and liquid nitrous oxide. When in this state they do not boil. Sometimes bubbles of steam are seen to rise and burst at the top of the globule, but these are always owing to some roughness of the surface, which prevents the steam from escaping in any other way; when the surface is smooth, no bubbles are observed.

If the temperature of the liquid be measured by means of a thermometer with a very small bulb, or a thermo-electric junction, it is always found to be below the boiling-point.

265. Freezing of Water and Mercury by means of the Spheroidal State.—This latter property enables us to obtain some very striking and paradoxical results. The boiling-point of liquid sulphurous acid is -10°C ., and that of liquid nitrous oxide is about -70°C . If a silver or platinum crucible be heated to redness by a powerful lamp, and some liquid sulphurous acid be then poured into it, this latter assumes the spheroidal state; and drops of water let fall upon it are immediately frozen. Mercury can in like manner be frozen in a red-hot crucible by employing liquid nitrous oxide in the spheroidal state.

These experiments are due to Boutigny, who called attention to them as remarkable exceptions to the usual tendency of bodies to equilibrium of temperature. The exception is of the same kind as that presented by a vessel of water boiling at a constant temperature of 100° over a hot fire, the heat received by the liquid being in both cases expended in producing evaporation.

266. The Metal not in Contact with the Liquid.—The basis of the entire theory of liquids in the spheroidal state is the fact that the liquid and the metal plate do not come into contact. This fact can be proved by direct observation.

The plate used must be quite smooth and accurately levelled. When the plate is heated, a little water is poured upon it, and assumes the spheroidal state. By means of a fine platinum wire which passes into the globule, the liquid is kept at the centre of the metal plate. It is then very easy, by placing a light behind the globule, to see distinctly the space between the liquid and the plate. The appearance thus presented may be easily thrown on a screen by means of the electric light.

The interruption of contact can also be proved by connecting (through a galvanometer) one pole of a battery with the hot plate, while a wire from the other pole is dipped in the liquid. The cur-

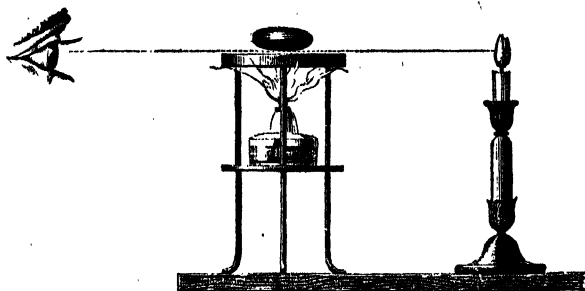


Fig. 255.—Separation between Globule and Plate.

rent refuses to circulate if the liquid is in the spheroidal state, but is immediately established when, on cooling the plate, the liquid begins to boil.

This separation is maintained by the rush of steam from the under-surface of the globule, which is also the cause of the peculiar movements above described.

In consequence of the separation, heat can only pass to the globule by radiation, and hence its comparatively low temperature is accounted for.

The absence of contact between a liquid and a metal at a high temperature may be shown by several experiments. If, for instance, a ball of platinum be heated to bright redness, and plunged (Fig. 256) into water, the liquid is seen to recede on all sides, leaving an envelope of vapour round the ball. This latter remains red for several seconds,

and contact does not take place till its temperature has fallen to about 150° . An active ebullition then takes place, and an abundance of steam is evolved.

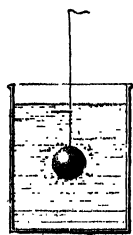


Fig. 256.—Red-hot Ball in Water.

If drops of melted sugar be let fall on water, they will float for a short time, though their density is greater than that of water (§ 79), contact being prevented by their high temperature. A similar phenomenon is observed when a fragment of potassium is thrown on water. The water is decomposed; its hydrogen takes fire and burns with a red flame; its oxygen combines with the potassium to form potash; and the globule of potash floats upon the surface without touching it, owing to the high temperature under

which it is formed. After a few seconds the globule cools sufficiently to come into contact with the water, and bursts with a slight noise.

267. Distillation.—Distillation consists in boiling a liquid and condensing the vapour evolved. It enables us to separate a liquid from the solid matter dissolved in it, and to effect a partial separation of the more volatile constituent of a mixture from the less volatile.

The apparatus employed for this purpose is called a still. One of the simpler forms, suitable for distilling water, is shown in Fig. 257.

It consists of a retort *a*, the neck of which *c* communicates with a

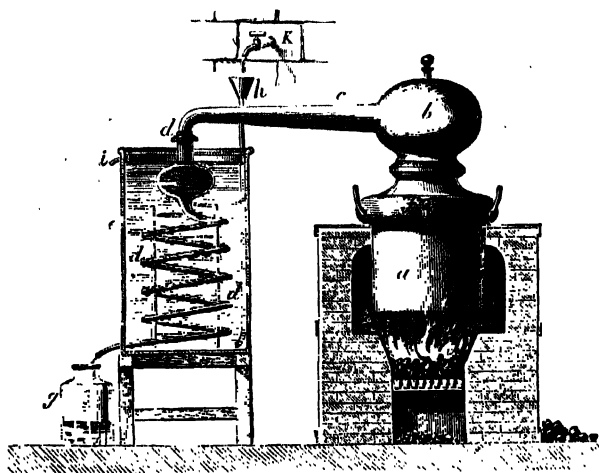


Fig. 257.—Still.

spiral tube *d d* called the *worm*, placed in the vessel *e*, which contains cold water. The water in the retort is raised to ebullition, the steam given off is condensed in the worm, and the *distilled water* is collected in the vessel *g*.

As the condensation of the steam proceeds, the water of the cooler becomes heated, and must be renewed; for this purpose a tube descending to the bottom of the cooler is supplied with a continuous stream of cold water from above, while the superfluous water flows out by the tube *i* at the upper part of the cooler. In this way the warm water, which rises to the top, is continually removed. The boiler is filled about three-quarters full, and the water in it can from time to time be renewed by the opening *f*; but it is advisable not to carry the process of distillation too far, and to throw away the liquid

remaining in the boiler when its volume has been reduced to a fourth or a fifth of what it was originally. . By exceeding this limit we run the risk of impairing the purity of the water by the carrying over of some of the solid matter contained in the liquid in the boiler.

269. Circumstances which Influence Rapidity of Evaporation.—In the case of a liquid exposed to the air, and at atmospheric temperature, the rapidity of evaporation increases with the extent of free surface, the dryness of the air, and the rapidity of renewal of the air immediately above the surface.

In the case of a liquid evaporated by boiling, the quantity evaporated in a given time is proportional to the heat received. This depends upon the intensity of the source of heat, the facility with which heat passes through the sides of the vessel, and the area of *heating surface*, that is to say, of surface (or more properly lamina) which is in contact with the liquid on one side, and with the source of heat on the other.

CHAPTER XXVII

MEASUREMENT OF THE MAXIMUM TENSIONS OF VAPOURS.

270. Tension of Aqueous Vapour.—The knowledge of the maximum tension of the vapour of water at various temperatures is important, not only from a theoretical, but also from a practical point of view, inasmuch as this tension is the motive force in the steam-engine. Experiments for the purpose of determining the values of this element have accordingly been undertaken by several experimenters in different countries. The researches conducted by Regnault are especially remarkable for the range of temperature which they embrace, as well as for the number of observations which they include, and the extreme precision of the methods employed. Next to these in importance are the experiments of Magnus in Germany and of Fairbairn and Tate in England.

271. Dalton's Apparatus.—The first investigations in this subject which have any pretensions to accuracy were those of Dalton. The apparatus which he employed is represented in Fig. 259. Two barometric tubes A and B are inverted in the same cistern H; one is an ordinary barometer, the other a vapour-barometer; that is, a barometer in which a few drops of water have been passed up through the mercury. The two tubes, attached to the support CD, are surrounded by a cylindrical glass vessel containing water which can be raised to different temperatures by means of a fire. The first step is to fill the vessel with ice, and then read the difference of level of the mercury in the two tubes. This can be done by separating the fragments of ice. The difference thus observed is the tension of aqueous vapour at zero Centigrade. The ice is then replaced by water, and the action of the fire is so regulated as to give different temperatures, ranging between 0° and 100°C , each of which is preserved constant for a few minutes, the water being at the same time

well stirred by means of the agitator *pq*, so as to insure uniformity of temperature throughout the whole mass. The difference of level

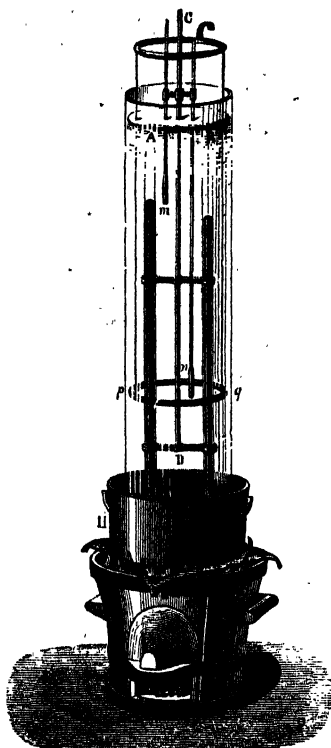


Fig. 259.—Dalton's Apparatus.

in the two barometers is read off in each case; and we have thus the means of constructing, with the aid of graphical or numerical interpolation, a complete table of vapour-tensions from 0° to 100° C. At or about this latter temperature the mercury in the vapour-barometer falls to the level of the cistern; and the method is therefore inapplicable for higher temperatures. Such a table was constructed by Dalton.

272. Regnault's Modifications.—Dalton's method has several defects. In the first place, it is impossible to insure that the temperature shall be everywhere the same in a column as long as that which is formed by the vapour at 70° , 75° , and higher temperatures. In the second place, there is always a good deal of uncertainty in observing the difference of level through the sides of the cylindrical glass vessel. Regnault employed this method only up to the temperature of 50° C. At this temperature the tension of the vapour is only

about 9 centimetres (less than 4 inches) of mercury, and it is thus unnecessary to heat the barometers throughout their entire length. The improved apparatus is represented in Fig. 260. The two barometric tubes, of an interior diameter of 14 millimetres, traverse two holes in the bottom of a metal box. In one of the sides of the box is a large opening closed with plate-glass, through which the necessary observations can be made with great accuracy. On account of the shortness of the liquid column it was very easy, by bringing a spirit-lamp within different distances of the box, to maintain for a sufficient time any temperature between 0° and 50° C.

The difference of level between the two mercurial columns should be reduced to 0° C. by the ordinary correction. We should also take into consideration the short column of water which is above the

mercury in the vapour barometer, and which, by its weight, produces a depression that may evidently be expressed in mercury by dividing the height of the column by 13.59.

To adapt this apparatus to low temperatures, it is modified in the following way. The upper extremity of the vapour barometer tube is drawn out and connected with a small copper tube of three branches, one of which communicates with an air-pump, and another with a glass globe of the capacity of about 500 cubic centimetres. In the interior of this globe is a small bulb of thin glass containing water, from which all the air has been expelled by boiling. The globe is several times exhausted of air, and after each exhaustion is refilled with air which has been passed over desiccating substances. After the last exhaustion, the tube which establishes communication with the air-pump is hermetically sealed, the box is filled with ice, and the tension at zero of the dry air left behind in the globe by the air-pump is measured; it is of course exceedingly

small. Heat is then applied to the globe, the little bulb bursts, and the globe, together with the space above the mercury, is filled with vapour. This form of apparatus can also be employed to measure tensions at temperatures up to 50° , the only difference being that the ice is replaced by water at different temperatures, allowance being made, in each case, for the elastic force of the unexhausted air.

In the case of temperatures below zero, the box is no longer required, and the globe alone is placed in a vessel containing a freezing mixture. The barometric tubes are surrounded by the air of the apartment.

In this case the space occupied by the vapour is at two different

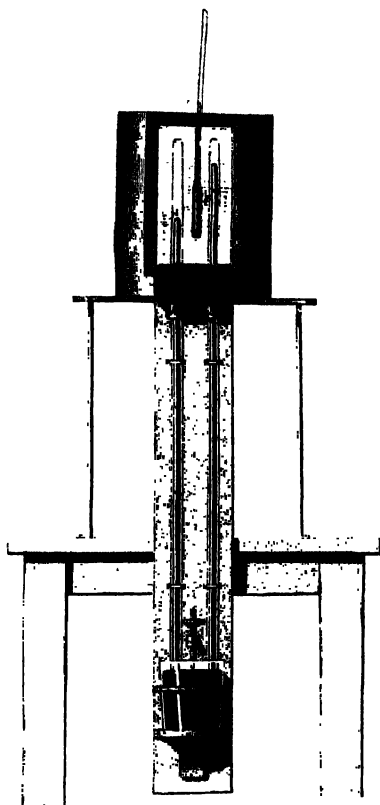


Fig. 260.—Modified Form of Dalton's Apparatus.

temperatures in different parts, but it is evident that equilibrium can exist only when the tension is the same throughout. But the tension of the vapour in the globe can never exceed the maximum tension for the actual temperature; this must therefore be the tension throughout the entire space, and is consequently that which corresponds to the difference of level observed.

In reality what happens is as follows:—The low temperature of the globe causes some of the vapour to condense; equilibrium is consequently destroyed, a fresh quantity of vapour is produced, enters the globe, and is there condensed, and so on, until the tension is everywhere the same as the maximum tension due to the temperature of the globe. This condensation of vapour in the cold part of the space was utilized by Watt in the steam-engine; it is the *principle of the condenser*.

Before Regnault, Gay-Lussac had already turned this principle to account in a similar manner for the measurement of low temperatures.

By using chloride of calcium mixed with successively increasing quantities of snow or ice, the temperature can be brought as low as -32°C . (-25.6°F .), and it can be shown that the tension of the vapour of water is quite appreciable even at this point.

273. Measurement of Maximum Tensions for Temperatures above 50° .—In investigating the tension of the vapour of water at temperatures above 50° , Regnault made use of the fact that the maximum tension of steam at the boiling-point is equal to the external pressure.

His apparatus consists (Fig. 261) of a copper boiler containing water which can be raised to different temperatures indicated by very delicate thermometers. The vapour produced passes through a tube inclined upwards, which is kept cool by a constant current of water; in this way the experiment can be continued for any length of time, as the vapour formed by ebullition is condensed in the tube, and flows back into the boiler. The tube leads to the lower part of a large reservoir, in which the air can be either rarefied or compressed at will. This reservoir is in communication with a manometer. The apparatus shown in the figure is that employed for pressures not exceeding 5 atmospheres. Much greater pressures, extending to 28 atmospheres, can be attained by simply altering the dimensions of the apparatus without any change in its principle. The manometer employed in this case was the same as that used in testing Boyle's law, consisting of a long column of mercury (§ 121).

In using this apparatus, the air in the reservoir is first rarefied until the water boils at about 50°C .; the occurrence of ebullition being recognized by its characteristic sound, and by the temperature

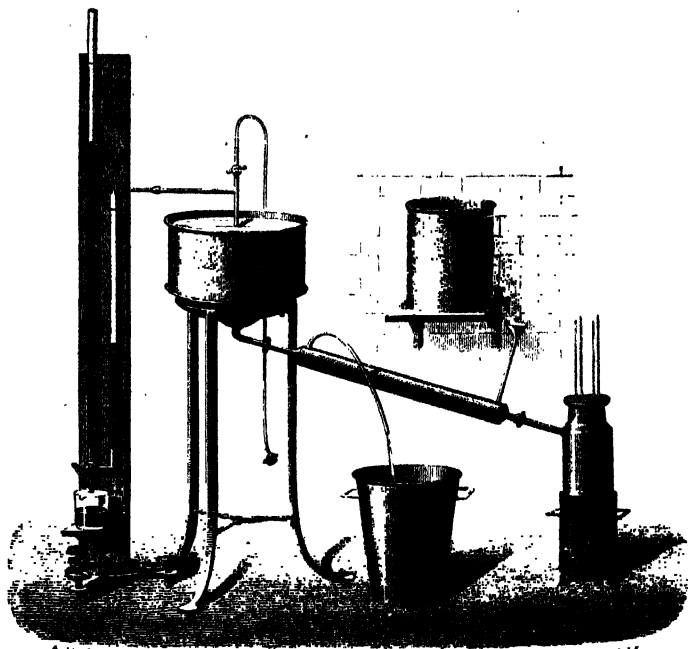


Fig. 261.—Regnault's Apparatus for High Temperatures.

remaining invariable. This steadiness of temperature is of great advantage in making the observations, inasmuch as it enables the thermometers to come into perfect equilibrium of temperature with the water. The tension indicated by the manometer during ebullition is exactly that of the vapour produced. By admitting air into the reservoir, the boiling-point is raised by successive steps until it reaches 100° . After this, air must be forced into the reservoir by a compression-pump.

The following is an abstract of the results thus obtained:—

Temperatures Centigrade.	Tensions in Millimetres of Mercury.	Temperatures Centigrade.	Tensions in Millimetres of Mercury.
-32°	0.32	5°	6.58
-20	0.93	10	9.17
-10	2.09	15	12.70
-5	3.11	20	17.39
0	4.60	25	23.55

Temperatures Centigrade.	Tensions in Millimetres of Mercury.
30°	31.55
35	41.82
40	54.91
45	71.39
50	91.98
55	117.47
60	148.70
65	186.94

Temperatures Centigrade.	Tensions in Millimetres of Mercury.
70°	238.09
75	288.61
80	354.64
85	433.04
90	525.45
95	633.77
100	760.00

	Tensions in Atmospheres.
100°	1
121	2.025
134	3.008
144	4.000
152	4.971
159	5.966
171	8.036

	Tensions in Atmospheres.
180°	9.929
189	12.125
199	15.062
213	19.997
225	25.125
239	27.534

274. Curves of Vapour-tension.—The comparison of these tensions with their corresponding temperatures affords no clue to any simple relation between them which might be taken as the physical law of the phenomena. It would appear that the law of variation of maximum tensions is incapable of being thrown into any simple expression—judging at least from the failure of all efforts hitherto made. An attentive examination of the above table will enable us to assert only that the maximum tension varies very rapidly with

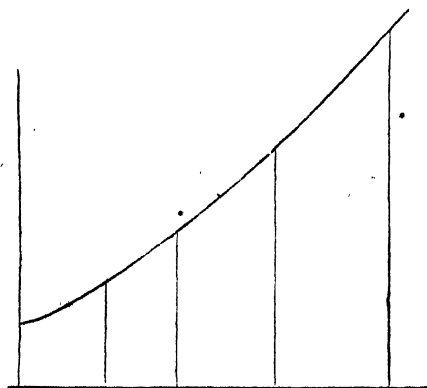


Fig. 262.

the temperature. Thus between 0° and 100° the variation is only 1 atmosphere, but between 100° and 200° it is about 15, and between 200° and 230° about 13 atmospheres.

The clearest way of representing to the mind the law according to which vapour-tension varies with temperature, is by means of a curve whose ordinates represent vapour-tensions, while the abscissæ represent the corresponding temperatures. Such a curve is exhibited in Fig. 262. Lengths proportional to the temperatures, reckoned from 0° C., are laid off on the base-line (called the line of abscissæ), and perpendiculars (called ordi-

nates) are erected at their extremities, the lengths of these perpendiculars being made proportional to the vapour-tensions. The scales employed for the two sets of lengths are of course quite independent of one another, their selection being merely a question of convenience. The curve itself is obtained by joining the extremities of the perpendiculars, taking care to avoid sudden changes of direction; and it not only serves to convey to the mind an idea of the amounts of vapour-tensions and their rates of variation at different temperatures, but also furnishes the readiest means of determining the vapour-tensions at temperatures intermediate between those of observation (see §88 B).

It will be noticed that the curve becomes steeper as the temperature increases, indicating that the tension increases faster than the temperature.

275. Empirical Formulæ.—Though all attempts at finding a rational formula for vapour-tension in terms of temperature have hitherto failed, it is easy to devise empirical formulæ which yield tolerably accurate results within a limited range of temperature; and by altering the values of the constants in such a formula by successive steps, it may be adapted to represent in succession the different portions of the curve above described.

The simplest of these approximate formulæ¹ is that of Dulong and Arago, which may be written—

$$\left(\frac{40 + C}{140}\right)^5 \quad \text{or} \quad \left(\frac{38 + F}{250}\right)^5,$$

and gives the maximum tension *in atmospheres*, corresponding to the temperature C° Centigrade, or F° Fahrenheit. This formula is rigorously correct at 100°C., and gives increasing errors as the temperature departs further from this centre, the errors amounting to about 1½ per cent. at the temperatures 80° C. and 225° C. Hence it appears that between these limits the maximum tension of aqueous vapour is nearly proportional to the fifth power of the excess of the temperature above —40° C.

276. Tensions of the Vapours of Different Liquids.—Dalton held that *the vapours of different liquids have equal tensions at temperatures equally removed from their boiling-points*. Thus the boiling-point of alcohol being 78°, the tension of alcohol vapour at 70° should be equal to that of the vapour of water at 92°. If this law were correct,

¹ For a more accurate formulæ, see Rankine on Steam-engine, p. 237.

it would only be necessary to know the boiling-point of any liquid in order to estimate the tension of its vapour at any given temperature; but subsequent experiment has shown that the law is far from being rigorously exact, though it is approximately correct for temperatures differing by only a few degrees from the boiling-points.

Regnault has performed numerous experiments on the vapour-tensions of some of the more volatile liquids, employing for this purpose the same form of apparatus which had served for determining the tensions of aqueous vapour. The following are some of his results:—

VAPOUR OF ALCOHOL.

Temperatures Centigrade.	Tensions in Millimetres.	Temperatures Centigrade.	Tensions in Millimetres.
-20°	3.24	+ 30°	78.52
0	12.70	100	1697.55
+10	24.23	155	6259.19

VAPOUR OF ETHER.

-20°	68.90	+ 30°	634.80
0	184.39	100	4953.30
+10	286.83	120	7719.20

VAPOUR OF SULPHIDE OF CARBON.

-20°	47.30	+ 30°	434.62
0	127.91	100	3325.15
+10	198.46	150	9095.94

277. Expression of Vapour-tension in Absolute Measure.—The maximum tension of a given vapour at a given temperature is, from its very nature, independent of geographical position, and should therefore, properly speaking, be denoted by one and the same number at all places. This numerical uniformity will not exist if the tension be expressed, as in the preceding sections, in terms of the length of a column of mercury which balances it. For example, in order to adapt Regnault's determinations to London, we must multiply them by the fraction $\frac{3457}{3456}$, inasmuch as 3456 millimetres of mercury exert the same pressure at London as 3457 at Paris. In general, to adapt determinations of pressure made at a place A, to another place B, we must multiply them by the fraction

$$\frac{\text{gravity at A}}{\text{gravity at B}}$$

If the length of mercurial column at 0°C. which balances a vapour-tension at a given place be multiplied by the value of g (denoting intensity of gravity) at that place, the product may be called the

absolute value of the vapour-tension; for the products thus obtained will be the same at all localities, This is the proper mode of treating determinations of vapour-tension made at various localities when it is desired to establish a rigorous comparison between them.

278. Laws of Combination by Volume.—It was discovered by Gay-Lussac, that when two or more gaseous elements at the same temperature and pressure enter into chemical combination with each other, the two following laws apply:—

1. The volumes of the components bear a very simple ratio to each other, such as 2 to 3, 1 to 2, or 1 to 1.

2. The volume of the compound has a simple ratio to the sum of the volumes of the components.

Ammonia, for example, is formed by nitrogen and hydrogen uniting in the proportion of one volume of the former to three of the latter, and the volume of the ammonia, if reduced to the same pressure as each of its constituents, is just half the sum of their volumes. Further investigation has led to the conclusion (which is now generally received, though hampered by some apparent exceptions), that these laws apply to all cases of chemical combination, the volumes compared being those which would be occupied respectively by the combining elements and the compound which they form, *when reduced to the state of vapour*, at such a temperature and pressure as to be very far removed from liquefaction, and consequently to possess the properties of what we are accustomed to call permanent gases.

It is obvious that if all gases and vapours were equally expansible by heat, the volume-ratios referred to in this law would be the same at all temperatures; and that, in like manner, if they were all equally compressible (whether obeying Boyle's law, or departing equally from it at equal pressures), the volume-ratios would be independent of the pressure at which the comparison was made.

In reality great differences exist between different vapours in both respects, and these inequalities are greater as the vapours are nearer to saturation. It is accordingly found that the above laws of volume-ratio often fail to apply to vapours when under atmospheric pressure and within a few degrees of their boiling-points, and that, in such cases, a much nearer fulfilment of the law is obtained by employing very high temperatures, or operating in inclosures at very low pressures.

278 A. Relation of Vapour-densities to Chemical Equivalents.—Chem-

ists have determined with great accuracy the combining proportions by weight of most of the elements. Hence the preceding laws can be readily tested for bodies which usually exist in the solid or liquid form, if we are able to compare the densities of their vapours. In fact, if two such elements combine in the ratio, by weight, of w_1 to w_2 , we have

$$v_1 = \frac{w_1}{d_1}, \quad v_2 = \frac{w_2}{d_2},$$

v_1, v_2, d_1, d_2 denoting the volumes and densities of the vapours of weights w_1, w_2 of the two substances.

Hence we have the equation—

$$\frac{v_1}{v_2} = \frac{w_1}{w_2} \cdot \frac{d_2}{d_1},$$

which gives the required volume-ratio of the vapours, if the ratio of their densities be known.

The densities themselves will differ enormously according to the pressure and temperature at which they are taken, but their ratio will only vary by comparatively small amounts, and would not differ at all if they were equally expansible by heat, and equally compressible. Hence comparison will be facilitated by tabulating the ratios of the densities to that of some standard gas, namely air, under the same conditions of pressure and temperature, rather than the absolute densities. This is accordingly the course which is generally pursued, so generally indeed, that by the *vapour-density of a substance* is commonly understood the relative density as measured by this ratio.

The process most frequently employed for the determination of this element is that invented by Dumas.

279. Dumas' Method.—The apparatus consists of a glass globe B, containing the substance which is to be converted into vapour.

The globe is placed in a vessel C, containing some liquid which can be raised to a suitable temperature. If the substance to be operated on is one which can be vaporized at 100°C ., the bath consists simply of boiling water. When higher temperatures are required, a saline solution, oil, or a fusible alloy is employed. In all cases, the liquid should be agitated, that its temperature may be the same in all parts. This temperature is indicated by the thermometer t .

When the substance in the globe has attained its boiling-point, evaporation proceeds rapidly, and the vapour escapes, carrying out

the air along with it. When the vapour ceases to issue, we may assume, if the quantity of matter originally taken has been sufficiently large, that all the air has been expelled, and that the globe is full of vapour at the temperature given by the thermometer, and at the external pressure H . The globe is then hermetically sealed at the extremity p of the neck, which has been previously drawn out into a fine tube.

280. Calculation of the Experiment. — As already remarked, the densities of vapours given in treatises on chemistry express the

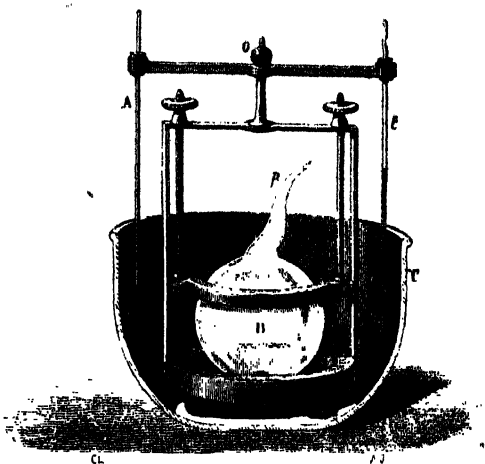


Fig. 283.—Dumas' Apparatus.

ratio of the weight of a given volume of the vapour to that of the same volume of air at the same temperature and pressure. In order to deduce this ratio from the preceding experiment, we must first find the weight of the vapour. This is done by weighing the globe with its contents, after allowing it to cool. Suppose the weight thus found to be W . Before the experiment the globe had been weighed full of dry air at a known temperature t and pressure h . Suppose this weight to be W' ; the difference $W - W'$ evidently represents the excess of the weight of the vapour above that of the air. If, then, we add $W - W'$ to the weight of the air, we shall evidently have the weight of the vapour. Now the weight of the air is easily deduced from the known volume of the globe. If V denote this volume at zero expressed in litres, the weight in grammes of the air contained in the globe at the time of weighing is

$$V (1 + Kt) 1.293 \times \frac{1}{1 + at} \cdot \frac{h}{760}$$

K denoting the coefficient of cubical expansion of glass, and a the coefficient of expansion of air. The weight of the vapour contained in the globe is consequently

$$A = W - W' + V (1 + Kt) \times 1.293 \times \frac{1}{1 + at} \cdot \frac{h}{760}$$

Let H be the pressure, and T the temperature at the time of sealing

the globe. The volume occupied by the vapour under these circumstances was $V(1+KT)$. The density of the vapour will therefore be obtained by dividing A by the weight of this volume of air at the same temperature and pressure. But this weight is

$$A' = V(1+KT) \cdot 1.293 \cdot \frac{1}{1+\alpha T} \cdot \frac{H}{760};$$

hence, finally, the required relative density is

$$D = \frac{A}{A'} = \frac{W - W' + V(1+Kt) \cdot 1.293 \cdot \frac{1}{1+\alpha t} \cdot \frac{h}{760}}{V(1+KT) \cdot 1.293 \cdot \frac{1}{1+\alpha T} \cdot \frac{H}{760}}.$$

The correctness of this formula depends upon the assumption that no air is left in the globe. In order to make sure that this condition is fulfilled, the point p of the neck of the globe is broken off under mercury; the liquid then rushes in, and, together with the condensed vapour, fills the globe completely, if no air has been left behind.

This last operation also affords a means of calculating the volume V ; for we have only to weigh the mercury contained in the globe, or to measure it in a graduated tube, in order to ascertain its volume at the actual temperature, whence the volume at zero can easily be deduced.

281. Example.—In order better to illustrate the method, we shall take the following numerical results obtained in an investigation of the vapour-density of sulphide of carbon:—

Excess of weight of vapour above weight of air, $W - W' = .3$ gramme; temperature of the vapour $T = 59^\circ$; external pressure $H = 752.5$ millimetres; volume of the globe at a temperature of 12° , 190 cubic centimetres; temperature of the dry air which filled the globe at the time of weighing, $t = 15^\circ$; pressure $h = 765$; $K = \frac{1}{38700}$.

The volume V of the globe at zero is

$$\frac{190}{1 + \frac{12}{38700}} = 189.94 \text{ cubic centimetres} = .18994 \text{ litre.}$$

The weight of the air contained in the globe is

$$.18994 \times 1.293 \cdot \left(1 + \frac{12}{38700}\right) \cdot \frac{1}{1 + 12 \times .00366} \cdot \frac{765}{760} = .23658 \text{ gramme.}$$

Weight of the vapour, .

$$.23658 + W - W' = .53358 \text{ gramme.}$$

The weight of the same volume of air at the same temperature and pressure is

$$18994 \times 1.293 \left(1 + \frac{59}{38700}\right) \cdot \frac{1}{1 + .00366 \times 59} \cdot \frac{752.5}{760} = 20019 \text{ grammes.}$$

The density is therefore .

$$\frac{.53658}{.20019} = 2.67.$$

Deville and Troost have effected several improvements in the application of Dumas' method to vapours at high temperatures. These temperatures are obtained by boiling various substances, such as chloride of zinc, cadmium, which boils at 860°C., or zinc, which boils at 1040°C. For temperatures above 800°, the glass globe is replaced by a globe of porcelain, which is hermetically sealed with the oxyhydrogen blowpipe. The globe itself serves as a pyrometer to determine the temperature; and since the weight of air becomes very inconsiderable at high temperatures, some heavier vapour, such as that of iodine, is substituted in its place. If we suppose, as we may fairly do, that at these high temperatures the coefficient of expansion of the vapour of iodine is the same as that of air, the temperature may easily be deduced from the weight of iodine contained in the globe. We subjoin a table of some relative densities of vapours obtained by this method:—

Water,	0.622	Phosphorus,	4.5
Alcohol,	1.6138	Cadmium,	3.94
Ether,	2.586	Chloride of aluminium,	9.347
Spirit of turpentine,	5.0130	Bromide of aluminium,	18.62
Iodine,	8.716	Chloride of zirconium,	8.1
Sulphur,	2.23	Sesquichloride of iron,	11.39

282. Limiting Values of Relative Densities.—In investigating the relative density of acetic acid vapour, Cahours found that it went on decreasing as the temperature increased, up to a certain point, after which no further change was observable. A similar circumstance is observed in the case of all substances, only in different degrees. The vapour of sulphur, for instance, has a relative density of 6.65 at 500°C., while at about 1000°C. it is only 2.23. This indicates that the vapours in question are more expansible by heat than air until the limiting temperatures are attained. Indeed it may be laid down as a general principle, that the nearer a vapour is to saturation the greater is the change produced in its absolute density by a given change whether of temperature or pressure. The limiting density-ratio is always that which it is most important to determine,

and we should consequently take care that the temperature of the vapour is sufficiently high to enable us to obtain it.

283. Gay-Lussac's Method.—Gay-Lussac determined the density of the vapour of water and of some other liquids by a method a little more complicated than that described above, and which for that reason has not been generally adopted in the laboratory. We proceed to describe it, however, on account both of its historical interest and of the importance of the question which it has assisted in solving.

A graduated tube divided into cubic centimetres, suppose, is filled with mercury, and inverted in a cast-iron vessel containing the same liquid. The inverted tube is surrounded by a glass envelope containing water, as in Dalton's apparatus. A small glass bulb containing a given weight w (expressed in grammes) of distilled water is passed into the tube, and rises to the surface of the mercury. The

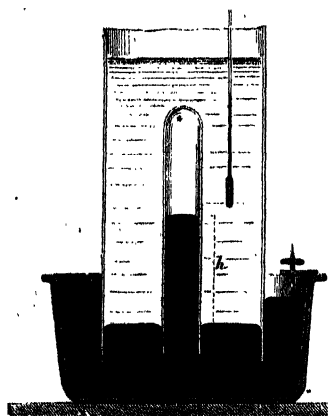


Fig. 264.—Gay-Lussac's Apparatus.

temperature of the apparatus is then raised by means of a fire below, the bulb bursts, and the water which it contained is converted into vapour. If the quantity of water be not too great, it is all converted into vapour; this is known to be the case when, at the temperature of about 100° , the mercury stands higher in the tube than in the vessel, for if there were any liquid-water present, the space would be saturated, and the tension of the vapour would be equal to the external pressure. This arrangement accordingly gives the weight of a known volume of the vapour of water.

This volume, which may at once be read upon the graduated tube, is equal to $V(1 + KT)$, V being the number of divisions occupied by the vapour. The temperature T is marked by a thermometer immersed in the water contained in the envelope. The tension of the vapour is evidently equal to the external pressure *minus* the height of the mercury in the tube.

In order to find the relative density, we must divide w by the weight of a volume $V(1 + KT)$ of air at the temperature T and pressure $H - h$, giving

$$V(1 + KT) \times 0.01293 \times \frac{1}{1 + \alpha T} \cdot \frac{H - h}{760}.$$

The relative density of the vapour of water, as thus determined by Gay-Lussac, is about $\frac{1}{8}$, or '625. Several recent investigations have given as mean result '622, which agrees with the theoretical density deduced from the composition of water.¹

284. Volume of Vapour formed by a given Weight of Water.—When the density of the vapour of water is known, the increase of volume which occurs when a given quantity of water passes into the state of vapour may easily be calculated. Suppose, for instance, that we wish to find the volume which a cubic centimetre of water at 4° will occupy in the state of vapour at 100°. At this temperature the tension of the vapour is equal to one atmosphere, and its weight is equal to '622 times the weight of the same volume of air at the same temperature and pressure. If then V be the volume in litres, we have (in grammes)

$$V \times 1.293 \times \frac{1}{1+100\alpha} \times .622 = 1,$$

whence

$$= \frac{1+100\alpha}{1.293 \times .622} = \frac{1.366}{.804246} = 1.698 \text{ lit.} = 1698 \text{ cubic centimetres.}$$

Hence we see that water at 4° gives about 1700 times its volume of vapour at 100° C.

The latent heat of evaporation is doubtless connected with this increase of volume; and it may be remarked that both these elements appear to be greater for water than for any other substance.

¹ Water is composed of 2 volumes of hydrogen, and 1 volume of oxygen, forming 2 volumes of vapour of water. The sum of the density of oxygen and twice the density of hydrogen is 1.244, and the half of this is exactly '622.—*D.*

